Accuracy of the series given in eof's

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1. Introduction

Northern Hemispheric extratropical EOF's of the 500mb level are discussed. Five sets of functions based on different samples are studied in their own sample as well as in an independent sample. Some methods of truncating the EOF-series are introduced. A case study is presented in order to find out how well the functions perform in synoptic term. The convergence properties of the eof-expansion and the stability of eof's is also discussed. Besides that eof's are a very effective method to compress data they seem to be very stable. Functions based on material from 1946 seem to work reasonably well with material from 1965.

2. Samples and functions

The functions discussed here are based on Monterey analyses of the 500mb level. The data are given in the FNWC grid in points north of 20°N, at 1404 points in total. The data are from the years 1946-70. In the studies six different samples are used. The properties of the samples are as follows:

Sample period	Sets of functions	Number of analyses	Mean analysis interval
1946-70 (excl. 1953-55,1960-61)	UNIF	11876	1 d (1946-52) 1/2 d(1956-70)
1946	F46	347	1 d
1970	F70	351	1 d
1969-70	F6970	351	2 d
1967-70	F6770	351	4 d
1965	independent test sample	351	1 d

The determination of the "universal" functions (UNIF's) is discussed by Karhila. The functions of small samples (351 analyses) can be computed in a usual way through the covariance matrix and its eigenvalues and -vectors. Functions (spatial elements) are orthogonal and normalized with the areal element:

$$(1) \quad \sum_i f_\nu(i)\, f_\mu(i)\, dA(i) = \delta_{\nu\mu},$$

where i refers to a grid point and $\sum_i dA(i) = 1$.
The time dependent coefficients can be calculated from:

$$(2) \quad C_\nu(t) = \sum_i \left[z(i,t) - zm(i)\right] \cdot f_\nu(i) dA(i),$$

where t refers to time and zm represents a long term mean field. Now the height of the 500mb level can be written:

$$(3) \quad z(i,t) = zm(i) + \sum_{\nu=1}^{N} C_\nu(t)\, f_\nu(i) + \text{residual}(i,t).$$

By using all the possible components in the expansion (3) we get an exact copy of the original analysis (residual $\equiv$ 0). However a far smaller number of components ($N \ll MAX$) is enough. Through descarding the high-indexed "noise components" it may be possible to get rid of the non-atmospheric noise included in 500 mb analyses. The point of truncation, i.e. finding the "suitable" number of components (N) will be discussed later on.

3. Methods of truncation

The only criterion on the point of truncation studied in meteorological applications seems to be the one used by Craddock (1969, 1970). This criterion is based on the LEV-graph (Fig. 1), where the natural logarithm of the eigenvalues is represented as the function of their ordinal numbers. According to the Craddock criterion the EOF-series should be truncated where the graph can be approximated with a straight line. By using this method for the sets of functions mentioned earlier we should take 140-160 components in small samples, but only about 90 components of UNIF's. The variance reduction due to accepted components would be about 99.5% of the total variance in small samples and some below 99% in case of UNIF's.

Some other methods of truncation are also applied. One is based on the size of the error variance in 500mb analyses. It is assumed that about 1% of the total variance is due to different errors. Thus 99% of the variance represents the true atmosphere. The eof-expansion should now be truncated when the components taken in the expansion account for 99% of the total variance. This suggests that about 100 components are required (Table 1).

The functions of small samples are tested in the independent data as well. Here quite a new method of truncation is developed. The data of the independent sample (1965) is divided into two parts: the "true" atmospheric part and the "non-atmospheric noise". This is made with the aid of UNIF's (UNIF's can be considered so general that this is thought to be possible). The method of truncation is based on how the expansion given by foreign functions (functions of foreign, small samples) describes the noise and fails to explain the true atmospheric information of the test sample. It is agreed to truncate the EOF-series where the series has described noise as much as true atmospheric information still remains unexplained. By using 100 UNIF-components to define the true atmospheric part we have to take about 130-150 components of foreign functions into the EOF-expansion. The required number of components depends a little on the sample the functions are based on: functions of a longer sample period are needed less.

One more method of truncation can be used, synoptic consideration. It will be discussed in the following section.

4. Case study

The representation ability of EOF's can be studied in synoptic terms as well. These considerations were carried out on the analysis of Feb. 10,1965 00 GTM (Fig. 2). This case was randomly chosen from that "blocking February" , when a strong blocking prevailed over the Atlantic west of Europe.

The low-indexed, large-scale components bring out the general features of the weather conditions, the large-scale flow systems. The high-indexed components build the small-scale waves. The more components are used in the expansion the better the resemblance to the original analysis will be.

Figures 3 - 5 show three UNIF-expansions of the case mentioned before. Fig. 3 represents the expansion of 80 components. Here the Atlantic blocking and large-scale troughs are clearly to be seen. However, many small-scale phenomena are still missing.

In the trough east of the Rockies there is not a tiny ridge to be seen as in the original analysis. The trough over Eastern America is not of the right shape. In the jet stream north-east of the Atlantic blocking there is no wave at all over Scandinavia. This wave was important, there was a surface low and a rain area connected with it. Generally small-scale waves seem to be missing, for example over Asia. The 80-component UNIF-expansion is still rather "smooth".

In the 120-component expansion (Fig.4) small waves have appeared. The trough over Eastern-America gets a better shape, there are more waves in Asia and the small Scandinavian wave can also be seen. In the trough near the Rockies the gradients seem to loosen, but no ridge can be seen. It cannot be seen in the 175-component expansion (Fig.5) either. In the original Monterey-analysis as well as in the German (DWD) analysis (Fig.7) that ridge does exist.

The analysis gives only an approximate representation of the atmosphere. Therefore it is questionable to require the eof-expansion to give an exact copy of the original analysis.
For instance, the Monterey analysis (Fig.2) and that of DWD (Fig.7) differ from each other over the Atlantic. The expansion of 80 components is just between them. At least two of these three variants must be wrong. Statistically the correct solution would be the mean of the three versions. Among them, the eof-solution is perhaps the one closest to the mean.

When we add more functions in the EOF-expansion, we get solutions closer to the Monterey analysis and further from the German analysis.

Similarly, if we use more than 175-components in order to represent the trough east of the Rockies, we may get a wrong result. The observational network is dense and the Monterey and DWD analyses are equal. Nevertheless the analyses may be suspect.

Thus when we add more and more functions, we add information, which is more and more questionable.

It is thus a matter of judgement to choose the point of truncation. In the present case it could be between 120 and 175 components.

The trough east of the Rockies will be later studied more closely. It is possible that quite new functions will be determined.

Finally the required number of components depends much on the purpose for which the eof-expansion is used. If a very accurate representation is required, nearly all determined components may be needed. Anyway the 120 expansion doesnot differ much from that of the 175. If we are interested only in the large-scale features (long range forecasting) we might do with 50 components or so, they account for 95% of the total variance.

The blocking case was expanded also with the aid of functions of small samples. They can represent the case very well even though it was independent data. As an example Fig. 6. where the case is represented with functions F46 by 120 components. If more then 140 components of foreign functions are taken into the expansion there will be some "extra" small waves. This suggests the functions have started describing some noisy phenomena. So it can be recommended not to take more than about 140 components of functions based on small samples when used in independent data.

5. Stability of EOF's

The stability considerations of EOF's have been mainly comparisons of functions pattern, of corresponding components from different samples. Functions are regarded as stable, if the patterns are similar. If this is considered the stability criterion, then only a couple of the first components of the functions studied here are stable. The correlation coefficients of the 1st components are well above 0.9, but already for the 3rd component they may be nearly zero.

The stability of the functions can be better understood as the ability of the functions to represent phenomena in an independent sample. This definition can be approached from two directions: statistical and meteorological.

Table 1. shows the cumulative variance reductions of different sets of functions in their own samples. The efficiency of EOF's can be seen clearly: 30 components account for over 90%, 100 components for about 98.5-99.5% and 200 components for over 99.5% of the total variance. Table 2 shows the corresponding variance reductions of different sets of functions in the test sample 1965, which was independent material for functions of small samples (but not for UNIF's). At the first glance we can see how well the functions of small samples can represent independent data: The variance reductions by 200 components are only 1% lower than in their own samples, and by 100 components about 2% lower. It's surprising that functions F46 are so effective, when we take into account the quality of the material they are based on. As a general feature we can see that functions of a longer period are a little more effective. When UNIF's were tested in two small independent samples (1953-55 & 1960-61, which were not included in the UNIF-sample) the variance reductions by 175 components were 99.61 and 99.71 %. That is more than in their own UNIF-sample !

These percentual figures are "mean" values, in some cases the EOF's are more effective in other cases less effective.

The stability can be understood in synoptic terms as well. As we can see in Fig. 6 the functions F46, which are based on data from 1946, can describe 500mb level conditions quite well in independent material from 1965. The Atlantic blocking and the troughs are quite well described. The quality and the density of the observational network in 1946 and 1965 were quite different. In 1946 the analyses could contain only large-scale features, but the EOF's (F46) are able to pick up the essential information from those analyses.

After all the similarity of patterns of components of different samples, even on different ordinal numbers may not be necessary. (The first component seems to be similar in all samples. The Atlantic blocking may need a couple of components to be built, but in different samples this feature is built by quite different patterns. The Atlantic blocking is so common that the functions of small samples were able to represent it. So the stability of the EOF's can be better understood as their ability to represent different phenomena in independent material. It seems to be that functions based on 1-year material can represent independent data fairly well, though components based on a longer period seem to be more effective. The UNIF's (based on 20-years material) seem to be so effective that they can be assumed to represent atmospheric conditions still in the 1980's (and 2000) as well as today, if there will be no dramatic changes in the atmospheric flow systems or in the observational network.

6. Residual and convergence studies

The accuracy of the EOF-series can be examined with the aid of the residuals. This consideration was carried out on the blocking case mentioned before with the aid of UNIF's. The values of residuals vary quite a lot at nearby points, so that the residual field consists of small cells. The mean absolute values of the residuals over the whole area were 15 and 10 m with 120 and 175 components respectively.

The convergence studies were carried out at seven grid points which were purposely chosen from areas with a different density of the observational networks. The convergence criterion is defined so that the EOF-series is considered to be converging, if the difference of the series and the original analysis permanently remains within a limit given beforehand when new components are added to the series. These considerations were carried out in the sample from 1965 and limits were 10, 20 and 30 m. With the error of 20 m we need about 70-100 components for convergence, but there were 10-40 days (from 351) when the series didn't converge using the determined 175 components. The convergence of the EOF-series seem to be better over areas with a sparse observational network. On the other hand a very large residual may be connected with an erroneous analysis.

Finally the different convergence of the EOF-series over areas of different observational networks will lead to the weighting of the functions. Functions should be weighted more over continents where a lot of information is available and less in vast marine areas where the information is inadequate (at least so far).

Table 1. The cumulative variance reductions of each set of functions in their own sample and the total variances of the samples.

COMP.	F46	F70	F6970	F6770	UNIF
1	63.70	58.77	58.26	56.67	57.69
2	67.80	63.45	62.15	60.33	61.49
3	70.89	67.11	65.37	63.36	64.25
4	73.58	70.18	68.21	65.90	66.62
5	75.80	72.42	70.50	68.31	68.83
6	77.60	74.43	72.52	70.59	70.94
7	79.22	76.28	74.45	72.56	72.82
8	80.74	77.92	76.06	74.35	74.64
9	82.15	79.45	77.48	76.01	76.18
10	83.43	80.86	78.89	77.54	77.65
20	91.06	89.29	87.94	87.25	86.76
30	94.66	93.30	92.46	92.05	91.51
40	96.57	95.50	94.93	94.66	94.16
50	97.69	96.81	96.42	96.20	95.76
60	98.41	97.64	97.38	97.21	96.82
70	98.86	98.21	98.02	97.90	97.54
80	99.16	98.61	98.46	98.37	98.05
90	99.38	98.90	98.78	98.71	98.41
100	99.53	99.11	99.01	98.96	98.69
110	99.64	99.27	99.19	99.16	98.89
120	99.72	99.40	99.34	99.31	99.06
130	99.79	99.50	99.45	99.42	99.18
140	99.83	99.58	99.55	99.52	99.28
150	99.87	99.65	99.62	99.60	99.36
160	99.90	99.71	99.68	99.66	99.43
170	99.92	99.75	99.73	99.72	99.48
180	99.94	99.79	99.78	99.76	99.51
190	99.95	99.82	99.81	99.80	99.54
200	99.96	99.85	99.84	99.83	99.56
VAR	23158	25301	24940	25081	25175

Table 2. The cumulative variance reductions of different sets of functions in the test sample 1965, which is independent for all but UNIF's.

COMP.	F46	F70	F6970	F6770	UNIF
1	57.12	56.33	57.22	57.73	57.95
2	58.83	59.56	59.95	60.84	61.48
3	61.10	61.53	61.97	62.57	64.11
4	63.24	63.21	63.54	64.70	66.52
5	64.73	65.05	65.47	67.09	68.63
6	66.16	67.28	67.81	68.75	70.34
7	68.06	69.02	69.73	70.45	72.16
8	69.77	70.56	71.35	72.26	74.12
9	71.21	72.43	72.48	73.66	75.84
10	72.56	73.87	74.39	75.55	77.57
20	82.58	82.96	84.24	85.14	86.85
30	88.57	88.04	89.57	90.01	91.48
40	91.66	91.37	92.55	92.86	94.06
50	93.54	93.42	94.23	94.50	95.63
60	94.81	94.76	95.53	95.66	96.64
70	95.68	95.74	96.33	96.43	97.33
80	96.35	96.51	96.96	97.02	97.86
90	96.87	96.99	97.36	97.44	98.23
100	97.27	97.37	97.70	97.76	98.51
110	97.57	97.66	97.95	98.01	98.72
120	97.89	97.91	98.16	98.23	98.90
130	98.16	98.10	98.32	98.41	99.03
140	98.35	98.27	98.48	98.54	99.14
150	98.51	98.41	98.60	98.66	99.23
160	98.68	98.53	98.71	98.76	99.31
170	98.81	98.64	98.80	98.85	99.36
180	98.91	98.73	98.88	98.93	99.38
190	99.00	98.81	98.95	99.00	99.38
200	99.07	98.88	99.01	99.06	99.38

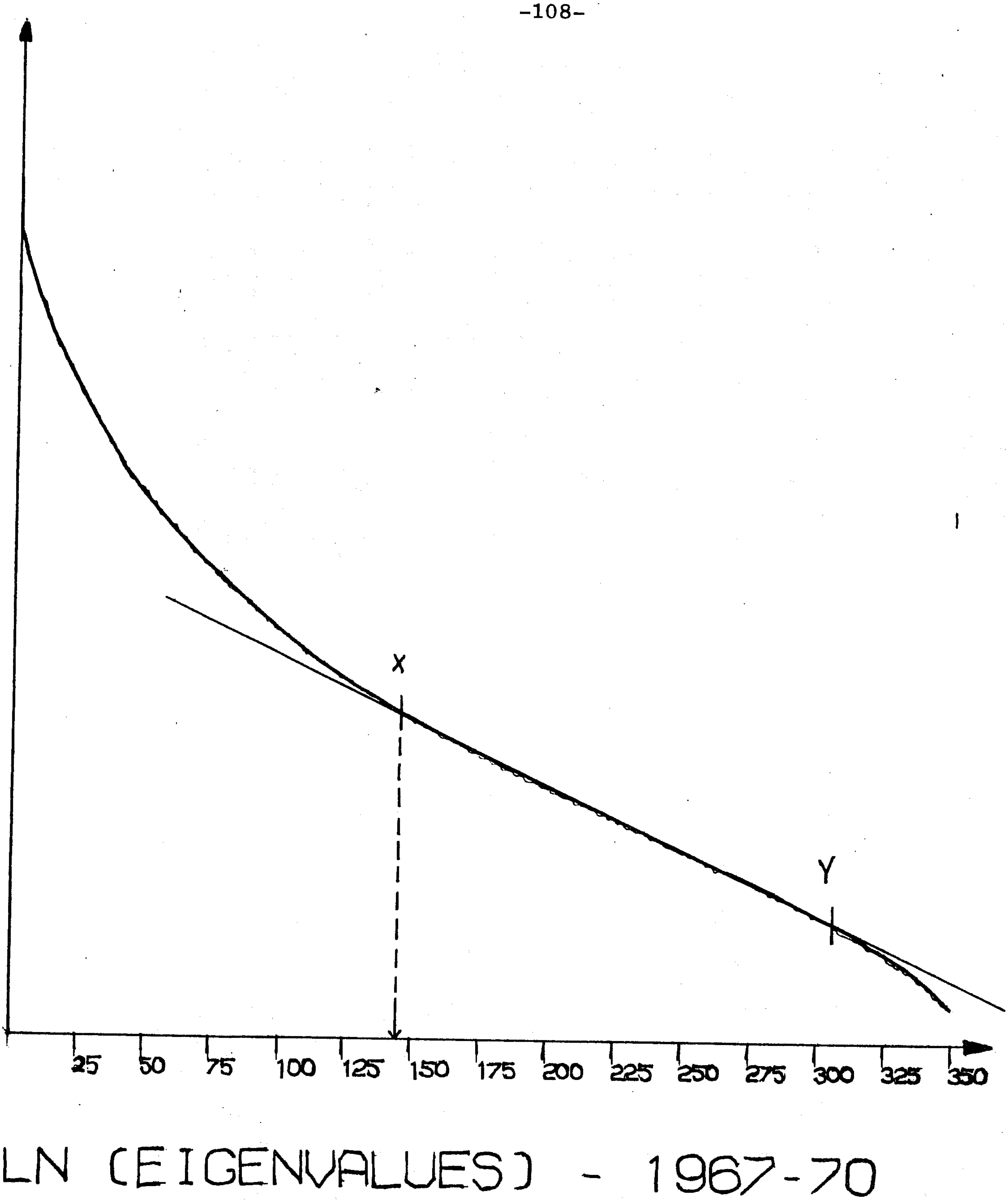

Fig. 1. The LEV-graph of functions F6770. The dashed line shows the point of truncation according to the Craddock criterion.

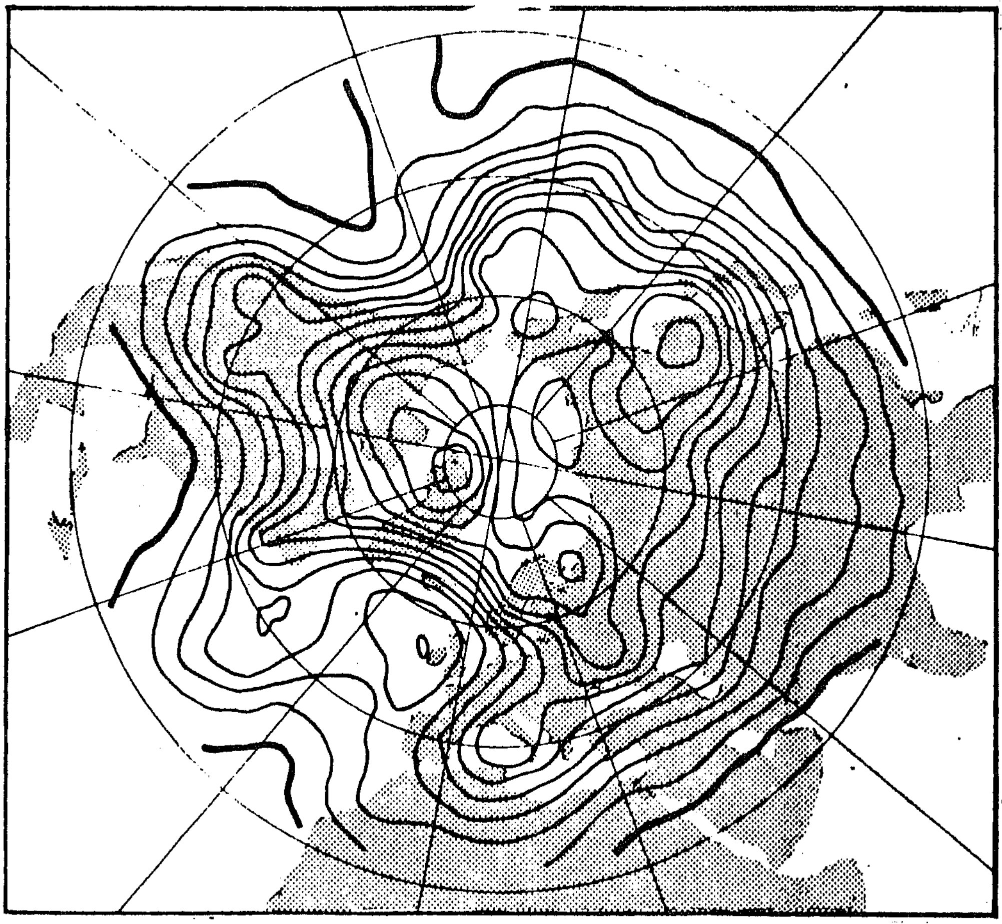

Fig. 2. The original Monterey analysis of Feb. the 10th, 1965 00 GMT. Heavy line is 5800 m, analysed with 80 m interval.

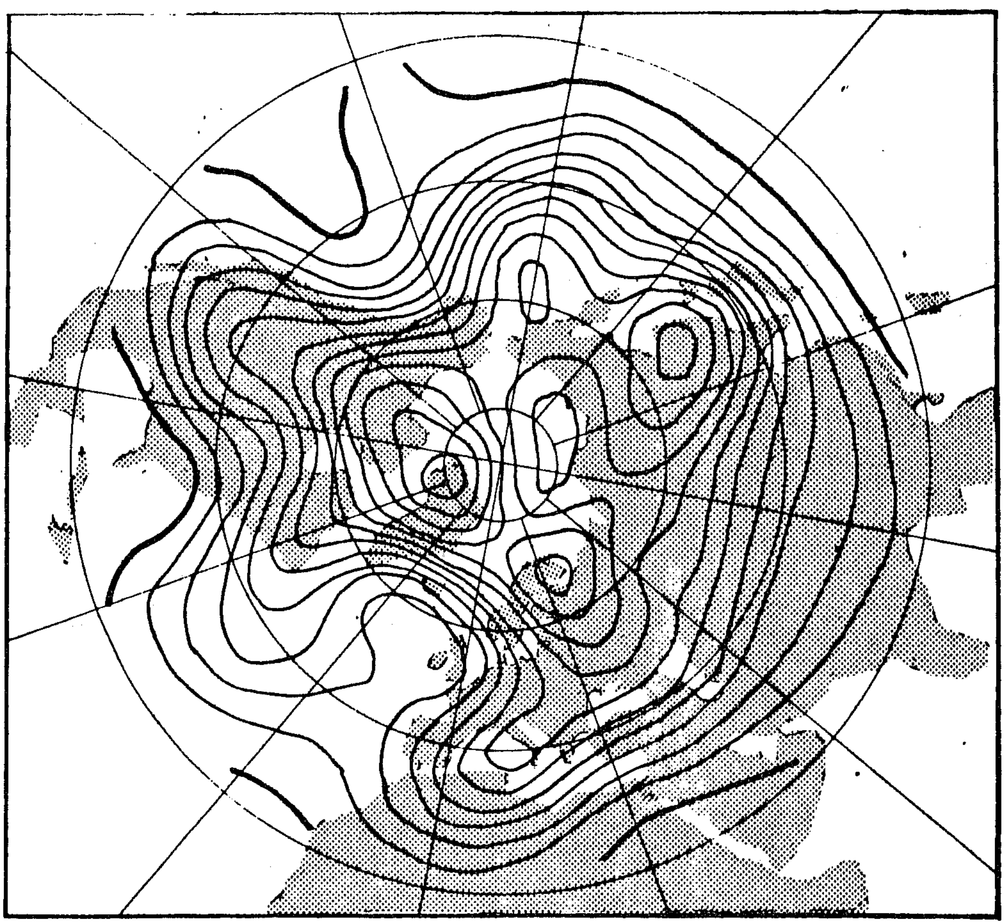

Fig. 3. UNIF-expansion of 80 components.

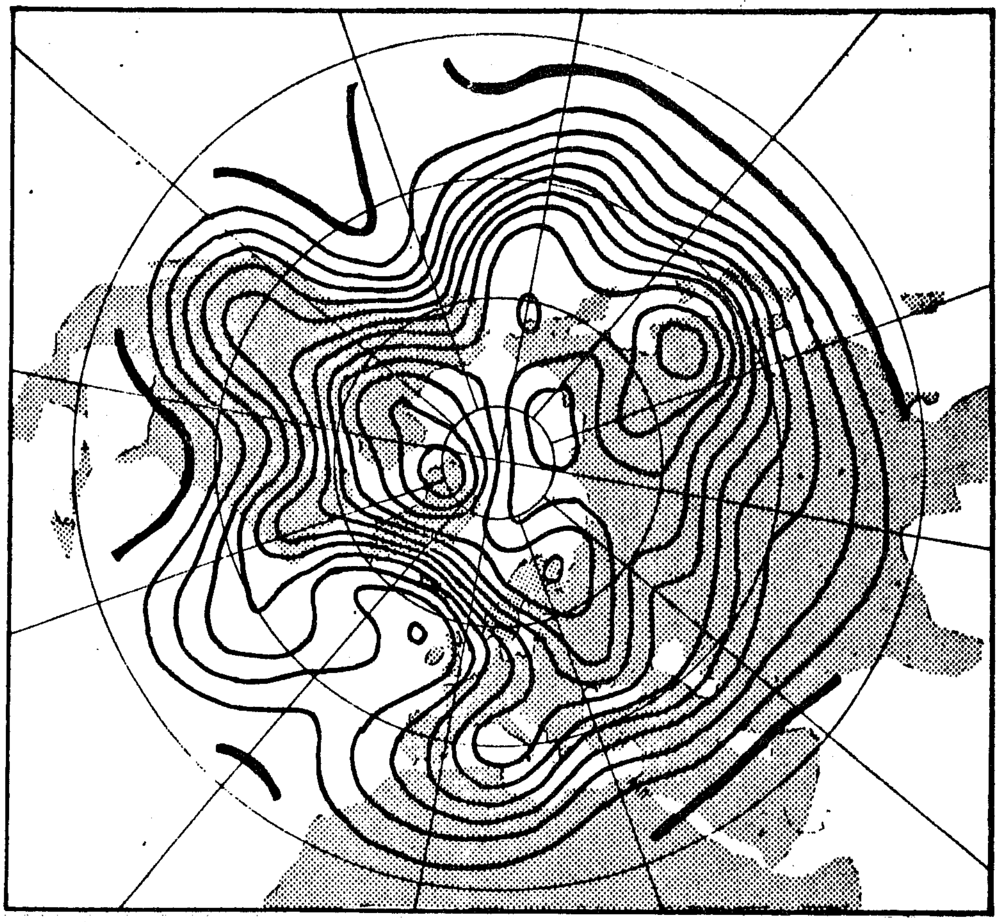

Fig. 4. UNIF-expansion of 120 components.

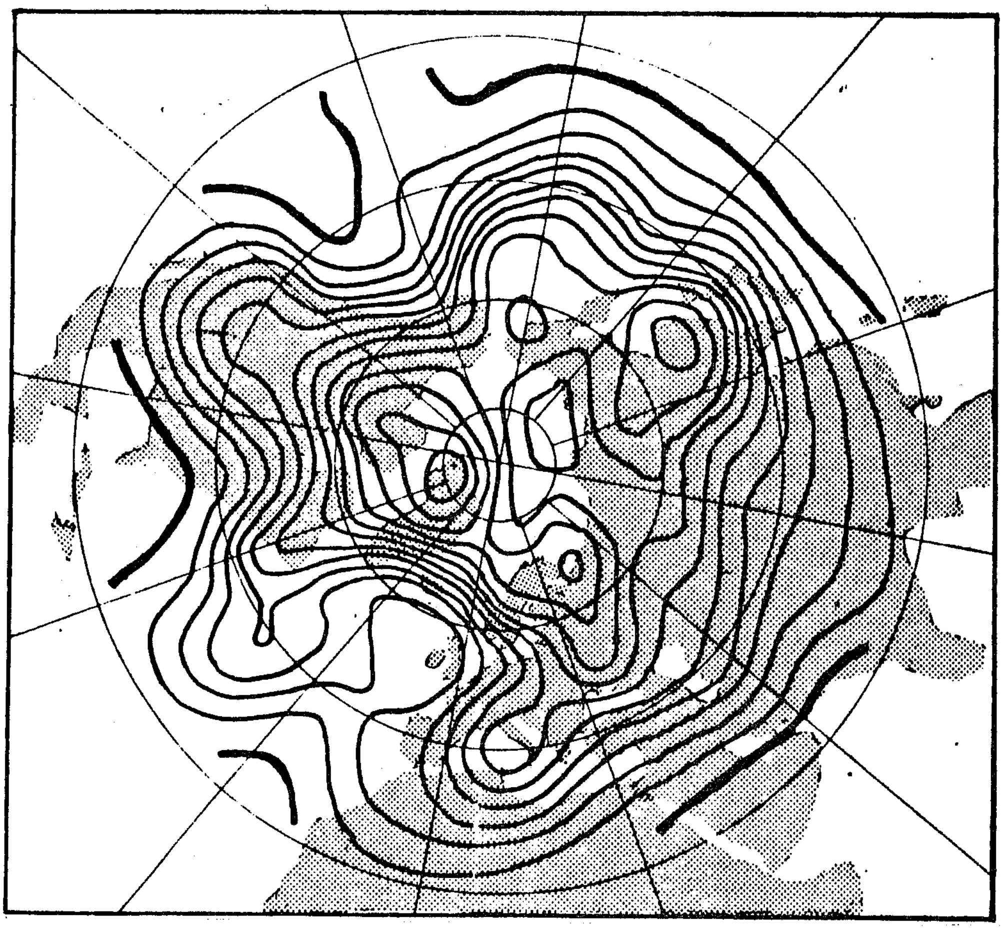

Fig. 5. UNIF-expansion of 175 components.

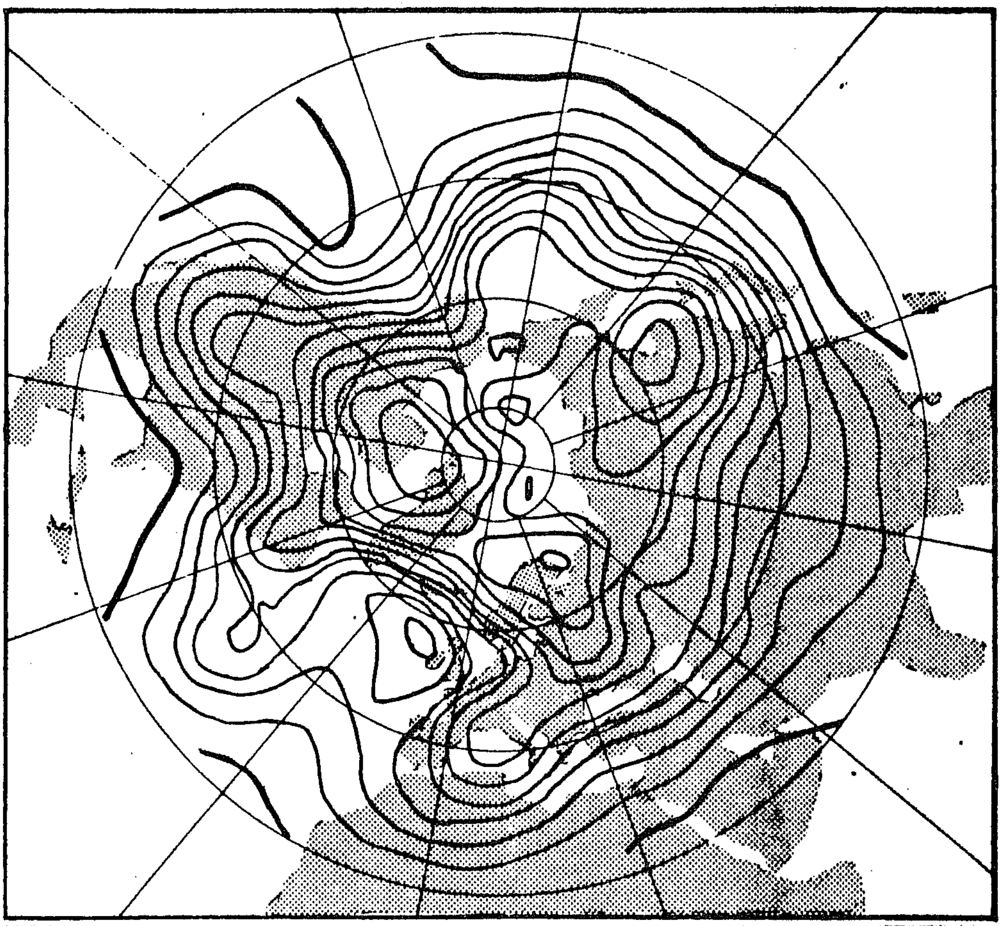

Fig. 6. F46-expansion of 120 components.

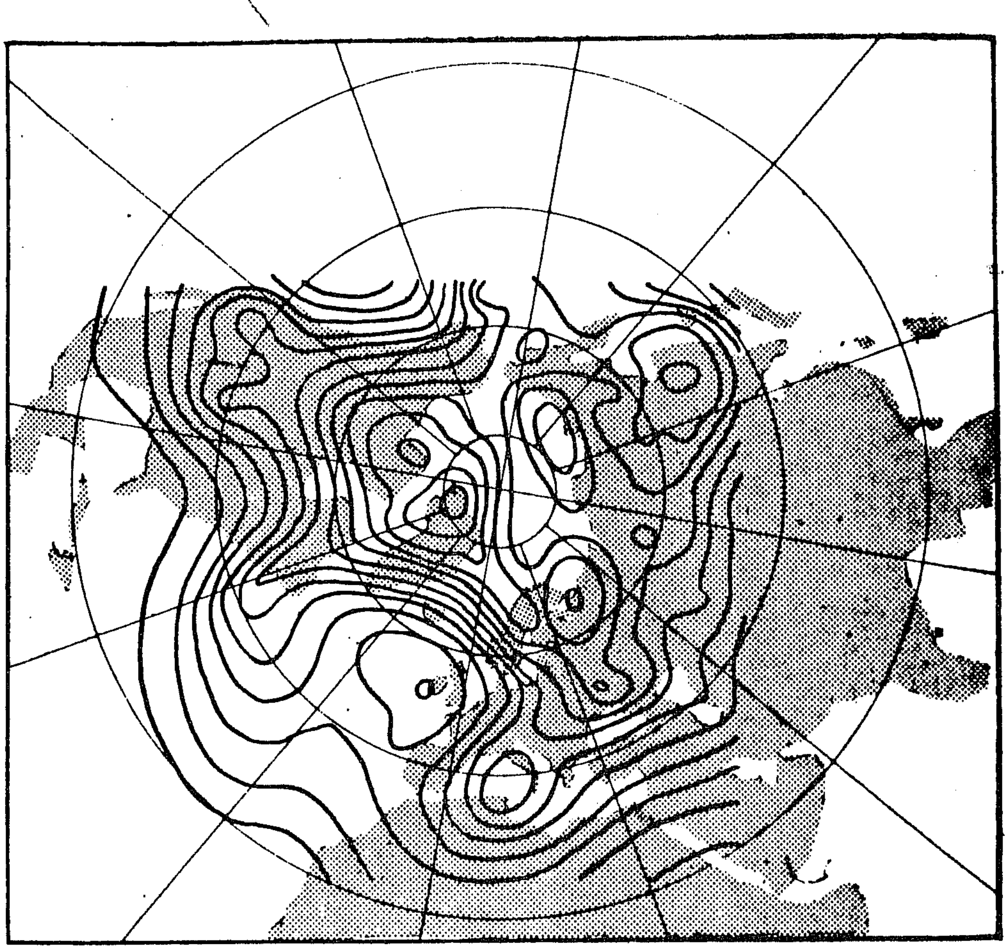

Fig. 7. The German (DWD) analysis of Feb. the 10th 1965 00 GMT.