PARAMETERIZATION OF CONVECTIVE PROCESSES

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1. Parameterization of Cumulus Convection

Though precipitating cumulus clouds have a vertical scale which is of the same order as that of synoptic systems in which they are embedded, they are separated from the synoptic systems by two or more orders of magnitude in their horizontal space scale and their time scale. The task of describing the statistical effects of cumulus ensembles in terms of synoptic scale variables, without any explicit calculation of the evolution of individual clouds in the ensemble, constitutes the task of cumulus parameterization.

In extratropical latitudes, the importance of cumulus parameterization lies mainly in the necessity to predict convective rainfall; the extent of this rainfall is determined by the large scale dynamics but the influence of the convective activity on the further evolution of large scale extratropical systems is usually of secondary importance. In the tropics, on the other hand, the very existence of many synoptic scale systems is directly attributable to the effects of cumulus convection. The cumulus ensembles exert their influence through their control of the budgets of heat, momentum and moisture.

The present state of cumulus parameterization involves much empiricism and many arbitrary procedures. Some advances in the theory have recently been made and when these are coupled with observational studies of the GATE data which are currently underway at many centres around the world, it is to be expected that some progress will result.

In the meantime, the lack of a comprehensive theory of cumulus parameterization holds up progress in Numerical Weather Prediction and limits our understanding of the general circulation of the tropical atmosphere and, indeed, of the atmosphere as a whole.

In these lectures, the theory of parameterization and the methods currently in use will be described. The theory of convectively driven circulations will then be reviewed.

1.1 Equivalent Potential Temperature and moist static energy

Two basic quantities used in convective parameterization are equivalent potential temperature and moist static energy. Here we derive these quantities and note the approximations involved in using either of them to describe the moist parcel ascent curve.

Consider a parcel containing unit mass of dry air with water vapour mixing ratio r_{so} before saturation. We examine the theoretical ascent curve as the parcel rises above the lifting condensation level without entrainment, retaining all condensed water and absorbing all the heat resulting from dissipation of vertical kinetic energy. The first law of thermodynamics gives:

$$dU + dW = (1+r_{so})\ d'Q \tag{1}$$

or

$$dH - Vdp = (1+r_{so})\ d'Q \tag{2}$$

where U = internal energy, H = enthalpy, V = volume, p = pressure and d'Q is the dissipative heating per unit mass.

Expanding the enthalpy into its dry air, water vapour and liquid water components, we have

$$\begin{aligned} H &= h_d(T) + r_s h_v(T) + (r_{so}-r_s)\ h_\ell\ (T,p) \\ &= h_d + r_s\ (h_v - h_\ell) + r_{so} h_\ell \\ &= h_d + Lr_s + r_{so} h_\ell \end{aligned}$$

Thus

$$dH = C_{pd}\ dT + d(Lr_s) + r_{so}[C_\ell\, dT + v_\ell\, dp]$$

where C_{pd} = specific heat of dry air at constant pressure, C_ℓ = specific heat of liquid water and v_ℓ = volume of liquid water. Hence (2) becomes:

$$(C_{pd}+r_{so}C_\ell)\ dT + d(Lr_s) - (V - r_{so}v_\ell)dp = (1+r_{so})d'Q \tag{3}$$

[The corresponding equation for the case where all condensed liquid water falls out is obtained by replacing r_{so} by r_s in (3)]. Regarding the volume occupied by liquid water as negligible compared to the total volume of the parcel, (3) can be approximated as:

$$(C_{pd}+r_{so}C_\ell)dT + d(Lr_s) - \frac{1}{\rho_d}dp = (1+r_{so})\ d'Q \tag{4}$$

Hence making use of the Clausius-Clapeyron equation ($de_s/dT = Le_s/RT^2$) we have

$$(C_{pd}+r_{so}C_\ell)\ \frac{dT}{T} + d(\frac{Lr_s}{T}) - \frac{R_d}{P_d}dp_d = \frac{1+r_{so}}{T}d'Q \tag{5}$$

Defining

$$\tilde{\theta}_e = T^{r_{so}C_\ell/C_{pd}}\ \theta_e$$

where

$$\theta_e = T(p_\theta/p_d)^{R_d/C_{pd}} \mathrm{Exp}(Lr_s/C_{pd}T)$$

we see that (5) can be written

$$d(\tilde{\theta}_e) = \frac{\tilde{\theta}_e}{C_{pd}T}(1+r_{so})\, d'Q \tag{6}$$

If the dissipative heating is neglected, so that the process can be considered reversible, we see that $\tilde{\theta}_e$ is a conserved quantity. If we make the further approximation of neglecting the term $r_{so}C_\ell$ in (5) [or r_sC_ℓ in the corresponding equation for the case of complete liquid fallout, this being the Rossby approximation], θ_e becomes a conserved quantity. θ_e is known as the equivalent potential temperature. [Note: with $r_{so} = 25 \times 10^{-3}$, we see that $r_{so}C_\ell/C_{pd} = 0.1$, so that the above approximation is accurate to about one tenth]

The vertical equation of motion for a parcel is

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + F_z \tag{7}$$

Hence we form the mechanical energy equation

$$\frac{d}{dt}\left[\frac{w^2}{2} + gz\right] = -\frac{1}{\rho}\left[w\frac{\partial p}{\partial z}\right] + wF_z \tag{8}$$

If we assume a steady state with negligible horizontal pressure gradient, and equate the negative work done by friction to the dissipative heating, (8) becomes

$$\frac{d}{dt}\left[\frac{w^2}{2} + gz\right] = -\frac{1}{\rho}\frac{dp}{dt} - \frac{d'Q}{dt} \tag{9}$$

Combining (9) and (4) we find

$$\frac{d}{dt}\left[\tilde{h}_s + \frac{w^2}{2}\right] = 0 \tag{10}$$

where

$$\tilde{h}_s = C_p'T + gz + L\tilde{q}_s$$

with

$$C_p' = (C_{pd} + r_{so}C_\ell)/(1 + r_{so})$$

$$\tilde{q}_s = r_s/(1 + r_{so})$$

Thus $(\tilde{h}_s + w^2/2)$ is a conserved quantity regardless of the dissipation.

The vertical kinetic energy $w^2/2$ is usually very small compared to $\tilde{h}_s$. Thus, replacing $\tilde{h}_s$ by the moist static energy h_s defined by

$$h_s = C_{pd}T + gz + Lq_s$$

we see that (10) can be approximated by

$$\frac{dh_s}{dt} = 0 \tag{11}$$

i.e. the moist static energy is approximately a conservative quantity.

We now derive criteria for positive buoyancy after finite displacements using h_s and θ_e, following Arakawa (1968). Suppose a parcel rises without entrainment from an initial level where its moist static energy is $h_s(o)$, to a height z. Since h_s is an approximately conservative quantity we have

$$h_s \equiv C_{pd}T_c + gz + Lq_s(T_c) = h_s(o)$$

(T_c = temperature of the cloud at height z).

We define a quantity h_e^* where

$$h_e^* = C_{pd}T_e + gz + Lq_s(T_e)$$

(T_e = temperature of the environment at height z,
q_s = saturation specific humidity).

Subtracting these two quantities we get

$$h_s - h_e^* = C_p(T_c-T_e) + L\left[q_s(T_c) - q_s(T_e)\right]$$

$$= C_p(T_c-T_e)(1+\gamma)$$

where

$$\gamma = \frac{L}{C_{pd}}\frac{\partial q_s}{\partial T}$$

i.e.

$$T_c - T_e = \frac{1}{C_p(1+\gamma)}\left[h_s(o) - h_e^*\right] \tag{12}$$

Thus as long as $h_s(o)$ remains greater than h_e^*, a non-entraining cloud will have positive buoyancy.

We can obtain a similar criterion involving θ_e by making a number of approximations. Regarding θ_e as conservative we have

$$\theta_e \equiv T_c\left(\frac{p_{oo}}{p_d}\right)^{\kappa_d} \mathrm{Exp}(Lr_s(T_c)/C_{pd}T_c) = \theta_e(0)$$

Define

$$\theta_e^* = T_e\left(\frac{p_{oo}}{p_e}\right)^{\kappa_d} \mathrm{Exp}\left[Lr_s(T_e)/C_{pd}T_e\right]$$

Then

$$\theta_e - \theta_e^* \simeq \left(\frac{p_{oo}}{p_e}\right)^{\kappa_d} \left[T_c\left(1 + \frac{Lr_s(T_c)}{C_{pd}\,T_c}\right) - T_e\left(1 + \frac{Lr_s(T_e)}{C_{pd}\,T_e}\right)\right]$$

$$= \left(\frac{p_{oo}}{p_e}\right)^{\kappa_d} \left[(T_c-T_e) + \frac{L}{C_{pd}}\left(\gamma_s(T_c) - r_s(T_e)\right)\right]$$

$$= \left(\frac{p_{oo}}{p_e}\right)^{\kappa_d} (T_c-T_e)\,(1+\gamma)$$

so that

$$(T_c-T_e) = \left(\frac{p_e}{p_{oo}}\right)^{\kappa_d} \frac{1}{1+\gamma} \left[\theta_{es}(o) - \theta_e^*\right] \qquad (13)$$

1.2 The Lapse Rate Adjustment Method of Parameterization

The lapse rate adjustment method, introduced by Manabe et. al. (1965), operates as follows:

(1) Dry Convective adjustment

(a) When the lapse rate of an unsaturated layer exceeds the dry adiabatic lapse rate, convection restores the layer to a neutral lapse rate of potential temperature.

(b) The kinetic energy created by convection is dissipated and converted into heat instantaneously.

(2) Moist Convective adjustment

(a) When a layer becomes saturated and the lapse rate exceeds the moist adiabatic lapse rate, convection restores the layer to a neutral lapse rate of equivalent potential temperature.

(b) The relative humidity never exceeds 100% as a result of the adjustment. All condensed water precipitates instantaneously.

(c) The kinetic energy created by the convection is dissipated instantaneously and the sum of total potential plus latent energy is conserved.

Mathematically the moist convective adjustment may be expressed as follows: δT and δr are determined by

$$\frac{\partial}{\partial p}\,\theta_e(T+\delta T,\ r+\delta r,\ p) = 0$$

$$r + \delta r = r_s(T+\delta T,\ p)$$

$$\int_{p_T}^{p_B} (C_p\ \delta T + L\delta r)\ \frac{dp}{g} = 0 \tag{14}$$

where p_B and p_T are, respectively, the pressures at the bottom and top of the unstable layer.

1.3 Variations on the Lapse Rate Adjustment Method

(1) Benwell and Bushby (1970) pointed out that Manabe's method, which involves a sudden change from making θ constant when there is no saturation to making θ_e constant when saturation occurs, leads to unacceptably large values of vertical motion in regions of convective adjustment. To get over this, they assumed that the lapse rate towards which adjustment takes place is the dry adiabatic lapse rate when the relative humidity is less than 50%, changing linearly to the saturated adiabatic lapse rate when the relative humidity is 100%.

(2) Kurihara (1973) changed the criterion for convective adjustment to depend not on the lapse rate of θ_e but on a lapse rate calculated from an entraining cloud model. The lapse rate is calculated from

$$d\left[m(C_pT+gz+Lq_s)_c\right] = dm\ (C_pT+gz+Lw)_e$$

where the subscripts c and e refer to the cloud and the environment, respectively, and dm is an entrained element of mass. The rate of entrainment E defined by

$$E \equiv \frac{1}{m}\frac{dm}{dz}$$

is assumed by Kurihara to be related to the cloud radius D by

$$E = 0.2/D$$

(2) Further he assumes that D is a function of the environmental humidity:

$$D = D_o \left(\frac{r_e}{r_{se}}\right)^{\frac{1}{2}}, \quad D_o = 500 \text{ m.}$$

(r_e = environmental mixing ratio, r_{se} = saturated value of r_e). Thus if r_e = 0 the radius of the cloud is zero and there is an infinite rate of entrainment, so that no convection is possible. When there is 100% relative humidity, the rate of entrainment corresponds to a cloud of radius 500 m. While this method may have some intuitive justification, it cannot be regarded as having a rigorous foundation. The method has been applied by Kurihara and Tuleya (1974) to the modelling of hurricanes. The results will be described in a later lecture.

1.4 Moisture Convergence Methods of Parameterization

The earliest models of tropical cyclones (Haque, 1952; Syono, 1953) took the release of latent heat in unstable moist convection to be proportional to the large scale vertical velocity in the interior of the atmosphere. However, Lilly (1960) showed that such a means of allowing for the heating leads to a growth rate which is strongly scale dependent, being maximum for the smallest scales - essentially it does not differentiate between the mechanism of individual cloud growth and the growth of large scale systems. The CISK theory (Ooyama, 1964; Charney and Eliassen, 1964; Ogura, 1964) was developed to overcome this difficulty. In the original CISK models, the heating in the interior was made proportional to the pumping of moisture out of the planetary boundary layer, the latter being related to the Ekman velocity

$$w_E = \frac{1}{2} D_E \zeta_g \sin 2\alpha$$

(D_E = Ekman depth = $\sqrt{2\nu^*/f}$, where ν^* is the eddy viscosity and f is the Coriolis parameter; ζ_g = geostrophic vorticity of the flow above the boundary layer, α = cross-isobar angle). The question of the vertical distribution of the heating was deferred, by simply taking a two-level model with the thermodynamic equation expressed at one level. By adopting this approach a flatter growth curve was obtained (the dynamics of CISK are examined in detail in a later lecture).

An advance on the original CISK parameterization was made by Kuo (1965) who set the heating proportional to the total convergence of moisture in a column of atmosphere and provided a scheme for determining its vertical distribution. Let I be the rate of convergence of water vapour in a unit column of atmosphere supplemented by the surface evaporation i.e.

$$I = -\int_o^{p_T} \nabla (\bar{V}\bar{q}) \frac{dp}{g} + C_D p_o V_o (q_o - q_a)$$

The heating and moistening of the conditionally unstable environment are then given by the Kuo scheme as

$$\dot{Q} = \xi \; C_p \; (T_c - T)$$

$$\partial_q = \xi \; (q_c - q)$$

where T_c and q_c correspond to the moist adiabat from the cloud base. The quantity ξ is such that all of I is used up in heating and moistening i.e.

$$\xi = \frac{L \; I}{\int_{p_T}^{p_B} \left[C_p (T_c - T) + L(q_c - q) \right] \frac{dp}{g}}$$

ξ was interpreted by Kuo as the fractional area of cloud and his original paper visualized the process physically as a mixing of cloud air with the environment after the cloud had formed.

In a more recent paper, Kuo (1974) argues that the compensating sinking outside the clouds, which recent observational papers have shown to be the predominant method of heating the environment, is automatically taken care of in his parameterization. He also modifies his method by introducing a factor b such that bI of the converged moisture goes into moistening the environment while (1-b)I is used in heating. No theoretical method is provided for determining b; this is left to observations.

A difficulty of Kuo's modified method insofar as it relates to the tropics has been pointed out by Cho (1976). Applying the method to a composite tropical wave, Cho has found that the factor b varies from one portion of the wave to another (the variation being from +.25 to -.52). Cho explains this as being due to the fact that when cumulus activity is decaying, rainfall exceeds moisture supply. If Kuo's method is to be successfully generalized, therefore, a method must be found for determining b.

1.5 Further Variations on Kuo's Method

Sundqvist (1970) used a variation of the Kuo scheme where I consisted of the moisture convergence between the surface and 900mb only. He distinguishes three separate phases in the parameterization

(i) When the environmental air is unsaturated: in this case the heating and moistening proceed essentially as in the Kuo scheme.

(ii) When the environmental air is saturated: in this case all the available moisture goes into heating and none of it goes into increasing the moisture content.

(iii) When the temperature and moisture content of the environment are the same as those of the cloud: in this case the heating is made independent of height and no further moistening takes place.

Barker and Kininmarth (1973) introduced a further variation on the Kuo scheme by setting

$$\dot{Q} = \xi c_p \left[(T_c - T) - \frac{L}{c_p} \sigma_u^* \right].$$

$$\delta_q = \xi \left[(q_c - q) + \sigma_u^* \right]$$

where σ_u^* is that part of the updraft condensate evaporated to the environment; this is prescribed empirically.

Krishnamurti et al. (1973), instead of defining I as the convergence of water vapour, define it as the convergence of moist static energy:

$$I = -\int \left[\nabla \cdot (\bar{h}\bar{V}) + \frac{\partial}{\partial p}(\bar{h}\bar{\omega}) \right] \frac{dp}{g}$$

where $h = C_p T + gz + Lq$

The distribution of $\dot{Q}$ and ∂_q with height is then the same as in the Kuo scheme except that now the proportionality factor is calculated from the conservation of h rather than of water.

1.6 Approaches to Parameterization which stress Detrainment and Compensating Sinking of the Environment

We shall here consider only those studies which are concerned with the effects of deep cumulus convection (for a study of the effects of shallow cumulus, see Betts, 1973).

Recent years have seen a more fundamental approach being taken to the interaction between a cumulus ensemble and the large scale environment than in the original parameterization studies. This has resulted in a greater stress being put on the compensating sinking in the environment outside the clouds. The first papers stressing this aspect of the problem were those of Arakawa (1968) and Pearce and Riehl (1968). Detailed analyses of the interaction between cloud ensemble and environment have been presented by Ooyama (1971), Ogura and Cho (1973), Yanai et al. (1973), Fraedrich (1973, 1974) and Arakawa and Schubert (1974). Of these, only the paper of Arakawa and Schubert has given a general closure method by which the effects of cumulus convection can be parameterized in a predictive model, though Ooyama has suggested a simple closure method applicable to tropical disturbances.

Here we shall briefly outline Ooyama's view of the cloud-environment interaction and then present a detailed analysis of the interaction as given by Ogura and Cho (1973). We shall then consider the quasi-equilibrium assumption of Arakawa and Schubert (1974).

In Ooyama's study, a cumulus ensemble is represented by a collection of independent buoyant elements where individual elements have only an instantaneous existence; no part of the atmosphere is occupied by the buoyant elements at any given time. A total time derivative following a mean parcel (regarded as a mathematical concept only) is defined by

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \bar{\underset{\sim}{V}} \cdot \nabla_H + \bar{\omega} \frac{\partial}{\partial p}$$

The total time derivative following the true motion in the environment (a physical concept) is given by

$$\frac{d_e}{dt} = \frac{\partial}{\partial t} + \bar{\underset{\sim}{V}} \cdot \nabla_H + \omega_e \frac{\partial}{\partial p}$$

The total vertical mass flux $\bar{\omega}$ due to both the convective mass flux and the vertical motion in the environment is given by

$$\bar{\omega} = \omega_c + \omega_e$$

The conservation equation for an intensive conservative quantity $\bar{\alpha}$ in the environment is

$$\frac{\partial \bar{\alpha}}{\partial t} + \nabla \cdot (\bar{\alpha} \bar{\underset{\sim}{V}}) + \frac{\partial}{\partial p}(\bar{\alpha}\omega_e) = D[\alpha] - E[\bar{\alpha}] + \bar{S}$$

where $D[\alpha]$ represents the source of α to the environment due to detrainment from buoyant elements, $E[\bar{\alpha}]$ represents a sink of $\bar{\alpha}$ from the environment due to entrainment into buoyant elements and $\bar{S}$ represents the non-convective sources of $\bar{\alpha}$ to the environment. [Note that due to the assumed instantaneous nature of the buoyant elements, no averaging process is involved in the above equation].

Using the mass continuity equation

$$\nabla \cdot \bar{\underset{\sim}{V}} + \frac{\partial \omega_e}{\partial p} = -\frac{\partial \omega_c}{\partial p}$$

the conservation equation for the environment can be written

$$\frac{d_e \bar{\alpha}}{dt} = \bar{\alpha} \frac{\partial \omega_c}{\partial p} + D[\alpha] - E[\bar{\alpha}] + \bar{S}$$

Ooyama shows that the mass conservation law for the ensemble of bubbles, multiplied by $\bar{\alpha}$ can be written

$$\bar{\alpha} \frac{\partial \omega_c}{\partial p} = E[\bar{\alpha}] - D[\bar{\alpha}]$$

Combining these two equations we have

$$\frac{d_e \bar{\alpha}}{dt} = D[\alpha - \bar{\alpha}] + \bar{S}$$

or

$$\frac{d\bar{\alpha}}{dt} = \omega_c \frac{\partial \bar{\alpha}}{\partial p} + D[\alpha - \bar{\alpha}] + \bar{S}$$

The above equation clearly shows how $d\bar{\alpha}/dt$ is determined by compensating sinking, detrainment and non-convective sources. To determine ω_c, α and the detrainment rate, a cloud model and a closure assumption are needed. Ooyama discusses a cloud model consisting of bubbles which have an entrainment rate inversely proportional to their radius. He does not give a general closure assumption.

We next consider the derivation of the budget equations for the environment following Ogura and Cho (1973). Their approach is similar to that of Arakawa and Schubert (1974). The assumed cloud model consists of a steady-state one dimensional plume with height independent radius entraining air from the surroundings. Unlike the approach of Ooyama, the derivation of the budget equations involves averaging over the areas of the environment and the clouds. The equations governing the conservation of the static energy S ($\equiv C_pT+gz$) and the specific humidity q are

$$\frac{\partial s}{\partial t} + \nabla \cdot (s\underset{\sim}{V}) + \frac{\partial}{\partial p}(s\omega) = Q_R + L(c - e) \tag{1}$$

$$\frac{\partial q}{\partial t} + \nabla \cdot (q \underset{\sim}{V}) + \frac{\partial}{\partial p}(q\omega) = -C + e \tag{2}$$

[Note: Eqn. (1) depends on the accuracy of the approximation $dp/dt \equiv \omega = -\rho g w$; see Betts, 1974 for a discussion]. Taking the horizontal average of (1) and (2) gives

$$\frac{\partial \bar{s}}{\partial t} + \nabla \cdot (\bar{s}\bar{\underset{\sim}{V}}) + \frac{\partial}{\partial p}(\bar{s}\bar{\omega}) = Q_R + L(\bar{C} - \bar{e}) - \frac{\partial}{\partial p}(\overline{s'\omega'}) \tag{3}$$

$$\frac{\partial \bar{q}}{\partial t} + \nabla (\bar{q}\bar{\underset{\sim}{V}}) + \frac{\partial}{\partial p}(\bar{\omega}\bar{q}) = -\bar{C} + \bar{e} - \frac{\partial}{\partial p}(\overline{q'\omega'}) \tag{4}$$

A spectrum of clouds is defined by the variable entrainment rate λ:

$$\lambda = \frac{1}{m}\frac{dm}{dz} = -\frac{p}{H}\frac{1}{m}\frac{\partial m}{\partial p} \tag{5}$$

where m is defined by $(-a_c\omega_c)$, a_c bring the cross-sectional area of the cloud in question. $H = RT/g$ is the scale height. Integrating (5) from the cloud base upwards, we find that

$$m(\lambda, p) = m_B(\lambda)\eta(\lambda, p) \tag{6}$$

where

$$\eta(\lambda, p) = Exp\left(-\int_{p_B}^{p_T} \frac{\lambda H}{p} dp\right)$$

and $m_B(\lambda)$ is the mass flux at cloud base for cloud-type λ.

Let $\sigma(\lambda)$ be the fractional area density function for the cloud spectrum, that is, $\sigma(\lambda_o)d\lambda$ gives the total fractional area occupied by the clouds with $\lambda_o - d\lambda/2 < \lambda < \lambda_o + d\lambda/2$. Then

$$\bar{\alpha} = \int \sigma(\lambda)\alpha_c(\lambda)d\lambda + \left[1 - \int \sigma(\lambda)d\lambda\right]\tilde{\alpha} \tag{7}$$

where α_c refers to the cloud and $\tilde{\alpha}$ to the environment.

Now

$$\overline{s'\omega'} = \overline{s\omega} - \bar{s}\bar{\omega}$$

$$= \int \sigma(\lambda)(s\omega)_c \, d\lambda + \left[1 - \int \sigma(\lambda)\, d\lambda\right] \widetilde{s\omega}$$

$$- \left\{ \int \sigma(\lambda) s_c \, d\lambda + \left[1 - \int \sigma(\lambda)\, d\lambda\right] \tilde{s} \right\} \left\{ \int \sigma(\lambda) \omega_c \, d\lambda + \left[1 - \int \sigma(\lambda)\, d\lambda\right] \tilde{\omega} \right\}$$

$$= \int \sigma(\lambda)(s_c - \tilde{s})(\omega_c - \tilde{\omega})\, d\lambda - \left[\int \sigma(\lambda)(s_c - \tilde{s})\, d\lambda\right]\left[\int \sigma(\lambda)(\omega_c - \tilde{\omega})\, d\lambda\right]$$

(after some cancellation of terms)

If $\sigma \ll 1$ and $|\tilde{\omega}| \ll |\omega_c|$, this reduces to

$$\overline{s'\omega'} \approx \int \sigma(\lambda)(s_c - \tilde{s})\, \omega_c \, d\lambda$$

$$= - \int_0^{\lambda_D(p)} m(\lambda)(s_c - \tilde{s})\, d\lambda \qquad (8)$$

Similarly

$$\overline{q'\omega'} = - \int_0^{\lambda_D(p)} m(\lambda)(q_c - \tilde{q})\, d\lambda \qquad (9)$$

Hence

$$\frac{\partial}{\partial p}\left(\overline{s'\omega'}\right) = -\frac{\partial}{\partial p}\int_0^{\lambda_D(p)} m(\lambda) s_c \, d\lambda + \frac{\partial}{\partial p}\left[\tilde{s}\int_0^{\lambda_D(p)} m(\lambda)\, d\lambda\right]$$

$$= - \int_0^{\lambda_D(p)} \frac{\partial}{\partial p}\left[m(\lambda) s_c\right] d\lambda - \frac{d\lambda_D}{dp} m(\lambda)\left[s_c - \tilde{s}\right]_{p_D}$$

$$+ \frac{\partial \tilde{s}}{\partial p}\int_0^{\lambda_D(p)} m(\lambda)\, d\lambda + \tilde{s}\int_0^{\lambda_D(p)} \frac{\partial m}{\partial p}\, d\lambda \qquad (10)$$

At the detrainment level we have

$$s_c(\lambda, p_D) = \tilde{s}(p_D) \qquad (11)$$

so that the second term above vanishes. Also the budget of static energy for cloud-type λ gives

$$\frac{\partial}{\partial p}\left[m(\lambda)\, s_c\right] = -\left[\frac{\lambda H}{p} m(\lambda)\tilde{s} + L\, C(\lambda, p)\right] \tag{12}$$

where C is the condensation rate. Making use of (11), (12) and (5), eqn.(10) becomes

$$\frac{\partial}{\partial p}\left(\overline{s'\omega'}\right) = \int_0^{\lambda_D}\left[\frac{\lambda H}{p} m\tilde{s} + LC\right] d\lambda + \frac{\partial \tilde{s}}{\partial p}\int_0^{\lambda_D} m(\lambda)\, d\lambda + \tilde{s}\int_0^{\lambda_D}\left(-\frac{\lambda H}{p} m\right) d\lambda$$

$$= \frac{\partial \tilde{s}}{\partial p}\int_0^{\lambda_D} m(\lambda)\, d\lambda + L\bar{C} \tag{13}$$

where $\bar{C} = \int_0^{\lambda_D(p)} C(\lambda, p)\, d\lambda$.

Similarly

$$\frac{\partial}{\partial p}\left(\overline{q'\omega'}\right) = \frac{\partial \tilde{q}}{\partial p}\int_0^{\lambda_D} m(\lambda)\, d\lambda - \bar{C} - \delta\left(\tilde{q}^* - \tilde{q}\right)_{P_D(\lambda)}$$

where δ is the detrainment rate.

Hence we have

$$\frac{\partial \bar{s}}{\partial t} + \nabla\cdot\left(\bar{s}\underset{\sim}{\bar{V}}\right) + \frac{\partial}{\partial p}\left(\bar{s}\bar{\omega}\right) = -M_c\frac{\partial \bar{s}}{\partial p} - L\bar{e} + Q_R \tag{14}$$

$$\frac{\partial \bar{q}}{\partial t} + \nabla\cdot\left(\bar{q}\bar{V}\right) + \frac{\partial}{\partial p}\left(\bar{q}\bar{\omega}\right) = -M_c\frac{\partial \bar{q}}{\partial p} + \bar{e} + \delta\left(\tilde{q}^* - \tilde{q}\right)$$

where $M_c = \int_0^{\lambda_D(p)} m(\lambda)\, d\lambda$ and it has been assumed that $\partial\tilde{s}/\partial p = \partial\bar{s}/\partial p$, $\partial\tilde{q}/\partial p = \partial\bar{q}/\partial p$. In order to determine the total evaporation $\bar{e}$, some consideration of cloud microphysics is necessary - specifically, a determination of the rate at which cloud-drops are converted to raindrops. In this way, the microphysics of clouds can have an influence on the large scale parameterization.

To determine the cloud mass flux M_c, a closure assumption is needed. The method proposed by Arakawa and Schubert (1974) depends on the assumption that a cloud buoyancy integral A(λ) remains constant as the large scale system evolves. $A(\lambda)$ is defined as

$$A(\lambda) = \int_{z_B}^{z_D(\lambda)} \frac{g}{c_p \bar{T}} \eta(z,\lambda)(s_c - \bar{s})\, dz$$

To see how this quantity arises, we consider the vertical equation of motion for a cloud parcel:

$$\begin{aligned} \frac{dw_c}{dt} &= -\frac{1}{\rho_c}\frac{\partial p}{\partial z} - g + F_z \\ &= \frac{\tilde{\rho} g}{\rho_c} - g + F_z \\ &= \frac{g}{\rho_c}(\tilde{\rho} - \rho_c) + F_z \\ &= \frac{g}{\rho_c}\left(\frac{p}{R}\right)\left(\frac{1}{\tilde{T}} - \frac{1}{T_c}\right) + F_z \\ &= \frac{g}{\tilde{T}}(T_c - \tilde{T}) + F_z \\ &= \frac{g}{c_p \tilde{T}}(s_c - \tilde{s}) + F_z \end{aligned}$$

Multiplying by $p_c w_c$ and integrating we find

$$\frac{d}{dt}\int_{z_B}^{z_D(\lambda)}\left(\frac{1}{2}\rho_c w_c^2\right)dz = \int \frac{g}{c_p \bar{T}}(s_c - \tilde{s})\rho_c w_c\, dz + \int F_z \rho_c w_c\, dz$$

i.e.

$$\begin{aligned} \frac{dK(\lambda)}{dt} &= \int \frac{g}{c_p \bar{T}}(s_c - \tilde{s}) M(\lambda)\, dz - \mathcal{D}(\lambda) \\ &\simeq A(\lambda) M_B(\lambda) - D(\lambda) \end{aligned} \tag{15}$$

where $D(\lambda)$ is the dissipation rate of the kinetic energy of clouds with entrainment parameter λ. We see that $A(\lambda)$ is a measure of the efficiency of kinetic energy generation.

It is an integral measure of the buoyancy force with the weighting function $\eta(z,\lambda)$. $A(\lambda)>0$ can be considered as a generalized criterion for moist convective instability. Because of entrainment, the criterion depends on cloud type. $A(\lambda)=0$ for all λ gives a neutral environment. Arakawa and Schubert show that

$$\frac{dA(\lambda)}{dt} = \int_0^{\lambda_{max}} \hat{K}(\lambda,\lambda') M_B(\lambda') d\lambda' + F(\lambda) \tag{16}$$

The integral measures the rate of stabilization of the cloud work function for cloud-type λ through the modification of the environment by cloud-types λ'. The large-scale forcing $F(\lambda)$ can be divided into two parts:

$$F(\lambda) = F_c(\lambda) + F_M(\lambda)$$

where $F_c(\lambda)$ represents the "cloud layer forcing" by the large scale motion and $F_m(\lambda)$ represents the "mixed layer forcing" by the large scale motion.

The closure assumption of Arakawa and Schubert states that for synoptic scale motions whose time scale is much larger than the cumulus time scale, a quasi-equilibrium state exists such that large-scale destabilization is continually offset by the stabilization resulting from cloud activity. The quasi-equilibrium assumption is stated as

$$\frac{dA(\lambda)}{dt} = 0$$

or

$$\int_0^{\lambda_{max}} \hat{K}(\lambda,\lambda') M_B(\lambda') d\lambda' + F_c(\lambda) = 0 \tag{17}$$

This is an integral equation which allows the cloud mass flux $M_B(\lambda)$ to be determined.

An important part of the Arakawa-Schubert theory is the treatment of the mixed layer, which intimately relates to $F_M(\lambda)$. In their theory, the mixed layer equations are given as

$$\begin{aligned}
\rho_M \frac{\partial s_M}{\partial t} &= -(\rho \underset{\sim}{V})_M \cdot \nabla s_M + \frac{1}{z_B}\left[(F_s)_0 + k\frac{\Delta s}{\Delta s_v}(F_{s_v})_0\right] + (Q_R)_M \\
\rho_M \frac{\partial q_M}{\partial t} &= -(\rho \underset{\sim}{V})_M \cdot \nabla q_M + \frac{1}{z_B}\left[(F_q)_0 + k\frac{\Delta q}{\Delta s_v}(F_{s_v})_0\right] \\
\rho_B \frac{dz_B}{dt} &= -(M_B - \rho_B \bar{w}_B) + \frac{k}{\Delta s_v}(F_{s_v})_0
\end{aligned} \tag{18}$$

where subscript M refers to mixed layer quantities, z_B=height of the mixed layer, $(F_s)_o$=flux of s at the surface, $(F_q)_o$=flux of q at the surface, $\Delta s=\bar{s}(z_{B+})-s_M$, $\Delta q=\bar{q}(z_{B+})-q_M$, $\Delta s_v=s_v(z_{B+})-s_{vM}$ (s_v = virtual temperature) and k is a constant taking a value between 0 and 1.

It is assumed that

$$s_c(z_B, \lambda) = s_M$$
$$q_c(z_B, \lambda) = q_M$$
$$h_c(z_B, \lambda) = h_M$$

so that the mixed layer equations (18) enter directly into the forcing of the cloud buoyancy integral.

As observational justification of the quasi-equilibrium assumption, Arakawa and Schubert have presented evidence that, for disturbances in the Marshall Islands area, $dA(\lambda)/dt$ is much smaller than $F(\lambda)$.

Applications of the Arakawa-Schubert scheme to the theory of CISK disturbances will be discussed in a later lecture.

Cumulus parameterization schemes which use the method of averaging over the cloud and environment but consider only a simple cloud type rather than an ensemble of clouds have been developed by Anthes (1977) and by Hayes (1977).

Anthes takes the contribution to the large scale due to cumulus convection as

$$\left(\frac{\partial T}{\partial t}\right)_c = \frac{L}{c_p}\overline{C^*} - \frac{\partial}{\partial p}(\overline{\omega' T'})$$

where

$$\overline{C^*} = -a\left[\omega_c \frac{\partial q_c}{\partial p} - \left(\frac{\partial q_c}{\partial t}\right)_e\right]$$

[a = cloud area, $(\partial q_c/\partial t)_e$ = dilution of the specific humidity of the cloud due to entrainment]. $(\partial q_c/\partial t)_e$ is calculated by assuming a constant entrainment rate inversely proportional to the cloud radius and "a" is taken as

$$a = \frac{(1-b)\,I}{\int\left[-\omega_c \frac{\partial q_c}{\partial p} + \left(\frac{\partial q_c}{\partial t}\right)_e\right]\frac{dp}{g}}$$

where I is the total moisture convergence in a column and b is specified as

$$b = \begin{cases} \left[\dfrac{1-RH}{1-RH_c}\right]^n, & RH \geq RH_c \\ 1, & RH < RH_c \end{cases}$$

RH is the relative humidity. n and RH_c are empirically prescribed quantities.

The flux $(\overline{w'T'})$ is calculated from the cloud model, with some consideration of the microphysics included.

Comparing the vertical distribution of the heating with that given by Kuo's scheme, Anthes found good agreement for clouds of large radius but poor agreement for clouds of small radius.

Hayes (1977) uses a single average cloud model which entrains and detrains at rates determined by averaging the observationally determined cloud ensemble data of Yanai et al (1973). The data of Yanai et al. were taken from observations in the Marshall Islands region. To make the data more applicable to middle latitudes, Hayes compresses the vertical scale by moving the tropopause from 225 mb. to 300 mb. The basic equations for the cloud model are

$$\frac{dM}{dp} = M(\delta - \gamma)$$

$$\frac{d}{dp}(Mh_c) = M(\delta h_c - \gamma h_e)$$

$$\frac{d}{dp}(Mr_c) = M(\delta r_c - \gamma r_e) + W_c$$

where M = cloud mass flux, δ = detrainment rate, γ = entrainment rate, h_c = moist static energy of cloud, h_e = moist static energy of environment, r_c = sum of humidity mixing ratio and cloud water mixing ratio, W_c = rain fallout.

The cloud-base level is defined as the lowest model level involved in convective processes in the grid square. It is taken to be the cloud base level of shallow or of deep convective activity in the previous timestep, whichever is lower, or if no convection is present, it is taken to be 900 mb. The cloud parcel is taken initially from the model layer below the cloud-base level, the temperature being increased by an initiating perturbation of 2^{o}K.

The cloud model equations are then integrated with the rate of formation of raindrops empirically prescribed. Parcel updraft velocity, rather then buoyancy, is used to determine the maximum cloud top level.

The criteria for convection to occur are that conditional instability prevail and that I > o where

$$I = -\bar{\omega}_{900} \quad \bar{r}_{900} + (dr/dt)\ \text{surface}\ .\ \Delta p$$

where Δp is the depth of the lowest model layer.

The fractional cloud cover at cloud base, $\alpha(p_o)$, is given by

$$\alpha(p_o) = \frac{L I \Delta t}{(C_p\, dT + L\, dq_s)\Delta p}$$

where dT and dq_s are the temperature and moisture increments required to produce a saturated parcel with an excess temperature of $2^{O}K$ at cloud base.

When applied to the 10-level model, the method is found to give better forecasts of convective precipitation than the modified lapse rate adjustment method of Benwell and Bushby (1970).

1.7 Parameterization of the Effects of Downdrafts

None of the parameterization schemes discussed so far has considered the effects of downdrafts within the clouds. From observations we know that, in fact, much of the return flow in deep cumulus convection occurs in the form of downdrafts which greatly modify the environment at low levels, and in particular the mixed layer (Betts, 1976; Seguin and Garstang, 1976 ; Zipser, 1969). In addition, downdrafts are important in initiating further convection.

The role of convective downdrafts in cumulus and synoptic-scale interactions has been investigated by Johnson (1976). He assumed that populations of cumulus clouds consist of individual cloud elements of various sizes, each possessing an updraft and downdraft which are modelled as steady-state entraining plumes. The large scale mass flux in the vertical is given by

$$\bar{M} = M + \tilde{M}_c$$

where $$\tilde{M}_c = M_u + M_d$$

where M_u and M_d are, respectively, the mass fluxes in updrafts and downdrafts. As in Ogura and Cho's study, it is assumed that the fractional mass entrainment rate is constant for each cloud, i.e.

$$\frac{1}{m_u(\lambda,z)} \frac{\partial}{\partial z} m_u(\lambda,z) = \lambda \qquad (1)$$

It is assumed that each updraft has an accompanying downdraft having the same entrainment rate, i.e.

$$\frac{1}{m_d(\lambda,z)} \frac{\partial}{\partial z} m_d(\lambda,z) = -\lambda \qquad (2)$$

Integrating (1) and (2) we have

$$m_u(\lambda,z) = m_B(\lambda)\, Exp\left(\lambda\left[z-z_B\right]\right) \qquad (3)$$

$$m_d(\lambda,z) = m_0(\lambda)\, Exp\left(\lambda\left[z_0(\lambda)-z\right]\right) \qquad (4)$$

where Z_B is the updraft originating level and $Z_o(\lambda)$ is the downdraft originating level. The latter level exists somewhere between mid-cloud and cloud top.

An average of any quantity α is now defined as

$$\bar{\alpha}(p) = \int_0^{\lambda_D(p)} \alpha_u(\lambda, p)\,\sigma_u(\lambda)\,d\lambda + \int_0^{\lambda_D(p)} \alpha_d(\lambda, p)\,\sigma_d(\lambda)\,d\lambda$$

$$+ \left[1 - \sigma_u(p) - \sigma_d(p)\right]\tilde{\alpha}$$

An analysis similar to Ogura and Cho's then leads to the equations

$$\frac{\partial \bar{s}}{\partial t} + \nabla\cdot(\bar{s}\bar{\underset{\sim}{V}}) + \frac{\partial}{\partial p}(\bar{s}\bar{\omega}) = -(M_u + M_d)\frac{\partial \tilde{s}}{\partial p} - L\bar{e}_u$$

$$+ \delta\left[s_u(\lambda_D, p) - \tilde{s}\right] + Q_R$$

$$\frac{\partial \bar{q}}{\partial t} + \nabla\cdot(\bar{q}\bar{\underset{\sim}{V}}) + \frac{\partial}{\partial p}(\bar{q}\bar{\omega}) = -\left[M_u + M_d\right]\frac{\partial \tilde{q}}{\partial p} + \bar{e}_u + \delta\left[q_u - \tilde{q}\right]_{p_D}$$

Johnson specifies that ratio of $m_o(\lambda)$ to $m_B(\lambda)$, i.e.

$$\varepsilon(\lambda) = m_o(\lambda)/m_B(\lambda)$$

so that the equations are now quite similar to those described earlier.

The terms $M_d\ \partial\tilde{s}/\partial p$ and $M_d\ \partial\tilde{q}/\partial p$ represent environmental lifting compensating cumulus downdraft mass flux. Their neglect will lead to predictions of excessive warming and drying in the lower troposphere.

The theory was applied diagnostically to disturbances over the western Pacific and over Northern Florida and it was concluded that convective scale downdrafts are indeed important contributors to the total cumulus transport of mass, heat and water vapour.

A closed parameterization scheme incorporating the effects of downdrafts in a numerical model was developed by Ceselski (1974) and applied to a real data situation. The initial mass flux in the clouds was equated to the large scale 900 mb. ascent, which was smoothed over 10 time steps (40 min.) to reduce the initiation of convection by non-meteorological gravity waves. Three possible cloud depths were allowed, each cloud being represented by a steady-state one-dimensional model. Updraft entrainment was assumed to be proportional to the vertical extend of the cloud model. For deep clouds, a percentage η (assumed fixed at 25%) of compensating downward mass flux was assumed. The scheme was applied to the study of a coasting easterly wave, with reasonable results.

1.8 Momentum Transfer by Cumulus Clouds

So far we have been considering the effects of cumulus clouds on the heat and moisture budgets of the environment. An important influence which has not been considered is the vertical transfer of horizontal momentum by the clouds.

Schemes for parameterizing the cumulus momentum transfer have been proposedby Schneider and Lindzen (1976) and by Anthes (1977). Schneider and Lindzen's scheme is derived as follows. Averaging the horizontal momentum equation over the cumulus ensemble gives

$$\frac{\partial\langle u\rangle}{\partial t} = -\langle V_H\rangle\cdot\langle\nabla_H u\rangle - \langle w\rangle\frac{\partial\langle u\rangle}{\partial z} + f\langle v\rangle$$
$$-\frac{1}{\rho}\left\langle\frac{\partial p}{\partial x}\right\rangle - \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\langle u'w'\rangle\right) - \langle\nabla_H\cdot(V_H\cdot u')\rangle$$

The horizontal eddy momentum flux divergence due to the clouds is assumed to be zero (i.e. $\langle\nabla_H\cdot(V_H' u')\rangle = 0$).

To obtain $\rho\langle u'w'\rangle$, we first examine $\rho\langle uw\rangle$.

$$\rho\langle uw\rangle = \rho\sum_i\left(\frac{\sigma_i}{A}u_{ci}w_{ci}\right) + (1-\sigma_c)\,\rho\,(uw)_e$$

Assuming that $\sigma_c \ll 1$ and $|u_{ci}| \leqslant |u_e|$, so that $u_e \sim \langle u\rangle$ and also that u_e is representative of the zonal velocity where vertical motion outside of the clouds is occurring gives

$$\rho\langle uw\rangle \simeq \rho\langle u\rangle\langle w\rangle + \rho\sum_i\left(\frac{\sigma_i}{A}u_{ci}w_{ci}\right)$$
$$-\rho\langle u\rangle\sum_i\left(\frac{\sigma_i}{A}w_{ci}\right)$$

Defining the cloud mass flux as $M_c = \rho\sum\frac{\sigma_i}{A}w_{ci}$ and u_c by $M_c u_c = \sum(\sigma_i/A)\,u_{ci}\,w_{ci}$ we arrive at

$$\rho\langle uw\rangle = \rho\langle u\rangle\langle w\rangle - M_c\,(\langle u\rangle - u_c)$$

Therefore

$$-\frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\langle u'w'\rangle\right) = \frac{1}{\rho}\frac{\partial}{\partial z}\left[M_c\left(\langle u\rangle - u_c\right)\right]$$

so that we have the cumulus friction in the form

$$\underset{\sim}{F}_c = \frac{1}{\rho}\frac{\partial}{\partial z}\left[M_c\left(\langle V_H\rangle - \underset{\sim}{V}_c\right)\right]$$

The quantity $\underset{\sim}{V}_c$ is not a conserved quantity because it is affected by cloud-scale horizontal pressure gradients. However, if the cloud vertical velocity is sufficiently large, Schneider and Lindzen argue that $\underset{\sim}{V}_c$ will be approximately conserved, and $\underset{\sim}{V}_c$ can be replaced by the environmental horizontal velocity at cloud base.

We note that no account has been taken in the above derivation of the effects of downdrafts, which can be expected to have an important influence on the momentum transfer. We also note that the eddy momentum flux by cumulus in the above scheme is

$$\rho\langle \underset{\sim}{V}'w'\rangle = -M_c\left(\langle V_H\rangle - \underset{\sim}{V}_c\right)$$

If $M_c > o$ and $\langle u\rangle > u_c$, where u_c is assumed to equal the wind at cloud base, it can be seen that the momentum flux is down the vertical gradient of horizontal wind. While this is probably the usual situation in reality, it should be noticed that cases of sustained up-gradient transfer of momentum have been found in examining squall lines over tropical continental areas (Moncrieff and Miller, 1976). Thus the above means of parameterizing momentum transfer may not always hold even qualitatively.

The parameterization of momentum transfer proposed by Anthes (1977) is similar to the above.

1.9 Assessment of Parameterization Schemes in Real-data Studies

There have been a number of studies in which various parameterization schemes have been compared by applying them in numerical prediction models with real initial data.

Elsberry and Harrison (1972) compared the parameterization schemes of Kuo (1965), Pearce and Riehl (1968) and Rosenthal (1968). (The Pearce and Riehl scheme involves the parameterization of w_e in terms of $\bar{w}$, $w_e = c\bar{w}$ where c is a constant determined from observations. Rosenthal's (1968) scheme is simpler than Kuo's; it involves a parameterization in terms of the boundary layer pumping of moisture only, with all the moisture going into heating the environment. The vertical distribution is similar to Kuo's). It was found that the Kuo and Pearce-Riehl schemes were unable to precipitate enough water; the Rosenthal scheme came closest to doing so.

Degtyarev and Sitnikov (1976) did a single-point comparison of the Kuo (1965) and Rosenthal (1968) schemes, with results similar to those quoted above: Rosenthal's method again gave rainfall rates closer to those observed.

Ceselski (1973) did a numerical experiment in which he tested six different formulations of the convective heating in a 48-hour forecast. Four of the schemes were based on variations of the Kuo (1965) method. The other two were the convective adjustment method and the Arakawa (1968) method, which considers subsidence of the environment and uses a closure method involving a time constant for adjustment. With the exception of two of the schemes which were variations on the moisture convergence method, all of the schemes give similar results. It was concluded that the reason why the convective adjustment method did not differ more from the other schemes was that there was no large area of saturated air in the initial data.

Edmon and Vincent (1976) have compared the Krishnamurti et al. (1973) scheme, which depends on the convergence of moist static energy, with the modified Kuo (1974) scheme. These schemes were applied to a convective situation in middle latitudes. It was found that the Kuo scheme with all the converged moisture going into heating (i.e. C=0) gave the best results. It was concluded that the criteria for convection of Krishnamurti et al. does not apply in middle latitudes.

A number of studies (Washington and Baumhefner, 1974; Hammarstrand, 1977; Tiedtke, 1977; Hollingsworth, 1977) have compared the Manabe et al. (1965) and Kuo schemes. The paper of Tiedtke dealt with the transformation of an unstable air mass at a single point over a period of time while the other studies integrated numerical models. The studies seem to agree that the Kuo scheme produces more realistic results. Washington and Baumhefner's comparison was for the tropics. It was found that the Kuo scheme produced more transient systems than the Manabe scheme. Hammarstrand examined results for middle and high latitudes using the primitive equation model at the University of Stockholm. She experimented with various partitionings of the moisture convergence between that going into heating and that going into moistening the environment, and with various values of the critical relative humidity necessary for convection in the Manabe scheme. The general conclusion of her work was that the convective precipitation pattern obtained with the Kuo scheme was closest to observation. The Manabe scheme suffered from the additional disadvantage of generating large amplitude gravity waves in the model. Tiedtke also concluded that the Kuo scheme gives better agreement with observation, giving heating and moistening up to considerably greater heights than the Manabe scheme. Hollingsworth found that, in a two-dimensional numerical simulation of a front within a developing baroclinic wave, the Kuo scheme gave a gradual build-up of the vertical circulation associated with the front while the process was unrealistically sudden with the Manabe scheme.

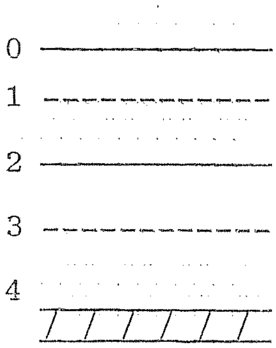
Miyakoda and Sirutis (1977) have compared long term integrations of a general circulation model using the Manabe (1965) and the Arakawa-Schubert (1974) schemes. It was found that the Arakawa-Schubert scheme gives a greater vertical spread of the condensational heating, that resulting from the Manabe scheme being weighted in the layer 900-500 mb.

2. Convectively Driven Circulations

2.1 The Intertropical Convergence Zone

Theoretical studies of convectively driven circulations provide a method of testing the validity of convective parameterization schemes, as well as being of great interest in themselves. Perhaps the simplest case of a convectively driven circulation which can be studied theoretically is that of a line-symmetric Intertropical Convergence Zone (ITCZ). The theory was presented by Charney (1971).

The ITCZ can be idealized as a line of cumulus convection having a width of from 200 to 500 km. extending for long distances in the zonal direction at a distance of from 5^{o} to 15^{o} away from the equator. It is usually situated in the Northern hemisphere and is furthest from the equator in the Northern summer.



To study the dynamics of the ITCZ in the most simplified manner, a two-level model is adopted as shown in diagram. Level 4 is chosen to be at cloud base (regarded as the top of the boundary layer), while level 2 is such that $\bar{p}_2 = \frac{1}{2}\bar{p}_4$. The remaining levels are chosen such that the intervening mean pressure intervals are equal.

Assuming the zonal component of motion is geostrophically balanced and regarding the Coriolis parameter as constant, the governing equations are (with $\partial/\partial x = 0$):

$$\frac{\partial u'}{\partial t} = f v' \tag{1}$$

$$f u' = -\frac{\partial}{\partial y}\left(\frac{p'}{\bar{\rho}}\right) \tag{2}$$

$$\frac{\partial p'}{\partial z} = -\rho' g \tag{3}$$

$$\frac{\partial}{\partial y}\left(\bar{\rho} v'\right) + \frac{\partial}{\partial z}\left(\bar{\rho} w'\right) = 0 \tag{4}$$

$$\frac{1}{\bar{\theta}}\frac{\partial \theta'}{\partial t} + \frac{N^2}{g} w' = \frac{Q'}{c_p \bar{T}} \tag{5}$$

where

$$N^2 = g\frac{\partial}{\partial z}(\ln\bar{\theta})$$ [N = Brunt-Väisälä frequency]

Now

$$\frac{\theta'}{\bar{\theta}} = \frac{c_v}{c_p}\frac{p'}{\bar{p}} - \frac{\rho'}{\bar{\rho}}$$

$$= \frac{c_v}{c_p}\frac{p'}{\bar{p}} + \frac{1}{\bar{\rho} g}\frac{\partial p'}{\partial z}$$

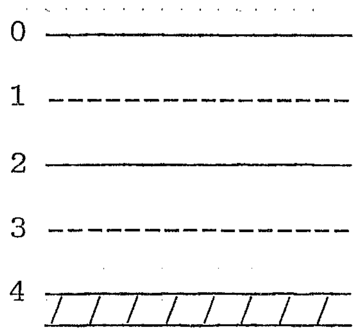
$$= \frac{1}{g}\frac{\partial}{\partial z}\left(p'/\bar{\rho}\right) - \frac{\partial}{\partial z}\left(\ln\bar{\theta}\right)\frac{p'}{\bar{\rho}}$$

$$\approx \frac{1}{g}\frac{\partial}{\partial z}\left(\frac{p'}{\bar{\rho}}\right)$$

since $\ln\bar{\theta}$ is a slowly varying function of z. Thus eqn. (5) can be written

$$\frac{\partial^2}{\partial t \partial z}\left(\frac{p'}{\bar{\rho}}\right) + N^2 w' = \frac{g}{c_p \bar{T}} Q' \qquad (6)$$

We now assume exponentially growing solutions of the form $u' = ue^{\sigma t}$ and adopt a 2-level model of the atmosphere as shown in the diagram.



Level 4 is chosen to be at cloud base while level 2 is such that

$$\bar{\rho}_2 = \tfrac{1}{2}\bar{\rho}_4$$

The remaining levels are chosen such that the intervening mean pressure levels are equal. Expressing (1) and (2) at levels 1 and 3 and denoting $\phi' = p'/\bar{\rho} = \phi e^{\sigma t}$ we find

$$\sigma u_1 = f v_1 \qquad (7)$$

$$\sigma u_3 = f v_3 \qquad (8)$$

$$f u_1 = -\partial\phi_1/\partial y \qquad (9)$$

$$f u_3 = -\partial\phi_3/\partial y \qquad (10)$$

Applying (4) at levels 1 and 3 and assuming $w_0 = 0$, we find [or taking $\Delta p = \bar{\rho}_1 g \Delta Z_{02} = \bar{\rho}_2 g H = \bar{\rho}_3 g \Delta Z_{24}$ where $H = \Delta Z_{13}$]

$$\frac{\partial v_1}{\partial y} - \frac{w_2}{H} = 0 \qquad (11)$$

$$\frac{\partial v_3}{\partial y} + \frac{w_2 - 2w_4}{H} = 0 \qquad (12)$$

We now apply the thermodynamic equation (6) at level 2, taking Q'_2 as given by the simplest method of parameterization, i.e. the boundary layer convergence of latent heat distributed uniformly with height; thus

$$Q_2' = \frac{L\rho_4 q_4 w_4'}{2\Delta p/g} \tag{13}$$

Thus (6) gives

$$\frac{\sigma}{H}(\phi_1 - \phi_3) + N^2(w_2 - \eta w_4) = 0 \tag{14}$$

where

$$\eta = \frac{Lq_4}{C_p\bar{T}H\frac{\partial}{\partial z}(\ln\bar{\theta})}$$

We take w_4 as given by the Ekman formula with the wind at level 4 equal to that at level 3; thus

$$w_4 = \frac{1}{2}D_E \zeta_y = -\frac{1}{2}D_E\frac{\partial u_3}{\partial y} = \frac{D_E}{2f}\phi_{3yy} \tag{15}$$

Adding (11) and (12) and making use of (15) gives

$$\frac{\partial}{\partial y}(v_1 + v_3) - \frac{2}{H}\left(\frac{D_E}{2f}\right)\phi_{3yy} = 0$$

Whence using (7)-(10)

$$\frac{\partial^2}{\partial y^2}\left[\phi_1 + \phi_3\left(1 + \frac{fD_E}{\sigma H}\right)\right] = 0$$

Assuming the perturbation quantities are zero at ∞, we can integrate this to give

$$\phi_1 + \phi_3\left(1 + \frac{fD_E}{\sigma H}\right) = 0 \tag{16}$$

If we define $y = (NH/f)y^*$, $\sigma = (fD_E/2H)\sigma^*$ eqn. (14) reduces, with the aid of (7)-(12) and (15) to

$$\phi_{3y^*y^*} = -\lambda_+^2\phi_3 \tag{17}$$

where

$$\lambda_+^2 = \frac{2 + 2\sigma^*}{\eta - (2 + \sigma^*)}$$

Outside the region of convection, $\eta = 0$ and the governing equation can be written

$$\phi_{3y^*y^*} = \lambda_-^2 \phi_3 \tag{18}$$

where

$$\lambda_-^2 = \frac{2+2\sigma^*}{2+\sigma^*}$$

(17) and (18) are solved subject to the boundary conditions

(1) $\phi_3 \to 0$ as $y^* \to \pm \infty$

(2) ϕ_3 continues at $y^* = \pm L$

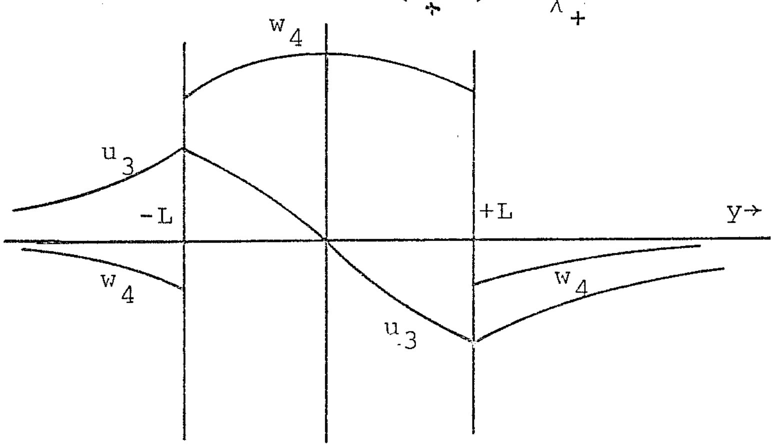
(3) V_3 continues, whence $\partial\phi_3/\partial y^*$ continues at $y^* = \pm L$

The (symmetric) solution is

$$\phi_3 = A \begin{cases} \mathrm{Cos}(\lambda_+ L)\,\mathrm{Exp}\left[\lambda_-(L-y^*)\right], & y^* > L \\ \mathrm{Cos}(\lambda_+ y^*), & -L \leq y^* \leq L \\ \mathrm{Cos}(\lambda_+ L)\,\mathrm{Exp}\left[\lambda_-(L+y^*)\right], & y^* < -L \end{cases}$$

and the corresponding eigenvalue relationship is

$$\mathrm{Tan}\,(\lambda_+ L) = \frac{\lambda_-}{\lambda_+} \tag{19}$$



The shape of the solution is shown in the diagram.

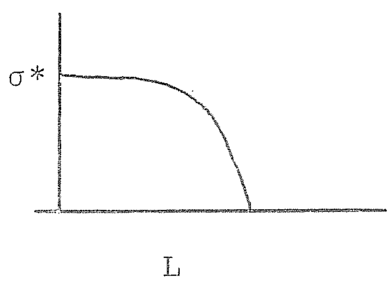
(19) can be rewritten

$$L = \sqrt{\frac{\eta-(2+\sigma^*)}{2+2\sigma^*}}\;\mathrm{Tan}^{-1}\sqrt{\frac{\eta-(2+\sigma^*)}{2+\sigma^*}} \tag{20}$$

Hence we see that a necessary condition for instability is

$$\eta > 2 \qquad (21)$$

Provided this inequality is satisfied, growth exists over a finite range of scales.



The maximum growth rate occurs for L = 0 and is given by

$$\sigma^*_{max} = (\eta - 2)$$

i.e.

$$\sigma_{max} = \frac{f D_E}{2H}(\eta - 2) \qquad (22)$$

Since $D_E = \sqrt{2\nu^*/f}$, where ν^* is the eddy viscosity, we see that $\sigma_{max} \propto f^{\frac{1}{2}}$, i.e. the larger the latitude the larger the growth rate tends to be. But η tends to decrease as the latitude increases. Thus σ_{max} will be greatest at an intermediate latitude. Charney used this argument to explain why the ITCZ would normally be found at a distance from the equator.

The above line-symmetric model of the ITCZ provides the simplest example of the dynamics of Conditional Instability of the Second Kind (CISK).

2.2 Non-linear Models of the ITCZ

A non-linear zonally symmetric model of the ITCZ was developed by Charney (1968). The heating was again parameterized in terms of the moisture pumping out of the boundary layer. The model extended from equator to pole and had two levels in the vertical. It was found that for weak heating in the ITCZ, the zonally symmetric circulation was basically a radiatively driven circulation. For strong ITCZ heating, the temperature levels in the tropics rose well above the radiative equilibrium values and the baroclinicity in middle latitudes was greatly increased. In order to have an ITCZ assume a steady position away from the equator in the model, it was necessary to have a convective heating function which decreased towards the equator. This was achieved by multiplying the heating by a factor $(\sin\phi/\sin\phi_o)^{\frac{1}{2}}$, where ϕ_o is a finite latitude, with the rationalization that the Ekman depth increases as $(\sin\varphi)^{-\frac{1}{2}}$ and that the air entering the clouds tends to come from a higher level, and to contain less moisture, as a result. In practice, the heating function often decreases towards the equator as a result of a sea-surface temperature minimum near the equator. Pike (1971) has shown in a multi-level primitive equation model that the ITCZ tends to reach a steady state at a latitude where the sea surface temperature is maximum.

Often the ITCZ does not appear as a line-symmetric feature but as a series of westward propagating wave disturbances. A study of the ITCZ waves in a numerical model was made by Bates (1970) who took the zonally symmetric model of Charney (1968) as basic state. It was found that waves grew as a result of barotropic instability on the zonal current in the neighbourhood of the ITCZ and maintained the barotropically unstable nature of the low level wind field by their thermodynamic affects on the mean flow even when they had reached finite amplitude. Since this study, our knowledge of the planetary boundary layer in the tropics has increased considerably, with the need to approach the problem afresh. Holton, Wallace and Young (1971) and Yamasaki (1971) showed that the boundary layer solution corresponding to a propagating wave disturbance has a singularity at a critical latitude where the Doppler shifted frequency of the wave equals the Coriolis frequency i.e. if we consider a wave disturbance with x and t- dependence of the form $\mathrm{Exp}[i(wt+kx)]$ superimposed on a zonal flow $\bar{u}$, the critical latitude exists where

$$\omega + \bar{u}k = \pm f$$

Chang 1973(a) has developed a model to test the hypothesis that the ITCZ develops at the critical latitude. Chang's is a multi-level boundary layer model with a single level above the boundary layer in which the latent heat is released.

No heating occurs in the boundary layer itself, which extends up to 5.5 km. The pressure force is assumed to remain independent of height throughout this 5.5 km layer. There is no a priori assumption about the latitudinal distribution of CISK heating in the model - the boundary layer dynamics coupled with the dynamics of the interior level determine where the ITCZ will occur. It was found that for motions which were asymmetric about the equator, the ITCZ developed 1°-3°N of a critical latitude which corresponds to the maximum Doppler shifted frequency of the waves. These results indicate that the critical latitude mechanism may play a role in determining the latitude of the ITCZ.

A multilevel zonally-symmetric model of the ITCZ has been developed by Schneider (1977). It was found that when the heating was parameterized in terms of the moisture convergence at low levels, the steady-state ITCZ was found at the latitude of maximum sea-surface temperature. The width of the ITCZ and the Hadley cell mass flux were found to be sensitive to the distribution of the cumulus heating in the vertical near cloud base. It was found that in addition to the Hadley call, a reverse Ferrel cell in mid-latitudes occurred.

2.3 Wave Disturbances Driven by Convection

Wave disturbances in the tropics, at least over the oceans, depend on the heating due to cumulus convection to maintain or increase their energy. In theoretical studies of wave motions, it is necessary for mathematical simplicity to assume that negative heating occurs over half the wave as a counterpart to the positive physical heating.

The dynamics of wave disturbances have been widely investigated and it has been found that the growth and structure of the waves depends very sensitively on the type of cumulus parameterization scheme adopted, as well as on the mean wind field and the treatment of friction. In the case where the simple CISK parameterization is used, i.e. the heating is set proportional to the boundary layer pumping, two essential elements in determining the dynamics are the formulation of the boundary layer pumping and the form of heating distribution in the vertical. Two types of CISK have been distinguished from each other, "Ekman-CISK" and the so-called "Wave-CISK". In Ekman-CISK the boundary layer pumping is due exclusively to frictional effects while in Wave-CISK, it is due exclusively to allobaric affects associated with the changing pressure field. Mathematically it is difficult to include friction if one wishes to study the latitudinal structure of the wave in the equatorial region. One must either solve numerically or assume a constant Coriolis parameter. Examples of studies of Ekman-CISK are those of Chang (1971), Chang and Piwowar (1974), Chang and Williams (1974) and Kuo (1975). In wave-CISK, where surface friction is neglected, it is possible to obtain analytical solutions for wave disturbances driven by convective heating on an equatorial beta-plane. Examples of wave-CISK studies are those of Hayashi (1970), Lindzen (1974) and Chang (1976). We shall here describe the study of Chang (1976).

The linearized equations of motion on an equatorial beta-plane in (x, y, z) coordinates; where $z = -H\ell n(p/p_o)$, can be written

$$i\omega\hat{u} - \beta y\hat{v} = -ik\hat{\phi} \tag{1}$$

$$i\omega\hat{v} + \beta y\hat{u} = -\partial\hat{\phi}/\partial y \tag{2}$$

$$\frac{\partial\hat{\phi}}{\partial z} = \frac{R\hat{T}}{H} \tag{3}$$

$$i\omega\hat{T} + \hat{\omega}\Gamma = \hat{Q}/c_p \tag{4}$$

$$ik\hat{u} + \frac{\partial\hat{v}}{\partial y} + e^{z/H}\frac{\partial}{\partial z}\left(e^{-z/H}\hat{\omega}\right) = 0 \tag{5}$$

where wave solutions of the form $e^{i(wt+kx)}$ have been assumed. (Γ is the static stability and H is a constant scale height).

Equations (1)-(5) may be combined into a single equation in $\hat{w}$ which may be separated into meridional and vertical structure equations by assuming that

$$\hat{\omega} = \sum_n Y_n(y)\,\omega_n(z)\,e^{z/2H}$$

The meridional structure equation is

$$\frac{d^2Y_n}{dy^2} + \left(\frac{k\beta}{\omega} - k^2 + \frac{\omega^2}{gh_n} - \frac{\beta^2y^2}{gh_n}\right)Y_n = 0 \tag{6}$$

where h_n, the equivalent depth, is the separation constant. Matsuno (1966) and Lindzen (1967) have shown that the solutions to (6) which satisfy the boundary conditions

$$Y_n \to 0 \quad \text{as} \quad |y| \to \infty \tag{7}$$

and lead to the frequency equation

$$\left(\frac{k\beta}{\omega} - k^2 + \frac{\omega^2}{gh_n}\right)\frac{\sqrt{gh_n}}{\beta} = 2n+1,\ n=-1,0,1,\ldots \tag{8}$$

are

$$Y_n(y) = \left[\frac{n}{1-\frac{k}{\omega}\sqrt{gh_n}} H_{n-1}(\xi) - \frac{1}{2\left[1+\frac{k}{\omega}\sqrt{gh_n}\right]} H_{n+1}(\xi)\right] e^{-\frac{\xi^2}{2}}$$

where $\xi = \beta^{\frac{1}{2}}(gh_n)^{-\frac{1}{4}}y$ and the Hermite polynomials $H_n(\xi)$ have values only for $n \geqslant 0$. The meridional velocity solution is $v_n \propto H_n(\xi)e^{-\xi^2/2}$.

The vertical structure equation is

$$\frac{d^2 w_n}{dz^2} + \lambda_n^2 w_n = \frac{Q_n'}{gh_n} \tag{9}$$

where

$$\lambda_n^2 = \frac{S}{gh_n} - \frac{1}{4H^2} \tag{10}$$

$$S = \frac{R}{H}\Gamma$$

$$Q_n' = \frac{R}{C_pH}\hat{Q}_n Y_n^{-1} \mathrm{Exp}\left[-z/2H\right]$$

The parameter λ_n is a measure of the vertical wavenumber and is generally complex for unstable waves- it is determined as an eigenvalue of (9) and w is then found from (8) and (10). The heating function Q' is assumed to have a white-noise distribution so that $\hat{Q}_n/Y_n$ is the same for all n. The subscript n is therefore dropped in the subsequent discussion.

The following boundary conditions are used to solve (9):

$$w = \begin{cases} 0, & z=0 & \text{(11a)} \\ C_1 e^{i\lambda z} + C_2 e^{-i\lambda z}, & z = z_t & \text{(11b)} \end{cases}$$

where

$C_1 = rC_2$ [If the stratosphere and the troposphere have the same static stability, r=0]

Here Z_t is the height of the tropopause and the condition (11b) results from the requirement that latent heating vanishes at Z_t. The parameter r is a reflection coefficient which, in the absence of vertical wind sheer, is given by the specification of the static stability distribution. If the heating is assumed to vanish below the cloud base (Z_c), the solution to (9) may be written by the method of Greens functions in the form

$$w(z) = -\left[\frac{re^{i\lambda z} + e^{-i\lambda z}}{\lambda(1+r)}\right]\int_{Z_c}^{Z_t} \sin\lambda z \, \frac{Q'}{gh}\, dz \,, \quad z \geq Z_t \tag{12a}$$

$$w(z) = -\frac{\sin\lambda z}{\lambda(1+r)}\int_{z}^{Z_t}\left(re^{i\lambda z} + e^{-i\lambda z}\right)\frac{Q'}{gh}\, dz + \frac{re^{i\lambda z} + e^{-i\lambda z}}{\lambda(1+r)}\int_{Z_c}^{z} \sin\lambda z \, \frac{Q'}{gh}\, dz, \quad Z_t > z > Z_c \tag{12b}$$

$$w(z) = -\frac{\sin\lambda z}{\lambda(1+r)}\int_{Z_c}^{Z_t}\left(re^{i\lambda z} + e^{-i\lambda z}\right)\frac{Q'}{gh}\, dz, \quad Z_c \geq z \geq 0 \tag{12c}$$

The heating function is specified as

$$Q'(z) = \tfrac{1}{2} m N S_t w_b e^{az'} \sin\pi z', \quad Z_t \geq z \geq Z_c \tag{13}$$

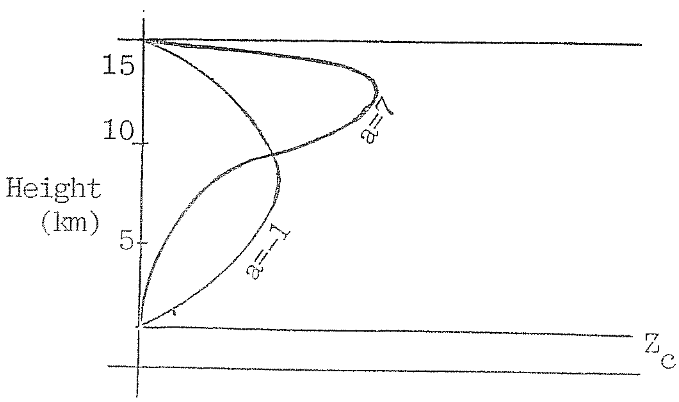
$$Q' = 0 \,, \quad z > Z_t \text{ or } z < Z_c$$

where

$$z' = \frac{z - Z_c}{\Delta Z}, \quad \Delta Z = Z_t - Z_c$$

The coefficient m specifies the strength of the heating and the factor ½ is introduced to take account of the fact that the amplitude of the Fourier component is one half of the heating maximum, as only positive condensation heating is permitted. The heating is proportional to the moisture convergence in the mixed layer which is represented by the vertical velocity w_b at the top of the mixed layer. The tropospheric value of the static stability S_t is included as a proportionality constant. The parameter a is used to vary the maximum heating level to test the sensitivity of the model. The coefficient N is a normalization factor so that the total amount of heat release weighted by density in a column would remain the same with different values of a.

Profiles of the heating multiplied by a density factor are shown in the diagram for various values of a.



By varying a between -1 and 7, a wide variation of vertical heating profiles can be obtained.

If the mixed layer with top at Z_b ($< Z_c$) is assumed to be the layer which provides the convergence for CISK, the stability characteristics are determined by evaluating (12c) at Z_b.

The solution is

$$\omega_b = -\frac{\sin\lambda z_b}{\lambda}\,\frac{\bar{m}\omega_b}{1+r}\,\frac{S_t}{g h_v}\,\frac{\pi\Delta Z}{2}\left[\frac{e^{-i\lambda z_c}(e^{q}+1)+r e^{i\lambda z_c}(e^{-q}+1)}{q^2+\pi^2}\right]$$

where $\bar{m}$ = mN and q = a-iλZ. Using (10) then leads to the stability equation

$$\Psi(\lambda) \equiv \frac{\pi}{2}\bar{m} z_b \Delta Z\left[\frac{\left(\lambda^2+\frac{1}{4H^2}\right)\left[(e^{q}+1)e^{-i\lambda z_c}+r(e^{-q}+1)e^{-i\lambda z_c}\right]}{(1+r)(q^2+\pi^2)}\right] + 1 = 0 \qquad (14)$$

where it has been assumed that $(\sin\lambda z_b)/\lambda \simeq z_b$. Eqn. (14) must be solved for the complex eigenvalue λ by numerical methods.

The conclusion may be summarized as follows:

(1) Tropical waves can be unstable to wave-CISK only if the vertical wavelength is comparable to or greater than the order of the vertical scale of heating.

(2) The growth rates of Rossby waves, mixed Rossby-gravity waves (n=0) and the larger zonal scale Kelvin waves (n=-1), while positive, remain small even for large amplitudes of the heating.

(3) The gravity modes and the shorter zonal scale Kelvin modes, which are usually not observed in the tropical atmosphere, are most unstable. (however these waves have very short periods and the type of parameterization used may not apply in that case).

The concern among numerical modellers which was raised by Lindzen's (1974) results, that very short vertical wavelengths can be excited by wave-CISK, can thus be relieved. It appears that his results were a consequence of assuming an unrealistic square-wave heating profile in the vertical.

In both the simple linear analysis of the ITCZ, which is an example of pure Ekman-CISK, and the analysis of wave-CISK according to Chang (1976), we have seen that the shortest scales of motion were the most unstable. Chang and Williams (1974) showed that in a quasi-geostrophic model, there is a short wave cut-off if the net heating at the top of the Ekman layer is zero; however they excluded the possibility of unstable gravity waves by their restriction of quasi-geostrophy. In an effort to produce a shortwave cut-off, Kuo (1975) introduced an "availability factor" τ/τ_o which multiplied the heating. τ_o was taken as 8 days to represent the time scale which it takes for the large scale moisture field to be replenished by evaporation. With this restriction on the heating, it was found that short gravity waves were suppressed and the maximum growth occurred for synoptic scale waves.

Koss performed an analysis of wave-CISK with a multilevel linear PE model on an f-plane, with slab symmetry assumed. The heating was assumed proportional to the boundary layer pumping, which contained both frictional and allobaric components. Keeping the total heating in the vertical constant, Koss experimented with various ways of partitioning the heat between levels, the boundary layer being assumed isothermal. It was found that the results were extremely sensitive to the vertical partitioning of the heating. For certain partitionings, a short wave cut-off was found. As less heat was released in the lower troposphere, there was a pronounced increase in the growth rate, along with a shift toward longer wavelength of the preferred wavelength for growth.

When surface friction was removed and the system was allowed to respond only to allobaric convergence in the boundary layer, the stability characteristics were markedly changed. In some cases where a cut-off previously existed, the cut-off now disappeared.

Koss' study clearly shows the need for a parameterization scheme which will allow the vertical partitioning of the heating to be determined by the dynamics of the waves themselves, without any external imposition of this sensitive function. It further shows the need for an accurate treatment of the boundary layer.

2.4 Application of the Arakawa-Schubert Parameterization Scheme to CISK

The Arakawa-Schubert parameterization scheme has been applied to CISK models by Israeli and Sarachik (1973) and Stark (1976). However it can be argued that the treatment of the mixed layer in these papers in unrealistic. In both cases it is assumed that the mixed layer has a positive static stability and that the mixed layer temperature increases in response to a compensating mass flux from the clouds. This treatment of the mixed layer differs greatly from that recommended by Arakawa and Schubert (1974). Since the moist static energy in the clouds at cloud base is assumed equal to the mixed layer moist static energy, the treatment of the mixed layer is of critical importance.

To avoid the difficulty of applying the full mixed layer equations in a linear model, Bates, Lasheen and Hanna (1977) assumed that the top of the mixed layer remains saturated at a fixed height and a fixed temperature while a disturbance develops. Observational evidence for the reasonableness of this assumption is found in tropical storms over the oceans, where it is known that mixed layer parcels flowing towards low pressure remain at constant temperature as a result of picking up sensible and latent heat from the surface. Since $q^* = \varepsilon e^*(T)/p$, where q^* is the saturation specific humidity, e^* the saturation vapour pressure and $\varepsilon = 0.622$, we see that

$$\frac{\partial h_M'}{\partial t} = L\left(\frac{\partial q^*}{\partial p}\right)_M \left(\frac{\partial p}{\partial t}\right)_M$$

$$= -\left(\frac{Lq^*}{RT}\right)_M \left(\frac{\partial \phi'}{\partial t}\right)_M \qquad (1)$$

where the subscript M refers to the mixed layer. Adopting $Z = -\ln(p/p_B)$ as vertical coordinate, the quasi-equilibrium assumption for the case of non-entraining clouds $(dA(o)/dt=0)$ can be written

$$\int_0^{Z_D} \left(\frac{\partial s_c'}{\partial t} - \frac{\partial s'}{\partial t} \right) dZ = 0 \qquad (2)$$

where the jump Δs in the static energy across the transition layer has been neglected. Since moist static energy is approximately conserved in non-entraining clouds we have

$$h_c' = h_M'$$

i.e.

$$s_c'(x,Z,t) + L(q_c^*)'(x,Z,t) = h_M'(x,t)$$

Taking total derivatives of this equation and equating coefficients of dt we obtain

$$\frac{\partial s_c'}{\partial t} = \left(\gamma \frac{\partial \phi'}{\partial t} + \frac{\partial h_M'}{\partial t}\right) \Big/ (1+\gamma)$$

where

$$\gamma = \frac{L}{c_p}\left(\frac{\partial q_c^*}{\partial T}\right)$$

Substituting from (1) then gives

$$\frac{\partial s_c'}{\partial t} = \left[\gamma \frac{\partial \phi'}{\partial t} - \left(\frac{Lq^*}{RT} \right)_M \left(\frac{\partial \phi'}{\partial t} \right)_M \right] \Big/ (1+\gamma) \tag{3}$$

We assume solutions of the form

$$\phi' = \phi(z)\, e^{ikx + \sigma t}$$

while slab-symmetry is imposed and the Coriolis parameter is regarded as constant. The flow is also assumed to be geostrophically balanced in the x-direction. The linearized momentum, hydrostatic and continuity equations then become

$$fv = ik\phi \tag{4}$$

$$\sigma v = -fu \tag{5}$$

$$\frac{d\phi}{dz} = RT \tag{6}$$

$$iku + \frac{dw}{dz} - w = 0 \tag{7}$$

In accordance with the analysis of Arakawa and Schubert, the thermodynamic equation can be written

$$\sigma s = (w_c - w) \frac{\partial \bar{s}}{\partial z} \tag{8}$$

But

$$w = \rho g \bar{\omega} / p$$

$$\therefore \quad w_c = g M_c / p = e^{z} M_c / H \rho_B \tag{9}$$

where $H = R\bar{T}_B / g$.

Assuming $\partial\bar{s}/\partial z$=constant, the above equations reduce to a single equation in ϕ:

$$\left(\frac{d}{dz} + \kappa\right)\left(\frac{d}{dz} - 1\right)\phi - \frac{\kappa H^2 \phi}{f} = 0 \tag{10}$$

where $\kappa = R/C_p$ and $H^2 = (k^2/f)\partial\bar{s}/\partial z$.

As upper and lower boundary conditions we take a lid condition and the Ekman pumping condition i.e.

$$W = 0 \quad \text{at} \quad Z = Z_D \tag{11}$$

$$W = -\frac{D_E}{2H}\frac{k^2}{f}\phi \quad \text{at } Z = 0 \tag{12}$$

In terms of ϕ these conditions can be expressed as

$$\left(\frac{d}{dZ} + \kappa\right)\phi = \frac{\kappa}{\sigma}\left[\hat{M} + F\phi\right], \; Z = 0 \tag{13}$$

$$\left(\frac{d}{dZ} + \kappa\right)\phi = \frac{\kappa}{\sigma}\left[\hat{M} e^{Z_D}\right], \; Z = Z_D \tag{14}$$

where

$$\hat{M} = \frac{1}{H}\frac{\partial S}{\partial Z}\frac{M_c}{\rho_B}$$

$$F = \mu^2 \frac{D_E}{2H}$$

The solution to (10) subject to (13) and (14) can be written

$$\phi = A_1 e^{\kappa\alpha_1 Z} + A_2 e^{\kappa\alpha_2 Z} \tag{15}$$

where

$$(\alpha_1, \alpha_2) = \left(\frac{1-\kappa}{2\kappa}\right)\left[1 \pm \left(1 + 4\kappa\left\{\frac{1+\mu^2/f}{(1-\kappa)^2}\right\}\right)^{\frac{1}{2}}\right]$$

$$A_1 = \left[\sigma(1+\alpha_2)\left(e^{\kappa\alpha_2 Z_D} - e^{Z_D}\right) + Fe^{Z_D}\right](\hat{M}/\Delta)$$

$$A_2 = \left[\sigma(1+\alpha_1)\left(e^{Z_D} - e^{\kappa\alpha_1 Z_D}\right) - Fe^{Z_D}\right](\hat{M}/\Delta)$$

with

$$\Delta = \sigma\left[\sigma(1+\alpha_1)(1+\alpha_2)\left(e^{\kappa\alpha_2 Z_D} - e^{\kappa\alpha_1 Z_D}\right) + F\left\{(1+\alpha_1)e^{\kappa\alpha_1 Z_D} - (1+\alpha_2)e^{\kappa\alpha_2 Z_D}\right\}\right]$$

We now return to the quasi-equilibrium assumption (2) which can be written in terms of ϕ as

$$\int_0^{Z_D} \left[\frac{\phi}{1+\gamma} + \frac{1}{K}\frac{d\phi}{dZ} + \frac{1}{1+\gamma}\left(\frac{Lq^*}{RT}\right)_M \phi_M \right] dZ = 0 \qquad (16)$$

Using mean data for the West Indies in the hurricane season it is found that $1/(1+\gamma)$ can be closely approximated as

$$1/(1+\gamma) = 1 + m\,(Z - Z_D) \qquad (17)$$

where $m=.42$ and $Z_D=1.8$. Substituting (17) and the solution (15) in (16), it is found that the growth rate is given explicitly by

$$\sigma = \left[\frac{M_1 - M_2}{M_2 M_3 - M_1 M_4}\right] F e^{Z_D} \qquad (18)$$

where

$$M_1 = \left(e^{K\alpha_1 Z_D} - 1\right)\left[1 + \frac{1}{\alpha_1}\left\{1 - \frac{m}{K\alpha_1}\right\}\right] + \frac{mZ_D}{\alpha_1} + \left(\frac{Lq^*}{c_pT}\right)_M Z_D\left(1 - \frac{mZ_D}{2}\right)$$

$$M_2 = \left(e^{K\alpha_2 Z_D} - 1\right)\left[1 + \frac{1}{\alpha_2}\left\{1 - \frac{m}{K\alpha_2}\right\}\right] + \frac{mZ_D}{\alpha_2} + \left(\frac{Lq^*}{c_pT}\right)_M Z_D\left(1 - \frac{mZ_D}{2}\right)$$

$$M_3 = (1+\alpha_1)\left(e^{K\alpha_1 Z_D} - e^{Z_D}\right)$$

$$M_4 = (1+\alpha_2)\left(e^{K\alpha_2 Z_D} - e^{Z_D}\right)$$

An examination of the solution (18) shows that it is in all cases negative i.e. the Arakawa-Schubert parameterization scheme allows no CISK growth to occur under the assumption of an isothermal mixed layer.

2.5 Combined Baroclinic Instability and CISK

The dynamics of combined baroclinic instability and CISK have been studied by a number of authors (e.g. Yamasaki 1969, 1971; Chang 1971). Here we shall briefly review the results of Yamasaki (1969).

Yamasaki adopted a multilevel linear model with slab-symmetry. The analysis was carried out for both a quasi-geostrophic and a primitive equation version of the model. The purpose was to study the combined effects of condensational heating, vertical shear of the trade easterlies and the β-effect.

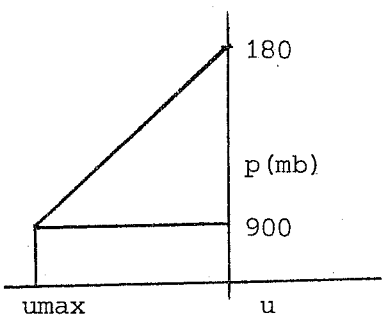
The heating was parameterized as

$$Q = -C_p \left(\frac{p}{p_o}\right)^{\kappa} S h \omega^*$$

where S is the dry static stability of the basic state, h is an empirical factor governing the vertical distribution of the heating and $\omega^*$is the vertical p-velocity at 900 mb. Surface friction was included by means of a linear drag:

$$\frac{\partial \vec{V}}{\partial t} + \cdots = -D\vec{V}$$

where D is zero at all levels except the lowest.



A mean wind field of the form shown in the diagram was prescribed.

Above the level p=180 mb, it was assumed that the motions were adiabatic.

Assuming wave perturbations of the form $e^{i(kx-kct)}$ four distinct types of solution were found:

1) A tropical cyclone mode (TC) of a scale of several hundred kilometres which occurred when the strength of the heating was pronounced in the lower troposphere. The growth rate of this mode was hardly influenced by the existence of the vertical wind shear. However, the amplitude was strongly suppressed in the upper troposphere by the effects of the shear. Surface friction was indispensable for the growth of this mode and it became more unstable as f was increased.

2) An unstable mode corresponding to an easterly wave (referred to as mode ES). Both the heating and vertical wind shear were important for its formation. Its scale was 2000-4000 km. Its growth rate was found to increase with increase of the vertical wind shear and with increase of f. Friction was found to be necessary for its growth when the wind shear was small. Basically this mode is a baroclinically unstable wave modified by the effects of heating.

3) The third mode (named mode E) is again a type of easterly wave, having a wavelength of 2000-4000 km. The growth rate was maximum for moderate shear and the mode was less unstable for higher latitudes.

The relative growth rate of modes ES and E depend on the vertical distribution of heat sources, the intensity of the shear, and the latitude.

4) The fourth mode (named HB) was unstable for a range of wavelengths from 2000 to 12000 km. Its horizontal scale was larger for large heat release in the upper troposphere. The vertical shear was found to decrease the growth rate, and the β-effect was found to be significant. It is an unstable Rossby wave which propagates westward relative to the zonal flow.

When the primitive equations were used, it was found that the small-scale gravity waves, which were excluded in the quasi-geostrophic model, became the most unstable.

2.6 Hurricanes

Hurricanes (known alternatively as typhoons and tropical cyclones) are the most extreme example of convectively driven circulations. They obtain their energy from the release of latent heat which must be continually supplied by evaporation from a warm ocean surface. As soon as they move over land, hurricanes start to decay. Three conditions are known to be necessary for the formation of hurricanes (a) A sea surface temperature greater than 26°C. (b) The initial disturbance must be far enough from the equator for the effects of the earth's rotation to become effective (87% of hurricanes form poleward of 20°). (c) The vertical shear of the mean wind must be small; otherwise the deep circularly symmetric circulation cannot get organized.

At their mature stage hurricanes often move into middle latitudes and end their existence as extratropical storms. It is doubtful if a medium range prediction model can ever hope to follow the life cycle of a hurricane which is in its initial stages when the forecast integration begins. The scale of the hurricane is too small to be resolved in any detail by the grid size of existing or foreseeable models. Nevertheless, the hurricane is of direct interest to the medium range modeller in that it has provided one of the main stimuli for the development of convective parameterization schemes. The original CISK theory, with the boundary layer pumping method of parameterization, was developed specifically with reference to hurricanes.

The observed features and the dynamics of hurricanes have been discussed in detail in the review article of Anthes (1974). One of the most remarkable features of the hurricane is the existence of an eye, with a diameter of from 5 to 50 km, in which subsiding motion occurs and the winds decrease to zero. The eye is surrounded by an eye wall in which intense convection occurs. In the region of the eye, the usual assumptions of cumulus parameterization studies, that the updrafts occupy a small fraction of the area and that the compensating sinking is uniformly distributed, may break down. It seems likely that the eye is a result of the compensating sinking resulting from the surrounding cumulus towers becoming concentrated near the geometric singularity which is the centre of the storm.

Numerical models of hurricanes have been developed by Yamasaki (1968(a), 1968(b), 1968(c)), Ooyama (1969), Rosenthal (1970), Sundqvist (1970), Anthes (1971, 1977), Kurihara and Tuleya (1974).

Ooyama's model, which used boundary layer convergence of moisture as the basis of parameterization, was the first to exhibit a life cycle of growth, maturity and decay. The structure and energy budget associated with his model were remarkably similar to those of hurricanes.

Several of the models demonstrated the importance of a high sea surface temperature in the development of the hurricane; for instance, Sundqvist found that, for a given stratification of the tropical atmosphere, the maximum swirling velocity did not exceed 25m/sec when the water temperature was 26°C, while a full fledged hurricane developed for water temperature of 27.5°C. Sundqvist also found that the rate of development of the hurricane depended strongly on how the heating was distributed with height.

Anthes (1977) used the cumulus parameterization scheme which involves a one-dimensional cloud model, described earlier. His model hurricane was somewhat larger than real storms in many respects. The vertical distribution of heating produced by his cumulus parameterization was typical of that which is necessary for tropical cyclone development. In the hurricane model experiments, the cumulus fluxes of heat and moisture cooled and dried the lower troposphere while they warmed and moistened the upper troposphere. An important consequence of this vertical redistribution of energy was the shifting of the total heating maximum to a higher level, a requirement for the development of a realistic hurricane structure. The low-level drying also acted to increase the storms intensity by increasing the total evaporation rate.

Kurihara and Tuleya (1974) developed a three-dimensional 11-level primitive equation model of a hurricane with Kurihara's (1973) modified lapse-rate adjustment method of parameterization. The development of a hurricane in which the surface pressure fell to 940 mb, with a sea surface temperature of 29°C, was simulated. Spiral bands in the hurricane were also simulated. The spiral bands behaved like internal gravity waves, forming in an area close to the centre and then propagating outwards. Kurihara and Tuleya concluded that the horizontal resolution of 20km, which was the finest in their model, was not adequate for the simulation of the details of the eye structure near the centre.

2.7 Simulation Experiments in which Cumulus Clouds are Explicitly Described

In two recent papers (Yamasaki, 1975; Rosenthal, 1977) simulation experiments have been described in which cumulus clouds are represented explicitly rather than being parameterized. Such an approach can, of course, only be adopted with a fine grid resolution. The aim of these experiments is to avoid the specification of arbitrary parameters which is a feature of all parameterization schemes.

Yamasaki's study used a two-dimensional representation of the clouds and surrounding environment. A mesoscale disturbance containing a number of cumulus clouds is studied. A horizontal mesh size of 200m is taken for the cloud active region, while a variable and larger mesh is used in the outer region. There is no assumption of hydrostatic balance. The clouds are initiated in a resting basic state by buoyancy perturbations separated by a distance of 4km in the inner region of the integration area. Cloud microphysical processes such as the autoconversion from cloud water to rain water, the collection and evaporation of cloud drops and the fall of rainwater are incorporated into the model using parameterizations which are similar to those of Kessler (1969). The time integration shows the development of a mesoscale disturbance in which cumulus clouds are formed one after another and the mesoscale disturbance is maintained for a period of about 15 hours.

Rosenthal (1977) simulates the development of a hurricane in an axially symmetric hydrostatic model in which the release of latent heat occurs totally in convective elements which are explicitly resolved on a 20km horizontal grid - in a sense, the model "parameterizes" itself. Discussing the early attempts at hurricane modelling, before the advent of CISK, Rosenthal claims that they were initialized in such a way that the instabilities of the small-scale motions were maximized, and that there was no initial large-scale system that might organize the small-scale convection. Rosenthal's solutions also showed large-amplitude small-scale features in the early stages of his integration. After a time, however, non-linear effects controlled the further growth of the small-scale elements. After 60 hours integration time the small-scale features had disappeared and the motion had come to resemble a hurricane, even having an eyewall feature. Lateral viscosity was found not to play an essential role in the dynamics. An essential feature of the model, which distinguishes it from the pre-CISK hurricane theories, is that water vapour is conserved.

The successful simulation of a hurricane in this way raises new questions about parameterization which seem likely to be the subject of debate for some time to come.

3. References

These are a general set of references, divided by subject area, and also include references additional to those referred to specifically in the text.

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