Models of Deep Convection and their Large-scale Interaction

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The figures for this paper can be found at the end of this report, on pages 187 - 199.

Convection in the atmosphere extends in scale from the smallest thermals and scattered boundary layer cumulus up to extensive fields of cumulonimbus in the form of squall lines and other mesoscale organizations characterized by the properties of the large scale flow in which they occur. These organized collections of cumulonimbus can be explicitly resolved by the latest prediction models, albeit minimally. Accurate parameterization of such organized systems will differ substantially from the isolated shower, for example, and not only will the appropriate transfer formulations differ but so also will the criteria for occurrence of particular forms of convection. Central to all parameterization schemes is some form of cloud model and considerations of the scales and organization in convection dictate the requirement for a number of different cloud models to accurately parameterize the convective processes in large scale models. It is on such models that this paper will be based.

Almost exclusively, discussion will refer to deep convection, i.e. cumulonimbus and not shallow cumulus. It has become common practice, particularly in the U.S., to refer to cumulus as including precipitating convection; however, this is rather misleading. Deep convection is characterized by the existence of structured updraughts and downdraughts driven by the precipitation. These convective circulations represent cloudscale organization of the flow dominated by advection and persisting for periods very much longer than the time taken for air to flow through the storm. This is to be contrasted with the turbulent, stochastic nature of shallow convection. These cumulonimbus circulations are organized by the large-scale wind shear, and as will be shown later, transport not only heat and moisture but have distinctive momentum transports also. The presence of cool precipitation downdraughts substantially effects these convective transports and contrasts markedly with the concept of between-cloud subsidence, warming and drying the environment. For cumulonimbus, the mass transports local to the cloud are thus much larger than the mean transports. This introduces a number of problems which make many analysis techniques less useful for cumulonimbus study than for nonprecipitating convection (Betts 1978).

The following sections will review convection models and modelling philosophy together with some results from a numerical model of deep convection; the basis for some theoretical models will then be described with the subsequent formulation of flux laws for deep convection which relate the flux structure to properties of the mean flow.

1. Modelling Review

Our understanding of cumulonimbus has advanced rapidly in recent years by the application of mathematical techniques, both analytical and numerical which, with suitable physical simplifications, allow mathematical models to be formulated. Simplified, idealised theoretical models give essential insight into dynamical mechanisms and guide the design and interpretation of numerical models while numerical modelling allows the systematic relaxation of many of the restrictions necessary for analytic tractability.

Convection models have been reviewed recently by Cotton (1975) and

Lilly (1979) who provide a wide-ranging bibliography and critique.

A number of modelling techniques are available:

i) Linear perturbation theory for convection

ii) Linear (forced) gravity wave theory

iii) Non-linear steady theory

iv) Numerical models (as an initial value problem)

v) Descriptive models (based on observations)

vi) Kinematic models (*ad hoc* basis)

Discussion will be restricted to iii) and iv) since these are the only techniques which permit investigation of the large amplitude convective interactions with the large scale.

It is self-evident that different models are developed for different purposes and as such it is difficult to compare and contrast models. However, in general model development has paralleled the vast increases in computing power made available during the last twenty years or so. Thus the earliest 1-D bubble or jet models were followed by fully-time dependent 1-D and axially symmetric 2-D models. These 2-D models (both axi-symmetrical and slab-symmetrical) have become increasingly complex and have studied a wide range of problems particularly of a microphysical nature.

Three dimensional numerical modelling is now possible together with the study of storm initiation and evolution to mature, quasi-steady systems. The simulated fields also provide data for studying the convective modification of the larger scale fields, as will be discussed later.

1.1 Problems and philosophy

The problems that occur in the numerical modelling of deep convection are shared to some extent by larger scale models. To model a typical cumulonimbus and its environment with a resolution of (say) 100 metres, and to model a reasonable representation of the dropsize spectrum of liquid water and associated microphysical and dynamical interactions, poses a problem at least one order of magnitude larger than practicable on present machines. Thus drastic measures have to be taken in order to formulate problems that can be tackled. Either major simplification to the physics or restrictions on the model size (e.g. 2-D) or both.

Space precludes a detailed examination of the numerical methods used in convective modelling. However, their uses and abuses are important and probably deserve more attention than modellers sometimes give, particularly to the conservative properties of the numerical schemes and their treatment of gravity waves. With few exceptions all models have been based on gridpoint representation with numerical schemes of a conventional type. One interesting departure from convention is the increasing use of the full equations as opposed to the 'anelastic' system from which acoustic waves have been eliminated. Very short time steps or some semi-implicit formulation are required, but it would appear that the computational penalties are not as severe as might be expected. The restricted domain of models means that boundary conditions, particularly those on the lateral boundaries, are a substantial problem, and there is scope for much improvement and new ideas. It is probably reasonable to say that at present the only really reliable lateral boundary conditions are either periodic — which are often physically unsatisfactory, or simple boundary conditions far enough away from the convection that only small perturbations in variables occur near the boundary. However, the most recent work using formulations specifically designed to transmit incident waves, look promising (e.g. Orlanski 1976).

The amount of physical detail that goes into a model is obviously very problem dependent. For cloud models the major concerns are processes involving water substance and the inclusion of turbulent or sub-gridscale motions. Consequently much research effort has been directed at the formulation and testing of microphysical parameterizations, some of enormous complexity. Likewise, the parameterization of turbulence is also an active and topical research field, and methods range from simple eddy diffusion-type closures to solutions of higher moment equations leading to much increased model complexity (e.g. Cotton 1975).

The inclusion of a large number of parameterizations into a fully three-dimensional model results in a cumbersome, expensive research tool which does not readily allow the modeller to gain insight into the sensitivity of the results to various parameters by repeating the simulations many times.

Alternatively, therefore, many models are run in a two-dimensional model allowing much more flexibility. This limitation to two-dimensions, however, imposes restrictions on the representativeness of the results and effectively precludes simulations using observed atmospheric wind soundings. Dynamical restrictions imposed by two dimensions are discussed by Moncrieff (1978). The modelling approach used in the model results which follow is a compromise retaining fully three-dimensional flows but restricting the parameterizations, hence retaining flexibility of use.

This compromise is based on certain fundamental assumptions inherent in the theoretical modelling also. Organized storms are viewed as dynamical systems dominated by advective rather than turbulent processes; also, only sufficient microphysics need be parameterized to model a gross configuration of heat sources and sinks within the system. Such assumptions support the use of relatively coarse finite-difference grids (of order 1 km) and relatively simple microphysical parameterisations whose basic aim is to allow liquid water trajectories to deviate from those of the air, enabling water condensed in the updraughts to escape and evaporate in downdraughts (e.g. Kessler 1969). Such resolution inevitably has detrimental effects; however, many of the fundamental properties of deep convection can be studied by this approach.

1.2 Some Model results

With these simplifications a 3-D model (described in Miller and Pearce 1974 and Moncrieff and Miller 1976) has been used to study deep convection in two principal ways. One is to investigate the dynamics of severe storms, their structure and motion and the relationship of these to their severity and longevity. The second has been to demonstrate and confirm the classification of deep convection into regimes or categories depending on the larger scale parameters such as vertical windshear and convective available potential energy, and to examine the way in which the convection modifies the larger-scale flow. A brief description of these regimes follows:

a) Transient

Fig.1a summarises schematically the behaviour of a cumulonimbus in essentially zero shear, typical of summer showers over land. These are short-lived systems, with the accumulation and evaporation of rainwater 'killing' the updraught and generating a downdraught (Byers and Braham 1949). Fig.1b shows this in a simulation.

b) Sheared

Under conditions of stronger vertical shear the developing updraught leans downshear. In the absence of a change of wind direction with height the convection behaves similarly to the transient regime. However, in more typica situations the rainwater can escape from the updruaht, allowing a more steady system. This regime is summarized schematically in Fig.2a, together with an example of the corresponding simulation in Fig.2b.

c) Tropical

A particularly well-organized quasi-steady form of convection is the tropical squall line. A comprehensive numerical, analytic and observational study of such systems has been published by Moncrieff and Miller (1976), Betts, Grover and Moncrieff (1976), Miller and Betts (1977), etc., and results will not be repeated here. These studies showed the unique nature of the steady convective overturning characterizing these systems. The convection propagates faster than the ambient flow, so that the updraught and downdraught air approaches the storm from the front and leaves to the rear. Again the updraught slopes downshear, but since the updraught air approaches from the opposite side of the storm, the precipitation does not fall into the updraught air and a quasi-steady circulation can exist. Figs. 3a and b clearly show this flow structure.

Since both the updraught and downdraught air leave the storm on the same side, it is relatively simple to examine the thermodynamic and dynamic changes effected by the convection.

TABLE 1

Pressure level (mb)	Δu ($m\,s^{-1}$)	Δv ($m\,s^{-1}$)	$\Delta\theta$ (K)	Δq ($g\,kg^{-1}$)	$\Delta\theta_e$ (K)
200	0.0	0.0	-0.0	0.0	0.0
300	0.1	0.1	-0.4	0.2	0.3
400	1.2	0.9	-0.7	1.2	3.2
500	1.3	2.0	1.4	3.2	11.2
600	-0.2	2.8	1.2	2.2	7.7
700	-0.6	-0.3	1.0	-0.8	-1.4
800	-2.5	0.2	0.6	-1.6	-4.2
900	-4.5	0.9	-1.9	-0.9	-4.5
1000	1.7	-2.9			

Table 1 gives an example of these changes. Not only are there substantial upward transports of heat and moisture, but also large momentum changes, particularly along the direction of storm propagation (u). These momentum changes correspond to an upgradient momentum transport enhancing the ambient shear and increasing the low level wind maximum that is typical of profiles in tropical latitudes. This dynamic modification will be discussed further in later sections.

d) 3-D Circulation

In general, particularly in midlatitudes, the wind field has directional as well as speed shear. Cumulonimbus in such shears have a correspondingly three-dimensional circulation. Simulation of an analysis of such a storm was carried out by Miller (1978) and Fig.4 shows the flow structure. The main point to be made from this is the interpretation of the flow as a combination of the previous examples of 'sheared' and 'tropical' circulations (Fig.5).

1.3 Downdraughts

In all the preceding simulations, the downdraught produced by the cumulonimbus played a crucial role. Downdraughts feature also in observational descriptions and the concept of a cooperation between up- and downdraughts is central to the understanding of storm dynamics. However, downdraughts play a more general rôle than this, which seems highly relevant to the parameterization problem. Cold outflows from cumulonimbus can propagate large distances and occupy large areas of the boundary layer; observational evidence of areas ~10^5 km^2 for a large system is typical with temperature deficits of 5 - 10^{o}C, a drying of 1 - 5 g kg^{-1} and velocity changes of up to 10 m s^{-1} (e.g. Miller and Betts 1977, Zipser 1969 and 1977). As such these outflows are important mesoscale phenomena directly resulting from cumulonimbus activity. Their presence drastically modifies the boundary layer fluxes and suppresses convection for hours.

Three distinct forms of downdraught have been identified in the simulation studies and supported by observations. There is the precipitation-driven downdraught discussed above with vertical velocities ~1-10 $m\,s^{-1}$; the dry 'compensating subsidence' featured in most parameterization schemes with vertical velocities of order $cm\ s^{-1}$; and a mesoscale downdraught with vertical velocities ~0.1-1.0 $m\ s^{-1}$ (Miller and Betts 1977).

1.4 Uses of numerical models

Numerical models of convection have a number of applications. Practically all models have been used to study cloudscale dynamics and cloud microphysics, and yet rarely has the numerical data been used for budget calculations of heat, moisture, momentum, etc., which would be invaluable for parametric interpretation. One-dimensional cloud models can be used directly in large-scale models as part of a parameterization scheme, but the difficulties or impossibility of incorporating the effects of shear, downdraughts and momentum transports in 1-D cloud models are a major limitation. The largest cloud models now model more than one cloud, thus permitting mutual interactions between cloud circulations, boundary layer and larger scale forcing fields. Such models will require sophisticated analysis methods, as used on present observational data sets.

The degree of detail in a cloud model that is necessary for use in a parameterization scheme is an open question, but schemes based on better, dynamically sound cloud models seems a logical step in improving the predictive skills of large-scale models. Such models and appropriate flux laws form the basis of the next sections.

2. Theoretical considerations

It is advantageous to represent the cloud models in analytic form, since in this way dynamical aspects can be more clearly identified and complicated simulation models more readily interpreted. Since analytic modelling requires a considerable degree of simplification, it is important that the dominant processes be retained. The most significant of these are as follows:

(a) The large-scale vertical shear Δu particularly its magnitude compared to the convective available potential energy $CAPE = \int_0^H g \frac{\delta\theta}{\theta} dz$, expressed as the ratio $R = CAPE/\frac{1}{2}(\Delta u)^2$

(b) Deep convection is distinctive from shallow (non-precipitation) convection mainly through the effect of potentially cold, cloud-scale downdraughts. Consequently the thermodynamic transports are very different from those in shallow convection.

(c) At least three regimes of deep convection exist; the thermodynamic transports are qualitatively similar in all regimes in the sense that potentially warm boundary layer air is transported aloft and replaced by potentially cold downdraught air; this is thermodynamically more efficient than shallow convection.

(d) Each regime has a fundamentally different flow structure, so the dynamical transports may be quite different in each regime.

2.1 The equation set relevant for deep convection

Since the deviations from a static reference state are small, it is valid to use the Boussinesq approximation to the Navier-Stokes equations

$$\frac{\partial \underset{\sim}{v}}{\partial t} + \underset{\sim}{v}\cdot\nabla\underset{\sim}{v} + \nabla\frac{\delta p}{\rho_0} - g\frac{\delta\theta}{\theta_0}\underset{\sim}{k} = \text{diffusion terms} \qquad \text{—(1)}$$

$$\frac{\partial}{\partial t}\delta\phi + \underset{\sim}{v}\cdot\nabla\phi + wB - Q = \text{diffusions term} \qquad \text{—(2)}$$

$$\operatorname{div}(\rho_0\underset{\sim}{v}) = 0 \qquad \text{—(3)}$$

where δp , $\delta\phi = \delta\theta/\theta$ are deviations of pressure and log-potential temperature from a static, mean state $p_0(z), \rho_0(z), \phi_0(z)$; $B = d\phi_0/dz$ is the static stability; $Q \simeq w\Gamma_s$ is the heat source in wet-adiabatic flow and Γ_s is the wet-adiabatic lapse rate; the flow is therefore driven by latent heat release. For the types of persistent organised deep convection considered here, the sub-cloud-scale diffusion may be neglected relative to the cloud-scale transports, and a steady state $(\partial/\partial t \equiv 0)$ may be assumed in a co-ordinate system moving with the cloud pattern. The flow is therefore stationary in <u>this</u> reference frame. One of the parameters to be determined is therefore this travel speed, since only in this reference frame can a stationary relative flow pattern be defined. The Boussinesq equations in the (primitive) form of Eqns. (1,2,3) can be re-expressed in conservative form, to define quantities C_1, C_2, C_3 which are constant along streamlines (trajectories for a steady state). These are total energy, entropy and vorticity (in two dimensions only),

$$C_1(\psi) = \tfrac{1}{2}\underset{\sim}{v}^2 + \frac{\delta p}{\rho_0} - \int_{z_0}^{z} g\,\delta\phi_p\,dz \qquad \text{—(4)}$$

$$C_2(\psi) = \delta\phi_p - \int_{z_0}^{z}(\Gamma_s - B)\,dz \qquad \text{—(5)}$$

$$C_3(\psi) = \frac{\eta}{\rho_0} - g\int_{z_0}^{z}\left.\frac{\partial\phi_p}{\partial\psi}\right|_z dz \qquad \text{—(6)}$$

where z_0 = the inflow reference level on a streamline ψ = constant

$\delta\phi_p$ = the deviation of ϕ following a parcel along ψ = constant

η = $\partial u/\partial z - \partial w/\partial x$ the y-component of vorticity.

Together with mass conservation, Eq.(3), Eqs. (4,5,6) are extremely useful in the following analysis.

2.2 The mid-latitude (steering-level) model

The formulation and solution of this model is given in detail in Moncrieff and Green (1972) (hereafter MG) and Moncrieff (1978). It is useful, however, to summarize the specified and calculated values as in Fig.6. The most important points to note are that the model is nonlinear so that the travel speed and the modification of both the thermodynamic and dynamic variables by the convection are calculated in terms of the large-scale flow variables such as the vertical shear and the parcel and environment lapse rates of potential temperature; this is advantagous when the flux laws are calculated later. In the case of constant shear and constant density, these modifications are particularly simple and the outflow and the travel speed are effectively determined by the value of the parameter $R = CAPE/\frac{1}{2}(\Delta u)^2$ where $CAPE$ is the convective available potential energy, or the positive area on a thermodynamic diagram and $CAPE = \int_0^H g\,\delta\phi_p\,dz$ where H is the level of zero buoyancy of the air originating at the surface $z_0 = 0$, Δu is the cloud layer vertical shear. It was shown by MG that

$$c - u_s = \Delta u \left(\frac{1 + \sqrt{1 + 4R}}{3 + \sqrt{1 + 4R}} \right) \qquad \text{--(7)}$$

where u_s is the surface wind-speed. Moreover, the regime can exist only for a limited range of R, namely $-\frac{1}{4} \leqslant R \leqslant 1$ (Moncrieff (1978)) so that $\Delta u/3 \leq c - u_s \leq 0.62\Delta u$ Consequently a sufficiently large shear is necessary for this regime of overturning to exist. The modification to the large-scale flow is shown in Fig.7. It is clear that a counter-gradient momentum transport has been effected, and the shear and kinetic energy of the large-scale flow enhanced. Moreover, the upper levels have been warmed directly by the latent heat release and the low levels cooled by evaporation; a distinctive three-layer structure has been imparted to the large-scale fields. This will be discussed in more detail later.

2.3 The tropical (propagating) model

The details of this model are contained in Moncrieff and Miller (1976) (hereafter MM). The specified and calculated values are shown in Fig.8, where in a similar fashion to the mid-latitude model, the modified outflow and travel speed are calculated in terms of large-scale (specified) inflow variables. Notice that since the modified and unmodified air is in this case totally on opposite sides of the storm, there is a non-zero net pressure change across the system, which has to be found, a feature of the arbitrary nature of the reference pressure in a system where outflow and inflow are on separate sides of that system. Although in principle the model can be solved for quite general inflow profiles, the case of constant density and zero shear is particularly useful since analytic solutions are obtainable:

$$c - u_M = 0.32\sqrt{CAPE} \qquad (8)$$

$$\delta u(z_1) = -(c - u_M)\left(1 + \sqrt{1-E}\right)\cosh\left(\frac{z_1}{FH}\right) \qquad (9)$$

$$\delta \phi(z_1) = \frac{2CAPE\left(1+\sqrt{1-E}\right)}{9H^2}\sinh\left(\frac{z_1}{FH}\right) \qquad (10)$$

where $-H/2 \leq z_1 \leq H/2$ and $E = \Delta p / \frac{1}{2}\rho(c - U_M)^2$ is a normalised pressure change at the mid-level $z_1 = 0$

where u_M is the environment wind at the mid-level $z = H/2$

In the more general case of constant shear (MM) :

$$c - u_M = F(R) \qquad \text{——(11)}$$

where $F = (c - u_M)/\sqrt{CAPE}$ is a form of Froude number. Significantly it can be shown (MM) that in order for the regime to exist $R \geqslant 3$ showing that it cannot exist if the environment shear is too large, contrasting with the midlatitude regime which necessarily requires a large shear.

The modification of the large-scale flow, given in Fig.9, is quite distinctive from that of the midlatitude case, particularly in the dynamical quantities. The momentum changes are quite different, reflecting the markedly distinctive dynamical structure of this regime; this shows that it is crucial to obtain the dynamical organisation of the flow before making statements about momentum transport. The transport of momentum is such as to effect a parabolic or jet-like profile, so that for a westward propagating system, easterly momentum is increased in mid-levels, and westerly momentum increased at top and bottom; momentum is both transported and generated, i.e. through the terms $\overline{\rho_0 u'w'}$ and $\partial p/\partial x$ respectively. This regime is therefore a direct method of maintaining the tropical easterlies, quite distinct from large-scale effects.

2.4 Energy budgets

It is useful to consider the energy budgets for the two models. These budgets are rather easily established since the outflow and inflow fluxes are readily determined from the model solutions. Taking the flux form of the steady Boussinesq energy equation

$$\nabla\cdot(\rho_o \underset{\sim}{v} \mathcal{E}) = 0 \qquad \text{——(12)}$$

where $\mathcal{E} = 1/2\, \underset{\sim}{v}^2 + \delta p/\rho_o - g\int_{z_o}^{z} \delta\phi_p \, dz'$

defines the (specific) kinetic energy generation, the net work done by the pressure field, and the net potential energy release respectively as

$$\Delta(\overline{uK}) = \frac{1}{V}\iiint \mathrm{div}(\rho_o \underset{\sim}{v} \tfrac{1}{2}\underset{\sim}{v}^2)\, dx\,dy\,dz = \frac{1}{H}\Big[\int_{\text{outflow}} u_1 K_1 dz_1 - \int_{\text{inflow}} u_o K_o dz_o\Big] \qquad \text{——(13)}$$

$$\Delta(\overline{u\delta p}) = \frac{1}{V}\iiint \mathrm{div}(\underset{\sim}{v}\, \delta p)\, dx\,dy\,dz = \frac{1}{H}\int_{\text{outflow}} u_1 \delta p_1 dz_1 \qquad \text{——(14)}$$

$$-\Delta(\overline{u\delta P}) = \frac{1}{V}\iiint \mathrm{div}\Big(\rho_o \underset{\sim}{v}\, g\int_{z_o}^{z} \delta\phi_p dz'\Big) dx\,dy\,dz = \frac{1}{H}\int_{\text{outflow}} \rho_1 u_1 \delta P_1 dz_1 \qquad \text{——(15)}$$

where $\delta P(z,z_0)=\int_{zo}^{Z} g\delta\phi_p dz$ is the potential energy released by each parcel along a trajectory and where it has been assumed that the outflow and inflow is over unit width in the y-direction. Now the righthand sides of Eqs.(13,14,15) can be evaluated directly from the solutions of the cloud models - the mean flow modification; this shows the powerful nature of the model formulation because the modifications are obtained directly from the solutions, a feature of the use of the conservation principles described earlier. Normally these would have to be determined for volume averages, an imprecise and difficult method, requiring the full knowledge of the internal flow structure.

First, using the solution obtained from the midlatitude model, it can be shown that

$$\Delta(\overline{uK}) = \frac{1}{4}\bar{\rho}\,\frac{\beta^4(\beta-1)}{(1+\beta)^3}(\Delta u)^3 = \frac{1}{2}\bar{\rho}_0\frac{\beta^3}{(1+\beta)^3}\,\mathrm{CAPE}\,\Delta u \qquad (16)$$

$$\Delta(\overline{u\delta p}) = \frac{1}{4}\bar{\rho}\,\frac{\beta^3(\beta-1)}{(1+\beta)^3}(\Delta u)^3 = \frac{1}{2}\bar{\rho}_0\frac{\beta^2}{(1+\beta)^3}\,\mathrm{CAPE}\,\Delta u \qquad (17)$$

and

$$-\Delta(\overline{u\delta P}) = \frac{1}{4}\bar{\rho}\,\frac{\beta^3(\beta^2-1)}{(1+\beta)^3}(\Delta u)^3 = \frac{1}{2}\bar{\rho}_0\,\frac{\beta^2(\beta+1)}{(1+\beta)^3}\,\mathrm{CAPE}\,\Delta u \qquad (18)$$

where $\beta(R) = (1+\sqrt{1+4R})/2$. As an example, for $R = 1$, $\Delta(\overline{uK})/-\Delta(\overline{u\delta P}) \simeq 0.62$ and $\Delta(\overline{u\delta p})/-\Delta(\overline{u\delta P}) \simeq 0.38$ showing that some 62% of the potential energy released by the overturning is directly available to enhance the kinetic energy of the large-scale flow, while 38% has been used to change the pressure field, and is not directly available for the mean flow, that is, it is the amount of energy necessary to drive the convection, and is presumably only available if dissipated by another process or dispersed, say by gravity waves.

Second, using the tropical model solutions, in the case of zero ambient shear, when analytic solutions are obtained:

$$\Delta(\overline{uK}) \simeq 0.034\,\bar{\rho}_0\,(\mathrm{CAPE})^{3/2} \qquad (19)$$

$$\Delta(\overline{u\delta p}) \simeq 0.029\,\bar{\rho}_0\,(\mathrm{CAPE})^{3/2} \qquad (20)$$

$$\Delta(\overline{u\delta P}) \simeq -0.063\,\bar{\rho}_0\,(\mathrm{CAPE})^{3/2} \qquad (21)$$

Again, about half the potential energy released is available for the large-scale flow, and about half is not directly available, since $\Delta(\overline{uK})/-\Delta(\overline{u\delta P}) \simeq 0.52$ and $\Delta(\overline{u\delta p})/-\Delta(\overline{u\delta P}) \simeq 0.48$

2.6 Inclusion in the large-scale equations

This aspect can be treated in more detail, but the following preliminary results are useful. Let $S = \bar{S} + S'$ be a scalar fluid property where $\bar{S}$ and S' are grid averages and sub-grid scale perturbations respectively. Then the relationship between the grid scale and sub-grid-scale variables is (conventionally) of the form

$$\frac{\partial \bar{S}}{\partial t} + \mathrm{div}(\bar{\rho}_0 \bar{\underset{\sim}{V}} \bar{S}) = -F_S' = -\frac{\partial}{\partial z}(\overline{w'S'}) - \nabla_h \cdot (\underset{\sim}{u}' S')$$
$$= -\frac{\partial}{\partial z}(\overline{w'S'}) - \frac{1}{L_\Delta}\Delta(u'S') \qquad \text{——(22)}$$

where L_Δ is the grid length. Now $\Delta(u'S')$ represents the horizontal eddy fluxes, usually neglected, and $\partial/\partial z\,(\overline{w'S'})$ is the flux convergence in the vertical. It is interesting that strictly speaking $\partial/\partial z(\overline{w'S'})$ is not all available for the large-scale flow in general, since some of the (heat) flux is required to drive part of the convective circulation. However, by using the models described in previous sections, it is not necessary to neglect $\Delta(u'S')$, and the part of $\partial/\partial z\,(\overline{w'S'})$ which is available is explicitly calculated. If one considers a layer of depth dz, into which a sub-grid-scale flux dF_S' is effected, then the net flux to the large scale can be defined as

$$dF_S' = \frac{a}{L_\Delta}\left(\rho_1 u_1 S_1\, dz_1 dy_1 - \rho_0 u_0 S_0\, dz_0 dy_0\right) \qquad \text{——(23)}$$

$$= \frac{a}{L_\Delta}\rho_1 u_1 \delta S_1\, dz_1 dy \qquad \text{(by continuity of mass) –(24)}$$

where $a \sim 1\%$ is the fractional area of the grid area L_Δ^2 occupied by deep convection. Then all of F_S' is available for the large scale flow since it is defined as being the difference between the outflow and inflow on that scale.

It is possible to obtain the large scale modification for both the midlatitude and tropical regimes, and by a judicious combination of these models to obtain more general representations. However, for the sake of illustration, consider the midlatitude model and the three layer model shown in Fig.10.

In the region $z_* \leq z_1 \leq H$, the solutions give

$$u_1(z_1) = \frac{\Delta u}{H}\beta^2(z_1 - z_*) \quad ; \quad \delta u_1(z_1) = \frac{\Delta u}{H}(\beta^2 - 1)(z_1 - z_*) \quad ; \quad \delta\phi_1(z_1) = (\gamma - \beta)(1+\beta)(z_1 - z_*) \qquad (25)$$

so the convective heat flux into the layer of depth dz_1 is given by

$$dF_H' = \frac{a}{L_\Delta} u_1 \delta\phi_1 \, dz_1 = \frac{a}{L_\Delta} \frac{\Delta u \, CAPE}{gH^4} \beta^2(1+\beta)(z_1 - z_*)^2 dz_1 \qquad (26)$$

$$\therefore \quad F_H'(z_1) = \frac{1}{3}\frac{a}{L_\Delta} \cdot \frac{\Delta u \, CAPE}{gH^4} \beta^2(1+\beta)(z_1 - z_*)^3 \qquad (27)$$

Similarly, for the other two layers

$$\therefore \quad F_H'(z_1) = \begin{cases} \frac{1}{3}\frac{a}{L_\Delta} \frac{\Delta u \, CAPE}{gH^4} \beta^2(1+\beta)(z_1 - z_*)^3 & z_* \leq z_1 \leq H \\ 0 & H - z_* \leq z_1 \leq z_* \\ -\frac{1}{3}\frac{a}{L_\Delta} \frac{\Delta u \, CAPE}{gH^4} \beta^2(1+\beta)(z_1 - H + z_*)^3 & 0 \leq z_1 \leq H - z_* \end{cases}$$

$$(28)$$

TABLE 2

Regime	Energetics	Momentum Transport	Heat Transfer
Midlatitude	$\frac{\Delta(\overline{uk})}{-\Delta(\overline{u\delta P})} \simeq$ 62% $\frac{\Delta(\overline{u\delta p})}{-\Delta(\overline{u\delta P})} \simeq$ 38%	Counter-gradient: no net generation Causes increased mean shear	downshear 3-layer form
Tropical	$\frac{\Delta(\overline{uk})}{-\Delta(\overline{u\delta P})} \simeq$ 52% $\frac{\Delta(\overline{u\delta p})}{-\Delta(\overline{u\delta P})} \simeq$ 48%	Counter-gradient: net generation Jet-like profile (no net shear)	downshear 2 layer form
Transient	$\Delta(\overline{uk})$ small $\Delta(\overline{u\delta P})$ large	Small	downshear 2 layer form

The most interesting aspect is the predominance of counter-gradient momentum transport, so momentum mixing in organised deep convection is inappropriate, contrasting with boundary layer eddies, where mixing of momentum probably does predominate. The results may be generalised for use in a parameterisation scheme for deep convection. One of the most important generalisations is the incorporation of a shallower downdraught, since observations and numerical simulations indicate that this is more realistic.

GENERAL CONCLUSIONS

The following are the most important points resulting from this work on organised deep convection.

(a) It is not necessary to have complex microphysical or sub-grid-scale turbulence parameterisation for studying organised deep convection models since dynamical processes dominate.

(b) Quite distinctive regimes exist in different environmental conditions, hence a single model is not sufficient for a general representation of convective processes.

(c) There are distinctive differences between shallow and deep convection, particularly the effect of cloud-scale downdraughts.

(d) Both thermodynamical and dynamical processes should be expressed in a consistent fashion. Momentum transports are distinctive in mainly being counter-gradient in form.

(e) Since shear, momentum transport and downdraughts are important, 1-D models are probably inadequate.

(f) More effective use needs to be made of three-dimensional model budgets to test and generalise the theoretical results.

(g) The combination of theoretical and numerical approaches provides a powerful technique for examining fundamental aspects of convection.

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