TECHNICAL MEMORANDUM

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Horizontal diffusion experiments with the ECMWF spectral model

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1. Introduction

Some experiments performed in ECMWF in order to tune the coefficient of a ∇^4 horizontal diffusion led to the conclusion that the optimum coefficient is presumably independent of truncation. This is an empirical result and theoretical justifications are still not clear.

2. Horizontal Diffusion in Numerical Prediction Models

Horizontal diffusion is included in most spectral models for two main reasons :

2.1 : As a representation of the physical effects of unresolved scales. It is commonly thought that coefficients for such horizontal diffusion should decrease with increasing resolutions.

2.2 : To avoid the so-called spectral blocking (Puri and Bourke , 1974) that is a spurious accumulation of energy in the waves near the truncation limit. However, the amplitude of this phenomenon decreases quite rapidly when the truncation increases, as shown by Puri and Bourke.

So, even for the medium range forecasting in ECMWF, in view of the fairly large truncations used, it seems very unlikely that we need any protection against spectral blocking.

3. Horizontal Diffusion used in ECMWF Spectral Model

For the description of the ECMWF spectral model we refer to Baede, Jarraud and Cubasch (1979).

Due to its particularly easy implementation in a spectral model and to its high scale selectivity we choose a linear ∇^4 horizontal diffusion on σ-surfaces (Baede et al. 1979).

For the prognostic fields X we write

$$\frac{X^{t+\Delta t}-X^{t-\Delta t}}{2\Delta t} = \left(\frac{\partial X}{\partial t}\right)^t - K\nabla^4 X^{t+\Delta t}$$

with $\left(\frac{\partial X}{\partial t}\right)$ = adiabatic and physical tendencies.

As in spectral space

$$(\nabla^2 X)_{n,m} = -\frac{n(n+1)}{a^2} X_{n,m} \qquad (a = \text{radius of the earth})$$

we get :

$$X^{t+\Delta t}_{n,m} = K_n \left[X^{t-\Delta t}_{n,m} + 2\Delta t \left(\frac{\partial X}{\partial t}\right)^t_{n,m}\right]$$

with $K_n = \dfrac{a^4}{a^4+2\Delta t\, K\left[n^2(n+1)^2\right]}$

For the divergence and vorticity fields, in order to avoid damping of uniform rotations (n=1) we take

$$K_n = \frac{a^4}{a^4+2\Delta t\, K\left[n^2(n+1)^2-4\right]} \qquad \text{so that } K_n = 1 \text{ for } n=1$$

All the prognostic quantities, including temperature, are diffused on σ-surfaces and this does not seem to lead to serious difficulties.

4. Experiments

4.1 Introduction

For all the experiments the model used the complete ECMWF physical package, described in Tiedtke et alii (1979). The only differences were the truncation and (or) the horizontal diffusion coefficient K.

In order to study the influence of K, two cases were selected : one winter case (15.2.1976) providing a rather good forecast up to 5 -6 days and a summer case (25.8.1975) providing a less good forecast. Some experiments were also performed on the 16.1.1979 case with a high pentagonal truncation P635874 (see fig:1.2).

Since significant differences do not appear before 3 days (and very often not before 4 or 5 days) we concentrated our attention on three different topics :

1: a synoptic evaluation of the maps from day 4 to day 10 or up to the point where the forecasts ceased to give useful guidance.

2: a comparison of the scores :RMS errors and correlations (in particular correlations of height of the 500 mb surface for wave-number 1-3 components, mainly because after 4 days this field contains an important part of the useful information).

3: a check in order to ensure that kinetic energy was reasonably well conserved in all the runs (less than 10% variation after 10 days) and in particular that the ratio between zonal and eddy kinetic energy was not becoming unrealistic.

4.2 Experiments with a P635874 truncation (fig:1.2)

Many experiments were performed with this truncation on the 16.1.1979 case providing a particularly good forecast, allowing us to say if an 8 day forecast is better than another one and not only "less worse".

Furthermore this case seems to be much more sensitive to horizontal diffusion than the two others (15.2.1976 and 25.8.1975).

Nine horizontal diffusion coefficients were used :

$K = \{0.5,\ 1.5,\ 3.0,\ 4.5,\ 7.,\ 9.,\ 13.,\ 17.\ \text{and}\ 30.\}.10^{14}\ m^4 s^{-1}$

Some results for these various K are presented in figures at the end of this report.

The day 4 maps (fig. 2 and fig. 4) show no important differences between them, while the day 8 maps (fig. 3 and fig. 5) look less similar. The maps for $K = 7.10^{14}$ look slightly closer to reality (mainly for 1000 mb).

The wavenumber 1-3 components of height for the 500 mb surface (fig:6.2) seem also to carry significant information longer in the range $K = 6 \rightarrow 14.10^{14}\ m^4 s^{-1}$.

Finally there is obviously a better partition of eddy and zonal kinetic energy after 10 days in the range $K = 7 \rightarrow 13.10^{14}\ m^4 s^{-1}$ (fig: 7.2a)

Some other experiments were carried out on the 15.2.1976 and 25.8.1975 situations : in both cases, as long as there was useful guidance in the maps they showed no significant differences. Furthermore, as shown in fig: 6.2 and 7.2b, these cases are probably less sensitive to diffusion tuning (also we might not have done experiments in a wide enough range of K).

4.3 Experiments with other truncations

(P282856. P404040. P314242. P312946. - fig: 1.2)

All these truncations have approximately the same number of spectral degrees of freedom.

Only two situations were taken into account (15.2.1976 and 25.8.1975). The largest number of tests was done with the two "traditional" truncations : P282856 (rhomboidal) and P404040 (triangular).

With the triangular one some first experiments showed that the coefficient originally chosen ($K = 48 \cdot 10^{14}\ m^4 s^{-1}$) was much too large since after 10 days the ratio between zonal and eddy kinetic energy was much bigger than in reality. This fact was obviously reflected in the maps.

Thus some new experiments were performed with smaller coefficients and there again the sensitivity to horizontal diffusion was rather small (as long as the forecasts were carrying useful information)and in fact it was almost impossible to choose a "better" coefficient in the range $K = 3 \rightarrow 17.10^{14}\ m^4 s^{-1}$.

Although less experiments were performed with the two other truncations, their sensitivity to horizontal diffusion seemed to be very similar to that of P282856 and P404040 (fig: 7.3 and 7.4).

One point to note is that there are more differences between two different equivalent truncations with the same diffusion coefficient than within a truncation between two experiments with two different coefficients (at least with two coefficients in the range K = 3 → 17.10^{14} $m^4 s^{-1}$) as shown in fig. 8 and fig. 9.

5. Conclusion

Although the number of situations and the range of coefficients used for some cases was insufficient, we can say that there is no fundamental reason to choose a particular coefficient rather than another in the range 3 to $17.10^{14} m^4 s^{-1}$ for the 4 small truncations. On the other hand the range 6 to $13.10^{14} m^4 s^{-1}$ seems to give better results for the high resolution model.

Therefore there is almost evidence that the horizontal diffusion has not necessarily to decrease when increasing resolution.

Furthermore, it appears that it is almost impossible to draw conclusions if we look only at 2 or 3 days forecasts. The influence of horizontal diffusion appears in a significant manner only after 4 to 5 days.

Finally, considering that this parameterisation might depend much more on other factors, like the physics used in the model (and maybe also on the season) ... it was decided to take the same coefficient (K = $7.10^{14} m^4 s^{-1}$) for all the truncations presently used.

Some finer tuning experiments might have to be planned on a larger number of cases (or on longer periods), but only when the model has reached a quasi-operational stage.

References :

Baede, A.P.M., Jarraud, M. Cubasch, U. (1979)	Adiabatic formulation and organisation of ECMWF's spectral model. ECMWF Technical Report No.15.
Daley, R. et alii (1976)	Short term forecasting with a multi level spectral primitive equations model. Atmosphere, 14, pp 98-134
Hoskins B.J. and Simmons, A.J. (1975)	A multi-layer spectral model and the semi-implicit method. Quart.J.Roy.Met.Soc., 101 pp. 637-655
Machenhauer, B. (1978)	The spectral method. (Numerical methods used in atmospheric models, Vol.II) (to be published).
Puri, K. and Bourke, W. (1974)	Implications of horizontal resolution in spectral model integrations. Month.Weath.REV. 102, No.5 pp. 333,347
Tiedtke et alii (1979)	ECMWF Model parameterisation of sub-grid scale processes. ECMWF Technical Report No.13.

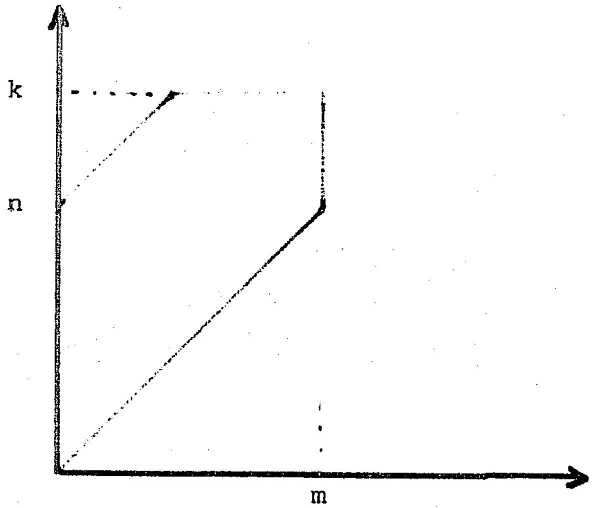

Fig. 1.1 Pentagonal Truncation $P_{m,n,k}$

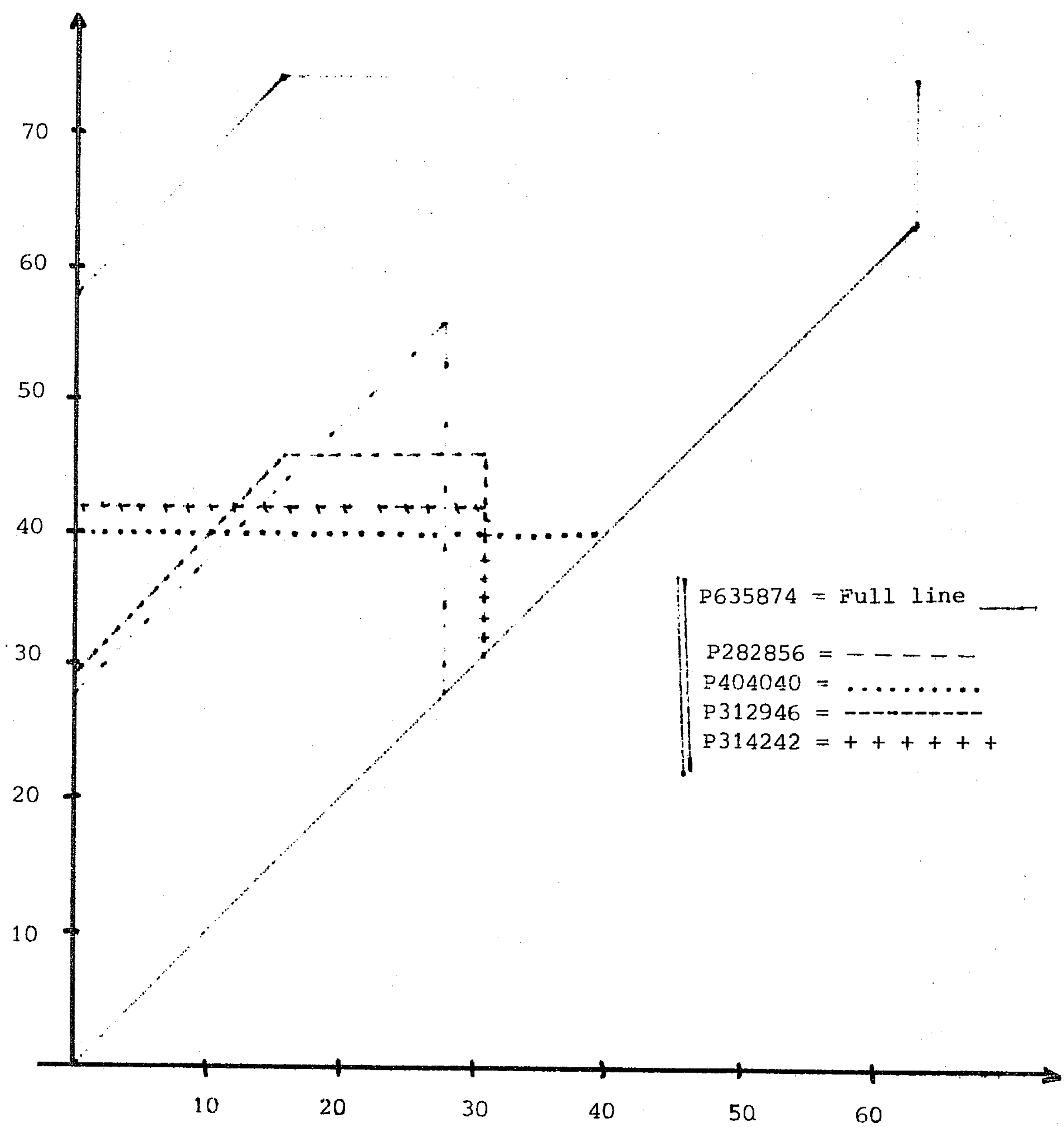

Fig. 1.2 The five truncations used in the experiment.

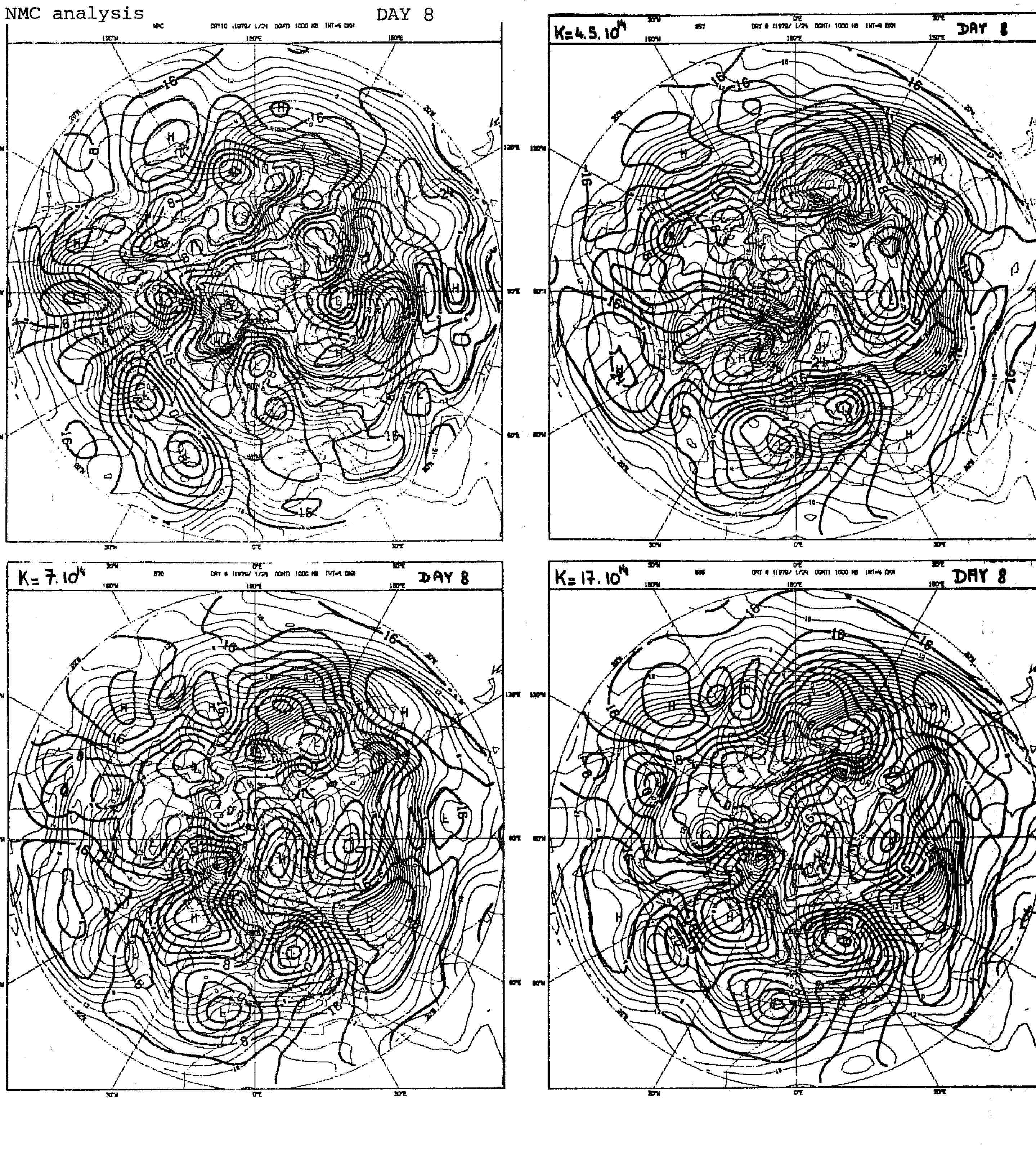

Figure 3: 1000mb day 8 forecast from 16.1.79.

(P635874 truncation)

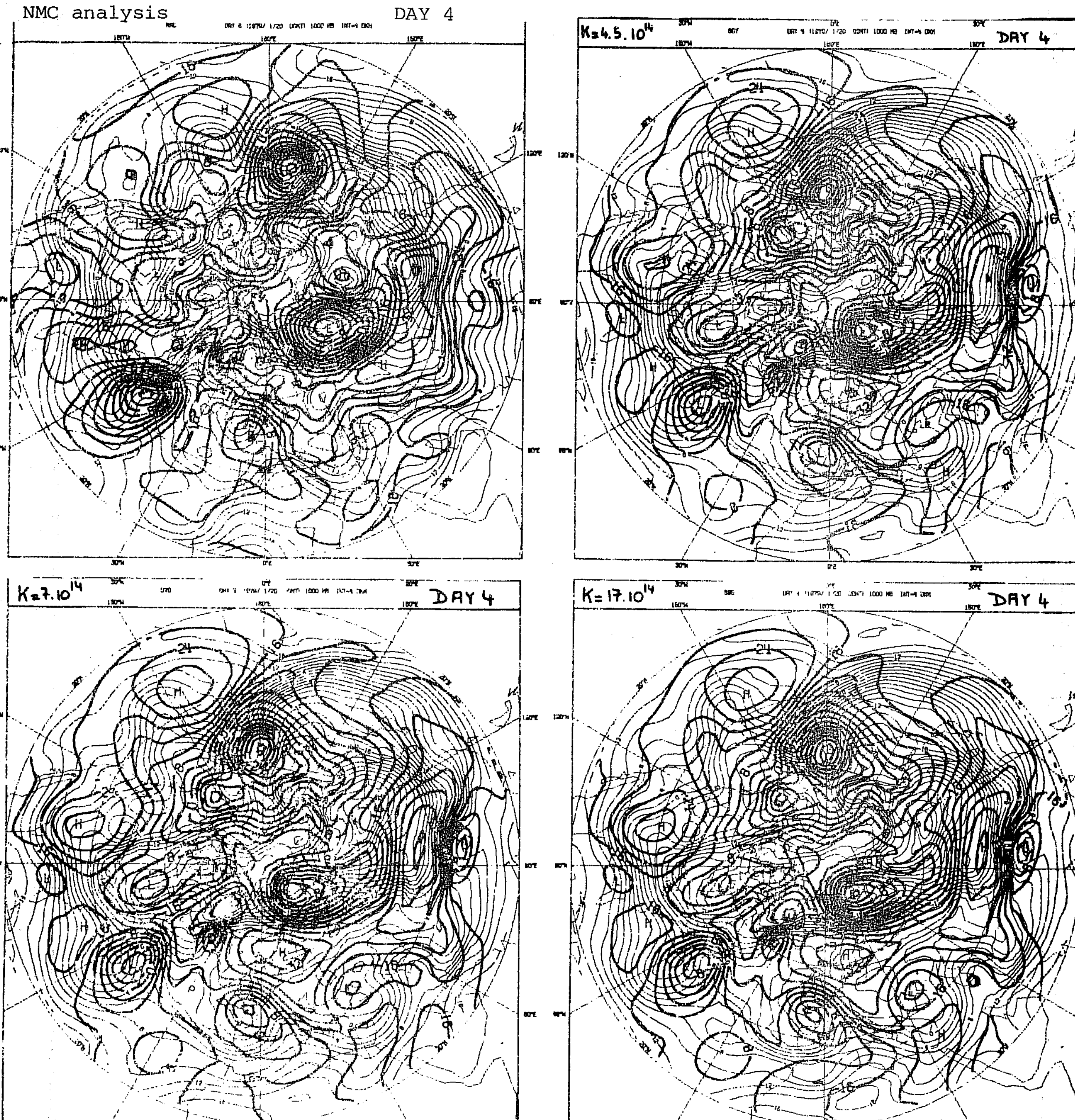

Figure 2: 1000mb day 4 forecast from 16.1.79.

(P635874 truncation)

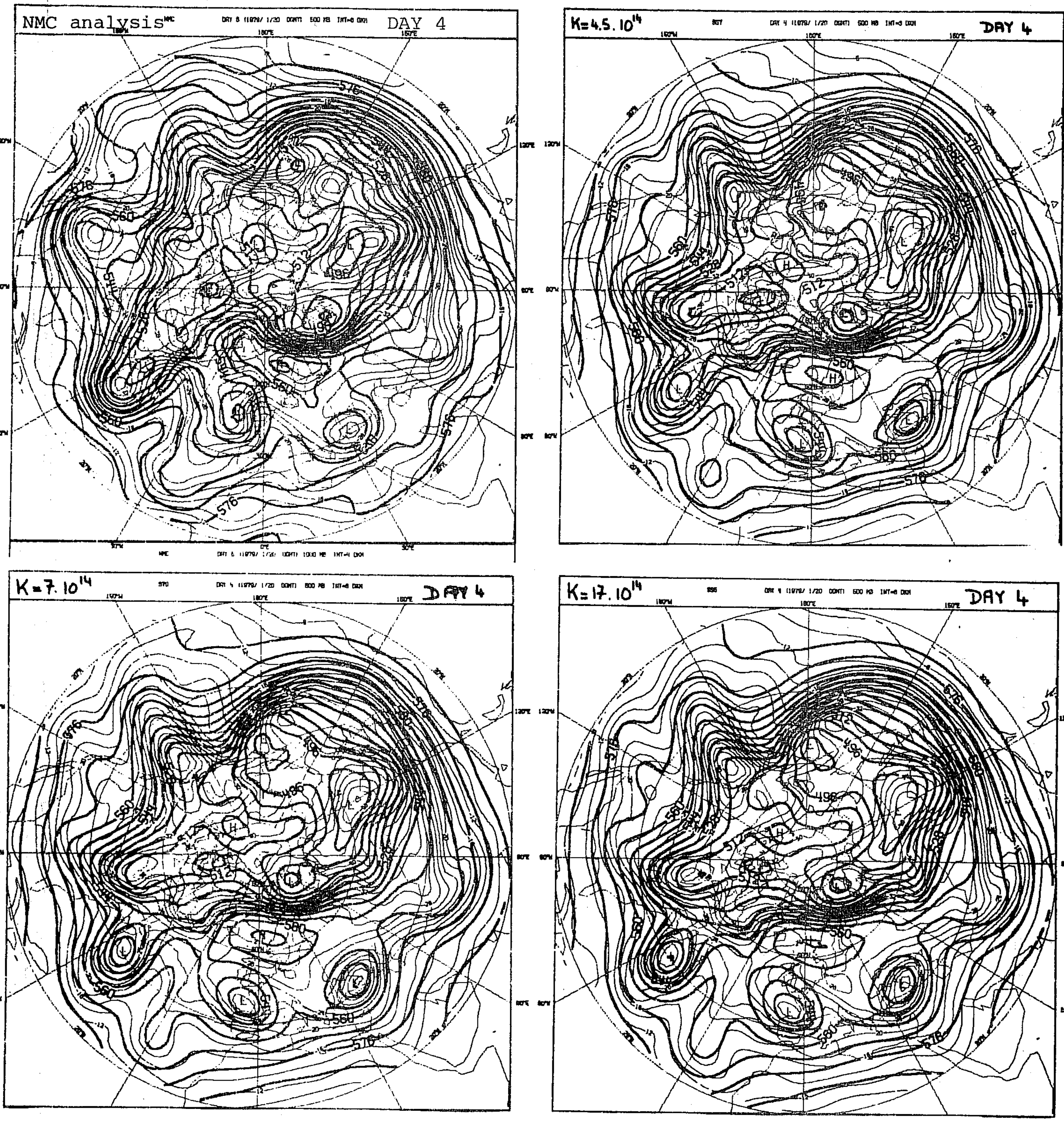

Figure 4: 500mb day 4 forecast from 16.1.79.
(P635874 truncation)

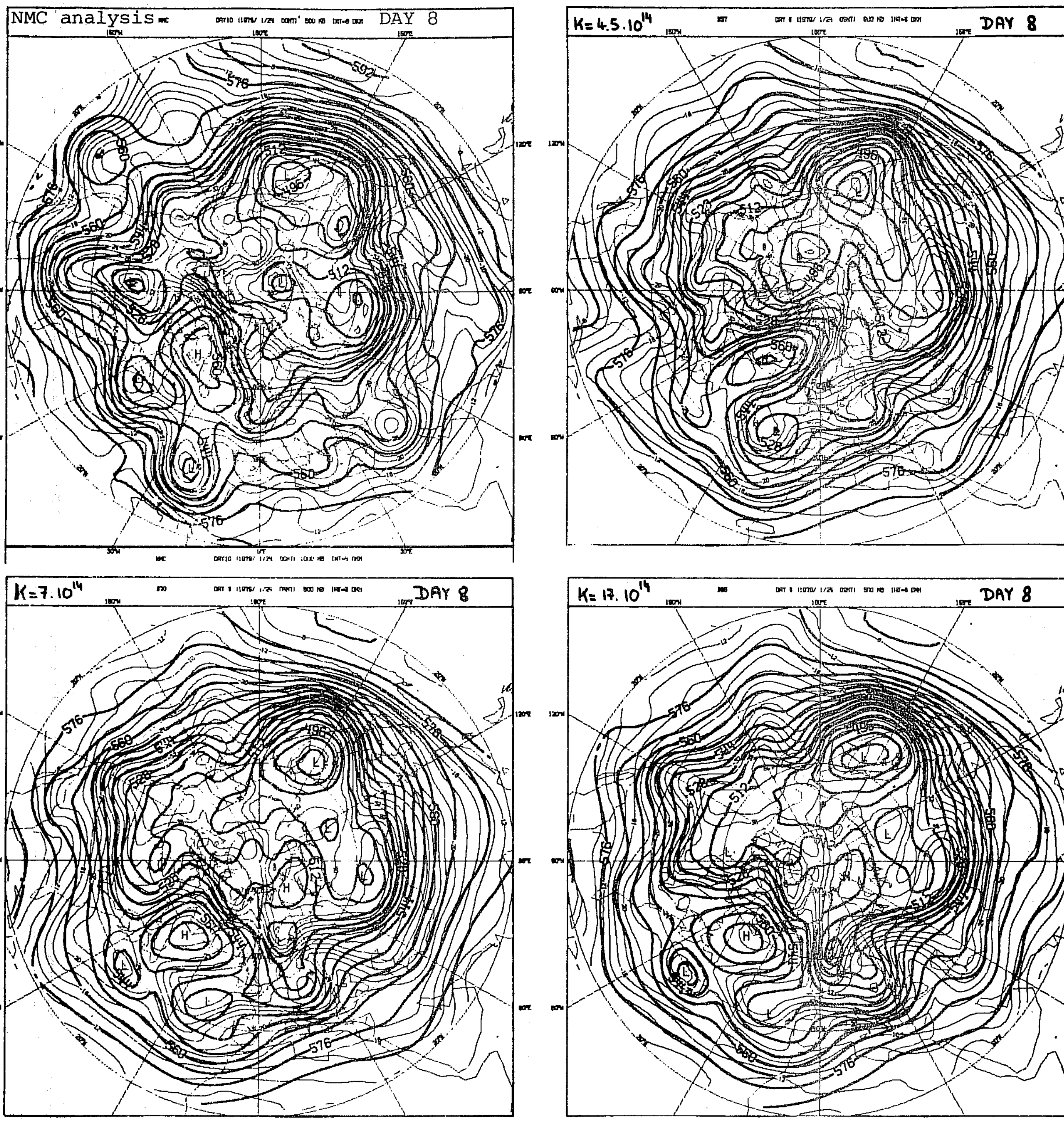

Figure 5: 500mb day 8 forecast from 16.1.79.
(P635874 truncation)

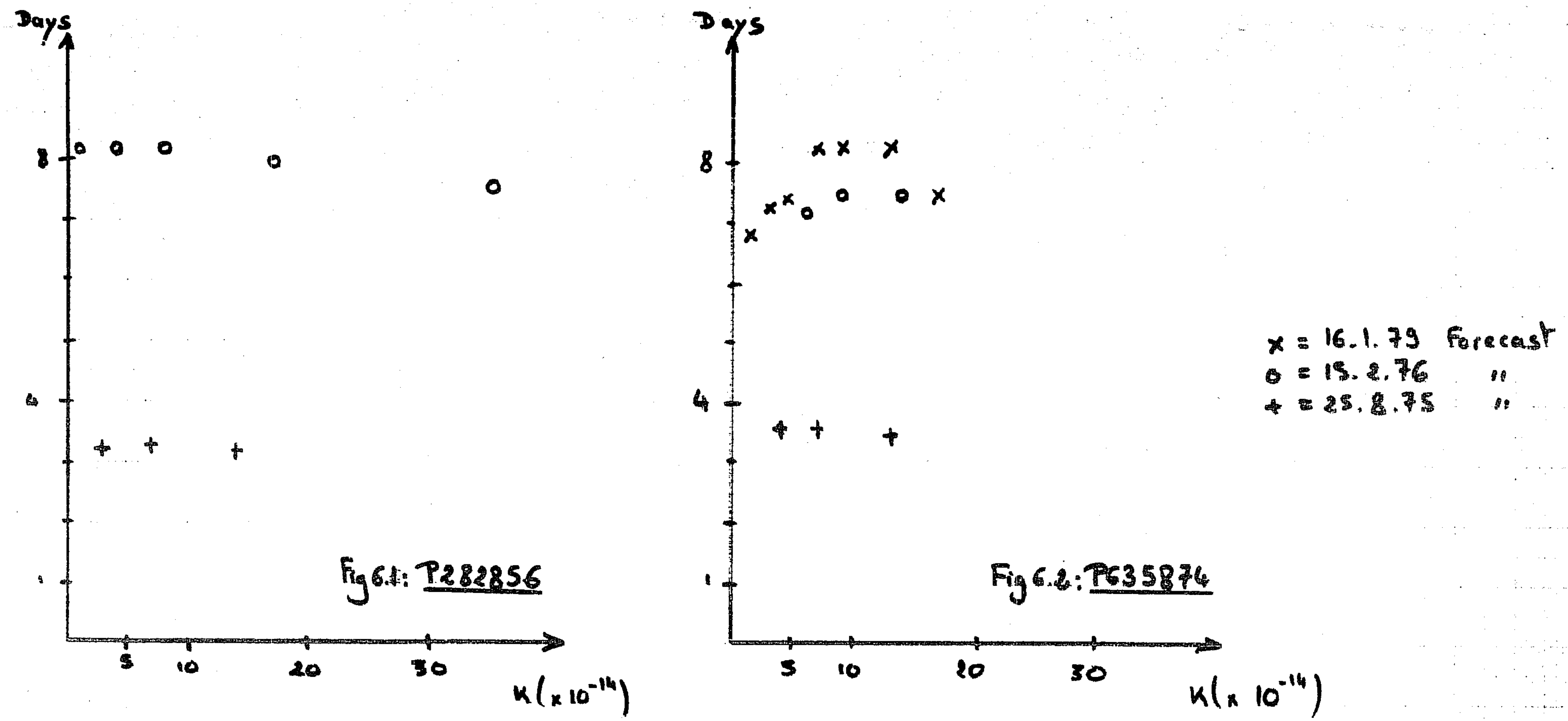

Figure 6: Time needed for the correlations of height of the 500mb surface (wavenumber components 1-3) to drop under 80% as a function of the horizontal diffusion coefficient K.

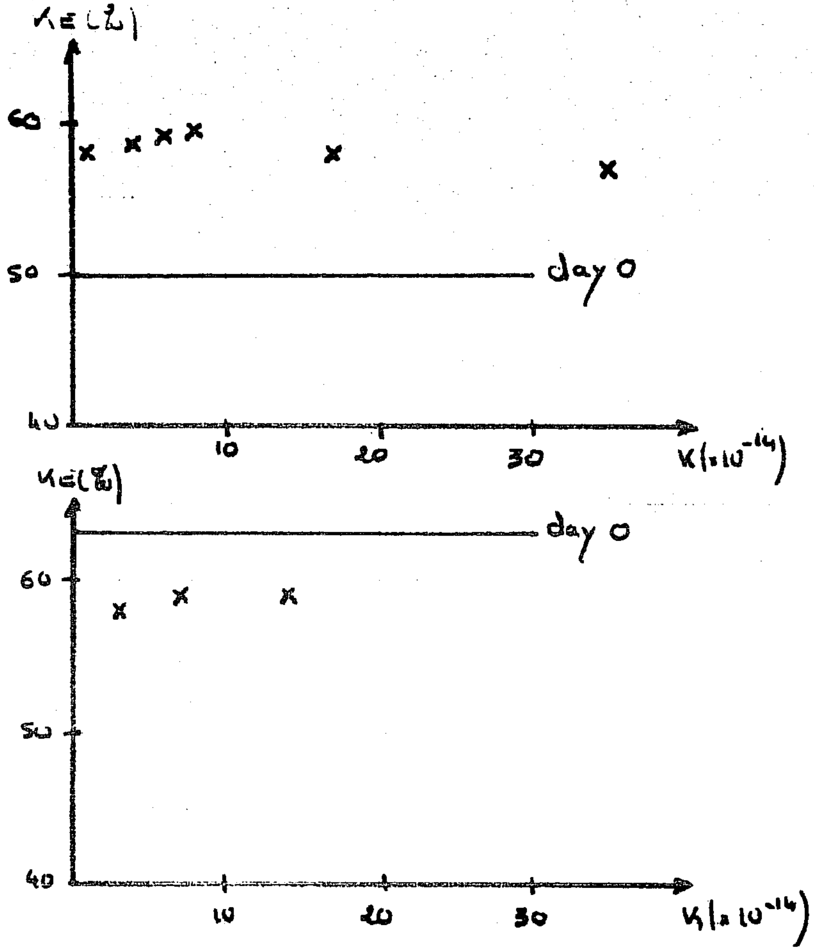

Fig. 7.1: P282856

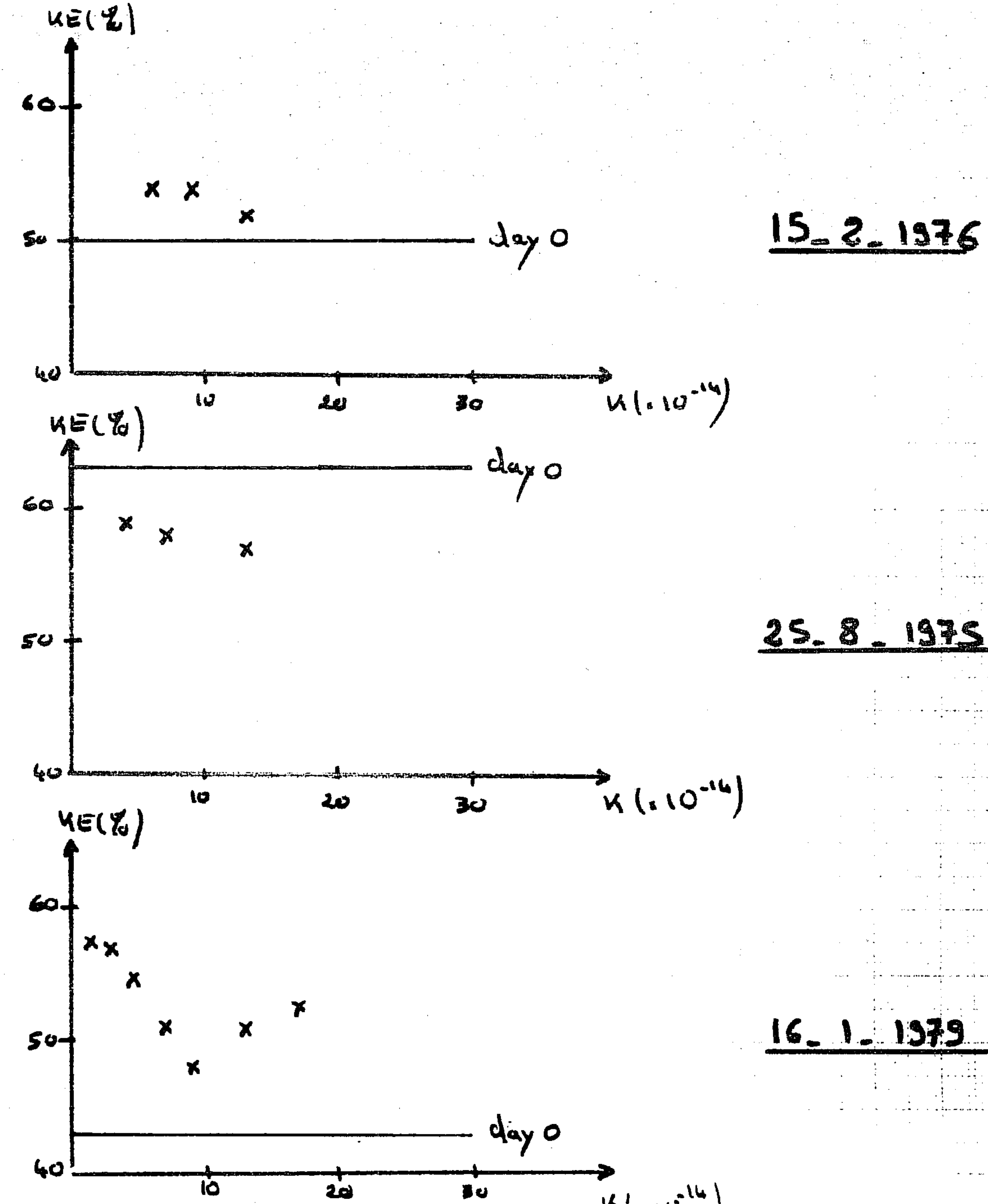

Fig. 7.2: P635874

$$KE = \frac{\text{Kinetic energy (zonal)}}{\text{Kinetic energy (total)}} \text{ at day 10}$$

as a function of K

(horizontal diffusion coefficient)

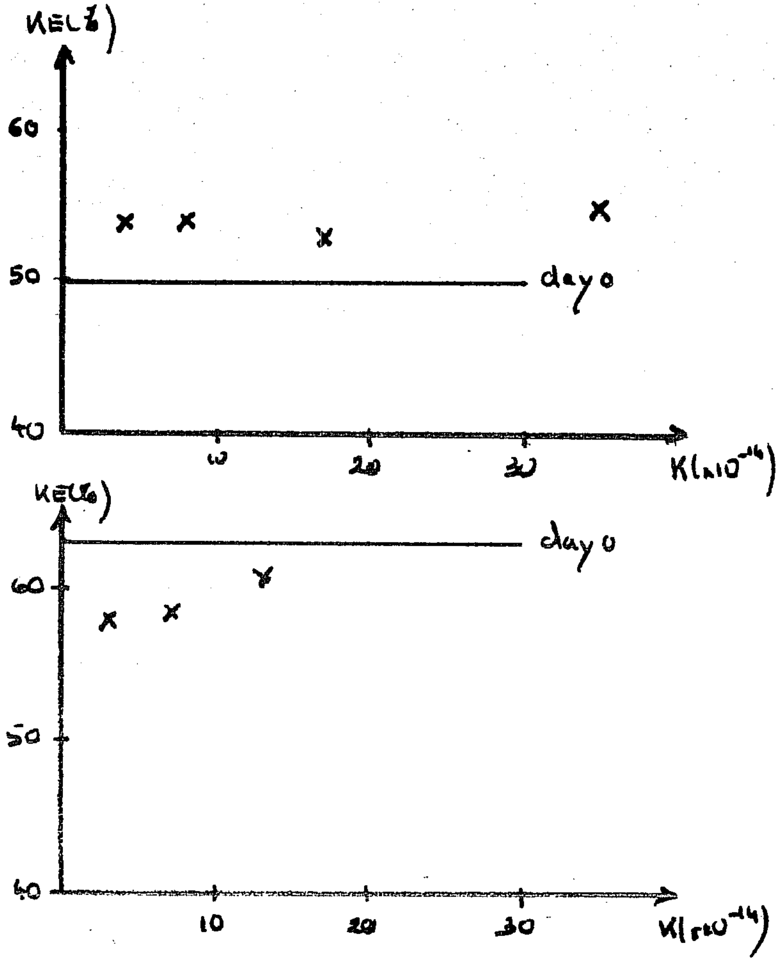

Fig.7.3: P404040

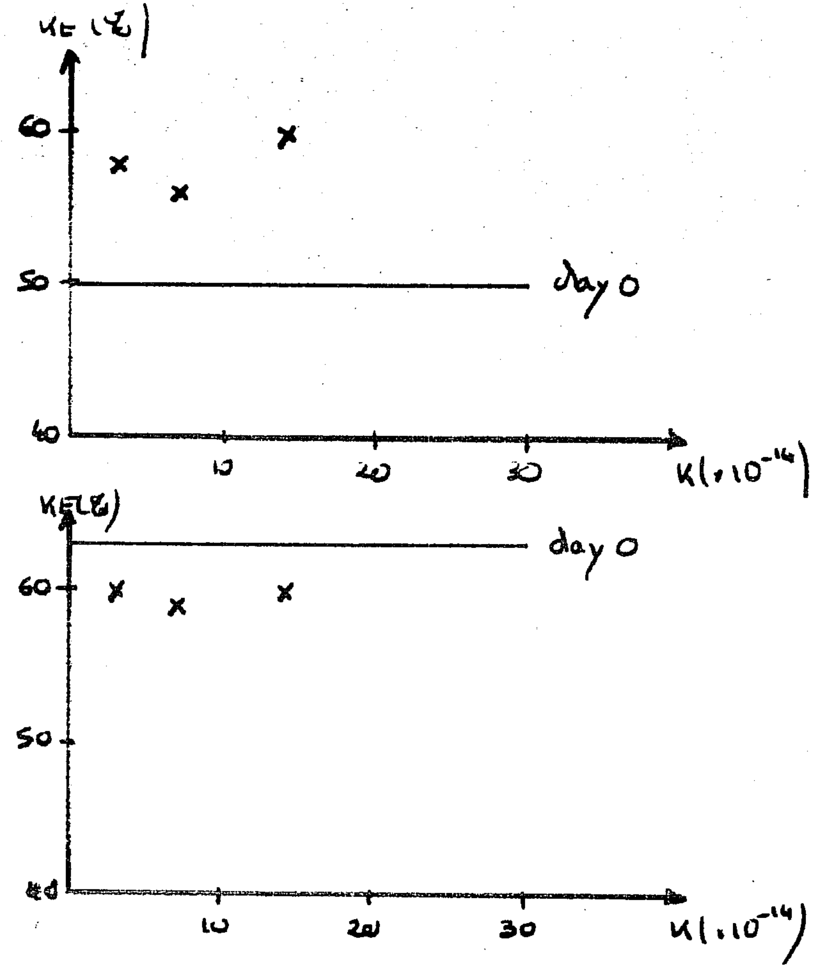

Fig. 7.4: P312946

15. 2. 1976

25. 8. 1975

KE = $\frac{\text{Kinetic energy (zonal)}}{\text{Kinetic energy (total)}}$ at day 10 as a Function of K (horizontal diffusion coefficient)

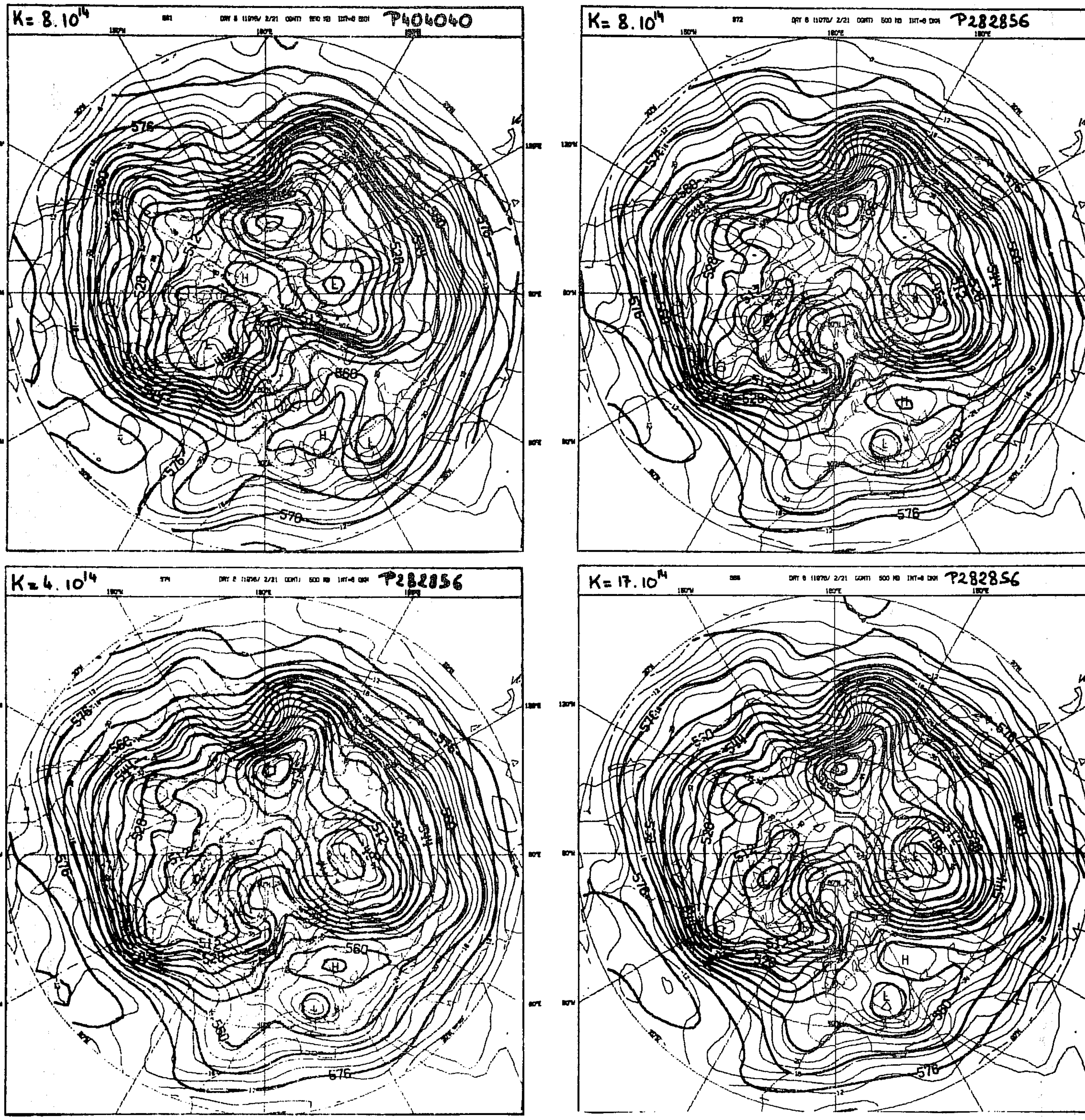

Figure 8: 500 mb day 6 forecast from 15.2.76.

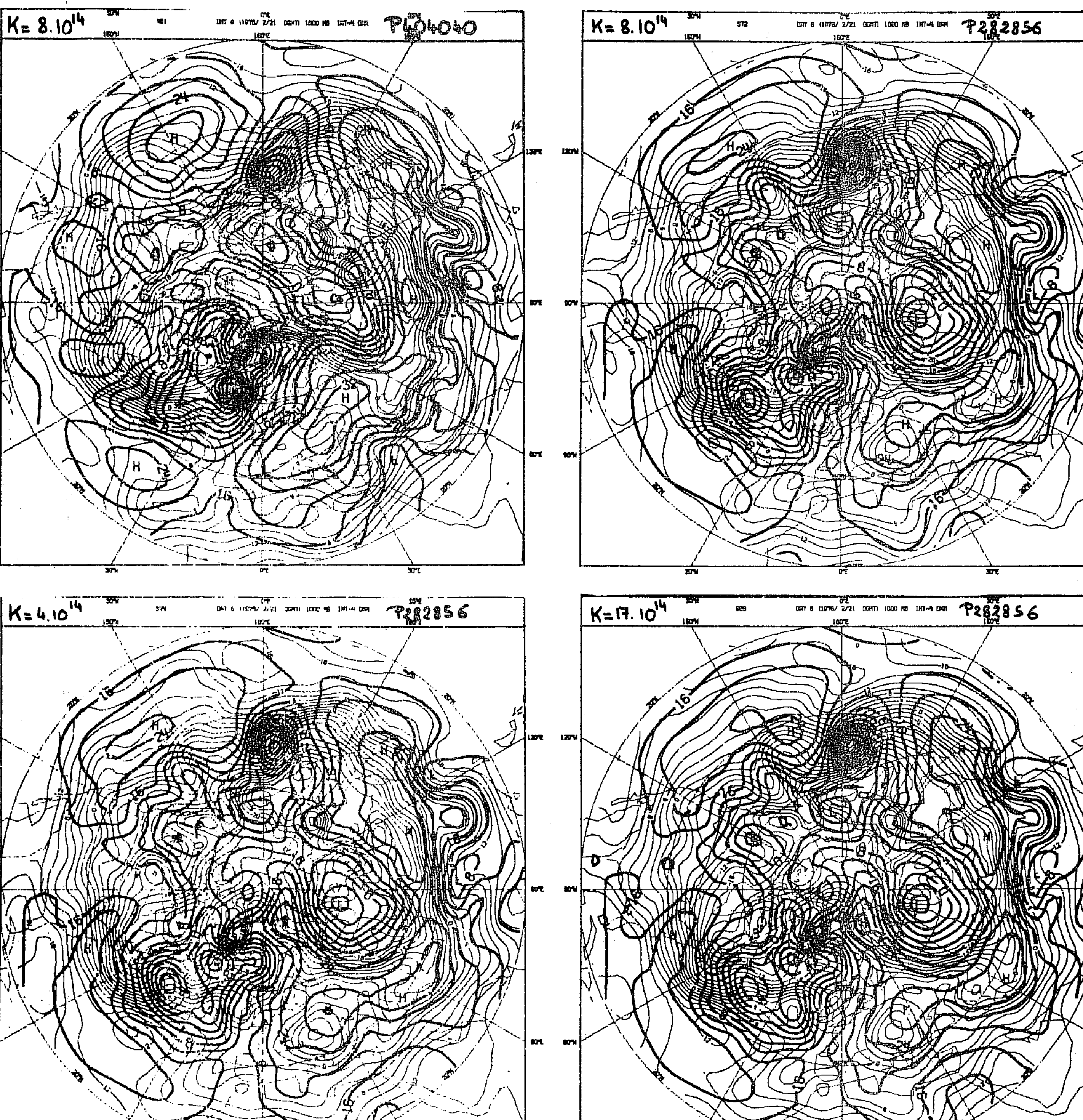

Figure 9: 1000mb day 6 forecast from 15.2.76.