INITIALISATION IN PRESENCE OF MOUNTAINS

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1 Introduction

The goal of initialisation is in general terms to get rid of, or at least to minimize, unrealistic effects in the subsequent time integration. Such spurious effects result primarily from an inproper adjustment between the wind and the mass fields, implying that large-amplitude gravity oscillations appear in the model atmospheres that are described by the so called primitive equations. It is a common experience that this type of contamination becomes enhanced when orography is incorporated in the models, indicating that, in this case, our knowledge of how to adjust wind and mass is insufficient, or/and the numerical treatment requires special attention. The last mentioned matter will be addressed in the sequel. Specifically, sigma type (i.e., terrain following coordinate) systems will be regarded, as the common use of these systems is increasing in the numerical modelling.

Truncation errors appearing in the pressure gradient force in the presence of orography have been studied and discussed by Phillips (1974), Sundqvist (1975) and Janjic (1977). Those papers deal primarily with truncation/aliasing problems that appear each time the pressure gradient force is evaluated in the course of a time integration. However, there is another type of truncation/consistency problem directly connected with the arrangment of the initial mass field. Namely, although we have both temperature and geopotential analyzed in the pressure system we can interpret only one of those fields to the sigma levels while the other field has to be evaluated from the hydrostatic relation used in the sigma system. As a consequence of the discretized vertical representation - either in the form of grid points or in the form of a truncated series expansion - the resulting pairs of temperature and geopotential in the sigma system may deviate from those in the p system at the corresponding pressure level and therefore the pressure gradient may become different in the two coordianate systems. In order to eliminate this last mentioned type of error Sundqvist (1976) suggested that the pressure gradient - instead of either temperature or geopotential - is interpolated from the p system analysis to the levels of the sigma system. The temperature then has to be solved from a set of differential equations obtained from the expression of the sigma system pressure gradient containing the proper hydrostatic relation between temperature and geopotential. This procedure suffers from certain mathematical problems however.

The intention of the present note is to discuss that vertical truncation error that affects the initial fields, in relation to the recent methods of normal mode initialisation (Machenhauer 1977; Daley 1978; Temperton and Williamson 1979). We shall begin with a brief recapitulation of how the truncation errors mentioned in the preceeding paragraph arise and present a definition of the two types. We will continue with a quantitative indication of the magnitude of the error arising from vertical truncation applying the vertical grid-structure of the ECMWF models. Qualitative presentation of the mathematical problems in solving for temperature from the pressure gradient expression will then follow before we discuss possible effects of the normal mode initialisation.

2 Origin of truncation errors appearing in the pressure gradient

Since we are focusing on the pressure gradient, which in the sigma system has the form

$$(1) \qquad G = \nabla_\sigma \phi + RT \nabla \ln p_s$$

we will essentially consider the surface pressure, p_s, and features of the temperature, T, and the geopotential, Φ, in the p- and sigma systems.

2.1 Representation of temperature along constant sigma

Consider a temperature distribution that is a function of pressure only

$$(2) \qquad T = T_0 + \sum_{n=1}^{N} \gamma_n z^n$$

$$z = \ln(p/p_0) \; ; \qquad p_0 = 100 \text{ kPa}$$

where T_0 and γ_n are constants. Hence if temperature varies linearly with lnp then N = 1, while the existence of for example a tropopause certainly requires N > 1. Let us now assume that we have a topographic field that simply can be described by one Fourier component for the surface pressure

$$(3) \qquad \ln(p_s/p_0) = z_s = \hat{P}_k (\cos kx - 1)$$

Taking the most straightforward sigma system, $\sigma = p/p_s$ we have

$$(4) \qquad z = \ln \sigma + z_s$$

Then inserting expressions (3) and (4) in (2) we obtain

$$(5) \quad T = T_0 + \sum_{n=1}^{N} \gamma_n \left(\ln \sigma + z_s \right)^n =$$

$$= T_0 + \sum_{n=1}^{N} \gamma_n \left[\ln \sigma + \frac{\hat{p}}{k} \left(\cos kx - 1 \right) \right]^n$$

Expansion (5) shows that the description of T along constant sigma requires not only the basic orographic wave number k but all wave numbers up to Nk. This temperature variation that we observe here is a projection of the vertical variation on a sigma surface because this is inclined relative to the pressure surfaces.

Using relation (5) for the temperature we can easily integrate the hydraostatic equation and hence obtain the geopotential at various sigma levels. We may then calculate the pressure gradient (which is zero in this particular case) in accordance with expression (1) between two grid point values along a sigma surface. We will find that for N = 0 and 1 the exact value of the pressure gradient can be obtained, while for $N > 1$ it is practically inconceivable to obtain the correct value (see Sundqvist 1975).

Note that the exact hydrostatic relation between temperature and geopotential is available in the above demonstration, implying that the truncation error that appears in the pressure gradient is a consequence of the variation of temperature and geopotential along the sigma (horizontal) surfaces. We may thus call this a horizontal truncation error in distinction to the error that will be demonstrated in the next subsection.

2.2 Temperature and geopotential in discrete vertical representation

We shall make the following demonstration concrete by adopting the hydrostatic relation and the algorithm for the pressure gradient of the ECMWF model (Burridge and Haseler 1977) which read respectively (see figure 1 for notations).

$$(6)\quad \phi_{k-\frac{1}{2}} = \phi_s + R \sum_{j=k}^{N} T_j (\Delta \ln \sigma)_j =$$

$$= \phi_{k+\frac{1}{2}} + R\, T_k (\Delta \ln \sigma)_k$$

$$(7)\quad G_k = \frac{1}{2} \nabla \left(\phi_{k-\frac{1}{2}} + \phi_{k+\frac{1}{2}}\right) + R\, T_k\, \nabla \ln p_s$$

The thickness between half integer levels is thus obtained by using the temperature defined at integer levels as the average temperature in the hydrostatic equation and those geopotentials are then used for the evaluation of the pressure gradient at integer levels.

Let us now consider two neighbouring gridponts with identical temperature soundings in the p system (the analysis) with temperature linearly decreasing with lnp up to a tropopause level above which the temperature is constant with height. We assume that the sigma surfaces are distributed and tilted as shown in fig 2. We may furthermore assume that the temperature has been interpolated from p to sigma with a high degree of accuracy so that we have practically the exact temperatures according to the given distribution in this case. Referring to the situation shown in fig 2 we easily realise that the thickness around level k will be obtained with the correct value at point A, whilst the thickness at point B will be underestimated because $T_k(B)$ is lower than the true average temperature for the layer at this point. The ultimate result of this situation is that an erroneous pressure gradient is obtained. It should be noted that, since the geopotential is obtained by a successive summation upwards, the error introduced at level k will be present at all the levels above k.

The error obtained in the pressure gradient in this case is also contributed to by the effects discussed in subsection 2.1, because the lapse rate varies with height ($N > 1$). However, in the specific case regarded in this subsection, the error in the pressure gradient at levels above k will solely be due to the geopotential error introduced at level k as a result of vertical truncation.

It is pertinent to note that the magnitude of the error considered here is a function not only of the vertical resolution but also of the angle between the sigma surfaces and the tropopause (or some other inversion) surface and how this latter one happens to be located relative to the sigma levels.

2.3 Quantitative results of vertical truncation

The assumed situation discussed in subsection 2.2 has been quantified applying the vertical structure of ECMWF's 9-level and 15-level models as well as a 30-level (equi-distant in sigma) model. The slope of the ground was 1800 m in 300 km, a few tropopause levels were adopted (around 26 kPa) and the lapse rate below the tropopause was about the same as in the standard atmosphere.
We shall not here go into individual numerical results but merely summarize the pressure gradient error, expressed in a geostrophic wind at latitude 45^o in the following way. For the three vertical resolutions 9, 15 and 30 levels, the typical error around and above the tropopause level is respectively about 12, 2 and 0.5 ms^{-1}.

It is also of interest to mention that the effects of the horizontal truncation (subsection 2.1) very clearly show up in the 30-level resolution; in fact at the sigma level(s) nearest to the tropopause level the error is about 2 m/sec in this resolution.

3 Some aspects on interpolation of the pressure gradient force

As indicated in the introduction, the truncation effects discussed in the preceeding subsection may be eliminated if the pressure gradient is interpolated from the p system to the sigma system. The initial gradient would then be given with the same accuracy as other individual variables that are interpolated. However, a differential equation then has to be solved in order to obtain the temperature (and subsequently the geopotential) explicitly. This equation is derived in the following way, employing the ECMWF's expressions for the hydrostatic relation and the pressure gradient force. Take the divergence of (7) with expression (6) inserted and equate this with the divergence (along sigma) of the interpolated p-system pressure gradient force. With a slight rearrangement of terms we then obtain

$$(8) \quad R \frac{(\Delta \ln \sigma)_k}{2} \nabla^2 T_k + R \nabla \cdot (T_k \nabla \ln p_s) = \left[\nabla_\sigma \cdot (\nabla_p \phi)\right]_k - \nabla^2 \phi_{k+\frac{1}{2}}$$

where the right hand side is known.

Unfortunately the Helmholtz type equation (8) becomes ill-conditioned when a high vertical resolution is employed, because a resonance phenomenon will appear in regions where the Laplacian of $\ln p_s$ is numerically large and positive, i.e. around the summits of mountains. This difficulty was marginally encountered (for a 5-level model) and discussed in the paper by Sundqvist (1976). A possible, more general application of this approach requires a thorough investigation of the problems indicated above.

4 Normal mode initialisation and the vertical truncation effects

Since the errors emanating from vertical truncation may have appreciable magnitude it is of interest to explore how those are affected by an initialisation procedure that follows after the interpolation from p to sigma. The following qualitative discussion of the role of normal mode initialisation in this context is meant to yield some ideas for quantitative experiments that could provide some insight into the questions.

Let us consider, as we have done in the preceeding sections, an atmosphere free of pressure gradients in the p system analysis but with a tropopause (or some other temperature inversion) along a constant pressure level. Then, after the interpolation to sigma, there exists a non-zero pressure gradient pattern around and above the temperature inversion. This pattern will consist of a wide range of horizontal scales since it has a close resemblance with the orography of the model.

Now, what will the resulting mass and wind fields be after the initialisation has been applied? It is possible that the smaller scales of the non-vanishing pressure gradient will be interpreted as gravity modes and are therefore removed. The large scale pressure gradient possibly contributes to the meteorological modes and consequently an appropriate wind field will be attached to those scales by the initialisation. In that case it might be of interest to perform a time integration in order to study the evolution of this motion field.

The truncation error that has been introduced exists only in a part of the vertical extent of the atmosphere, implying that not only the external mode but also internal ones are involved. It would thus be interesting to find out how the effects of the initialisation are distributed on the various vertical modes. In order to find out if the error becomes suppressed or spread out in the vertical, the pressure gradients existing before and after the initialisation should be compared.

Results from experiments along the above notions ought to provide indications of the magnitude and characteristics of the problem. Although the normal mode initialisation may yield some reduction of those spurious pressure gradients, it seems probable that specific measures are required for achieving satisfactory results. Since a sufficiently high vertical resolution is a remedy, an assessment of the necessary number of levels in this respect is desirable. Regarding the case of coarser vertical resolutions which generally may be expected to give rise to unacceptably large errors, hence methods combined with the initialisation ought to be investigated.

As a principle example, a procedure employing the variational technique (Daley,1978) could be considered. Then in the function to be minimized, the difference between the presure gradient calculated from the sigma system variables and the one obtained from interpolation could be incorporated.

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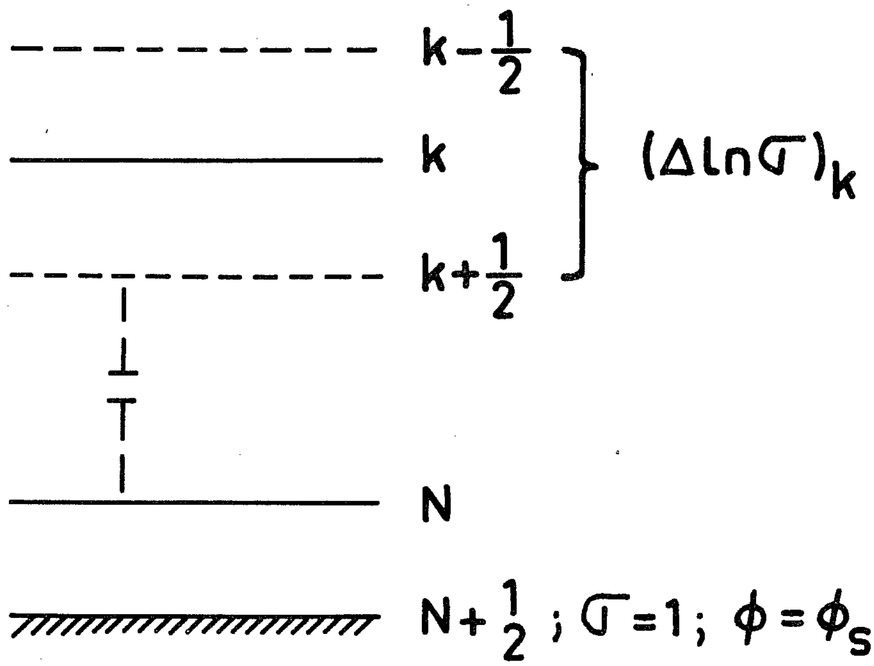

Fig. 1. Simplified display of vertical grid notations of the ECMWF models.

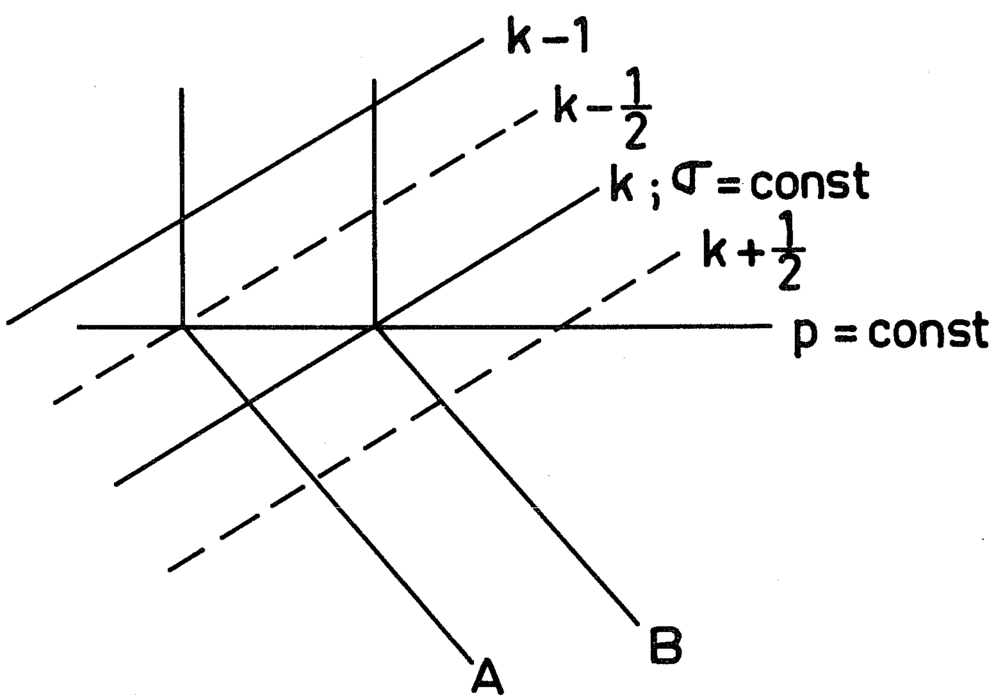

Fig. 2. Sketch showing assumed state for demonstration of the vertical truncation.