TECHNICAL MEMORANDUM

22

Packing and truncation errors in fields retrieved from spherical harmonic coefficients

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In the ECMWF archives, the fields of geopotential, temperature, humidity and u, v and ω wind components are stored as spherical harmonic coefficients, and these are packed 4/60-bit word. The fields are fitted with spherical harmonics up to triangular truncation T80. For the analysis fields, all T80 coefficients are stored in the archives, while for the forecast only the T40 subset of coefficients is stored.

This paper describes a study of the effects of packing and truncation on the accuracy of various fields derived from spherical harmonic coefficients.

1. u-VELOCITY AT 500 MB

1.1 Packing errors

The effect of packing was determined by comparing the maximum absolute error and rms error at each latitude for grid point fields derived from T80 coefficients which had been

(i) packed 4/word

(ii) packed 3/word

(iii) packed 2/word

The grid point field derived from T80 coefficients which had not been packed was taken to be the correct field. (Since the original u grid point field is staggered relative to the derived u field, it is not possible to compare the new and old fields directly to get error estimates).

In the examples below, I will generally give 2 tables; the first showing the worst errors at any latitude, and the second showing the errors at the equator. For all fields selected, the errors were smallest in equatorial regions, while the maximum errors occurred sometimes at high latitudes and sometimes in mid-latitudes. Accordingly, it is easier to get an impression of the relative error levels of the different packing and truncation methods by comparing them in equatorial regions.

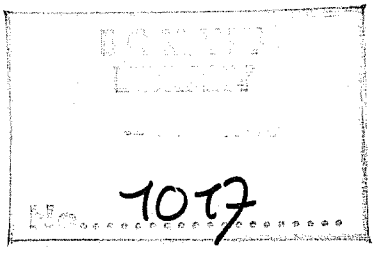

Table 1. Worst errors of 500 mb u due to packing

Packing	Max abs error	Latitude	rms error	Latitude
4/word	10.1131 m/sec	88.125°N	4.5160 m/sec	88.125°N
3/word	0.3039 m/sec	88.125°N	0.1328 m/sec	88.125°N
2/word	0.0002 m/sec	88.125°N	0.0001 m/sec	88.125°N

Table 2. Errors of 500 mb u at equator due to packing

Packing	Max abs error	rms error
4/word	0.0453 m/sec	0.0124 m/sec
3/word	0.0019 m/sec	0.0004 m/sec
3/word	0.0000 m/sec	0.0000 m/sec

The large errors with the 4/word packing are confined to a region close to the north pole. The maximum absolute error is 5 m/sec at 86.25°W, 3 m/sec at 84°N, 2m/sec at 80°N and less than 1 m/sec everywhere south of 73°N.

1.2 Truncation errors

The effects of truncation were investigated by inspecting grid point fields which had been retrieved at different truncations from spectral coefficients which had never been packed. Tables 3 and 4 show the errors for T60/T80 (i.e. the grid point field retrieved at triangular truncation T60 from coefficients which were originally fitted at T80), T40/T80, T40/T40, T30/T30 and R20:40/T40 (where R20:40 is the field retrieved by the summation

$$\sum_{m=0}^{20} \sum_{n=m}^{41}$$

of the product of the spectral coefficients with the Legendre polynomials. This is the form in which the Research Department verification programs retrieve the grid point fields).

Table 3. Worst errors of 500 mb u due to truncation

Truncation	Max abs error	Latitude	rms error	Latitude
T60/T80	5.2695 m/sec	88.125°N	2.4675 m/sec	88.125°N
T40/t80	18.5911 m/sec	88.125°S	10.7790 m/sec	88.125°S
T40/T40	5.6298 m/sec	31.875°N	1.6388 m/sec	88.125°N
T30/T30	7.2268 m/sec	31.875°N	2.2782 m/sec	46.875°N
R20:40/T40	7.1185 m/sec	31.875°N	1.6388 m/sec	88.125°N

Table 4. Errors of 500 mb u at equator due to truncation

Truncation	Max abs error	rms error
T60/T80	0.3530 m/sec	0.1469 m/sec
T40/T80	0.7588 m/sec	0.2671 m/sec
T40/T40	0.8762 m/sec	0.3134 m/sec
T30/T30	1.6622 m/sec	0.6574 m/sec
R20:40/T40	1.2136 m/sec	0.4177 m/sec

For all fields except velocities, the fields retrieved at truncation T40/T40 were identical to those for T40/T80. The u velocity however showed significant errors in polar regions when retrieved at a truncation other than that originally fitted.

1.3 Divergence and vorticity

The errors caused by packing the coefficients of u were significantly reduced by storing instead the coefficients of divergence and vorticity. u and v coefficients can be derived from vorticity and divergence coefficients using the relations

$$U_{mn} = a\left\{-\frac{1}{n}E_{mn}Z_{m,n-1} - \frac{im}{n(n+1)}D_{mn} + \frac{1}{n+1}E_{m,n+1}Z_{m,n+1}\right\}$$

$$V_{mn} = a\left\{\frac{1}{n}E_{mn}D_{m,n-1} - \frac{im}{n(n+1)}Z_{mn} - \frac{1}{n+1}E_{m,n+1}D_{m,n+1}\right\}$$

where U_{mn} = coefficients of $u\cos\phi$

V_{mn} = coefficients of $v\cos\phi$

Z_{mn} = coefficients of vorticity

D_{mn} = coefficients of divergence

$$E_{mn} = \left\{\frac{n^2 - m^2}{4n^2 - 1}\right\}^{\frac{1}{2}}$$

and a = radius of earth.

Tables 5 and 6 show the errors due to packing for u fields retrieved at T80 truncation from T80 fitted divergence and vorticity coefficients. They are much smaller than the corresponding values in Tables 1 and 2.

Table 5. Worst errors due to packing in 500 mb u derived from vorticity and divergence

Packing	Max abs error	Latitude	rms error	Latitude
4/word	0.0187 m/sec	88.125°N	0.0116 m/sec	88.125°N
3/word	0.0005 m/sec	88.125°N	0.0004 m/sec	88.125°N

Table 6. Errors at equator due to packing in 500 mb u derived from vorticity and divergence

Packing	Max abs error	rms error
4/word	0.0046 m/sec	0.0019 m/sec
3/word	0.0001 m/sec	0.0000 m/sec

However, the errors caused by retrieving the u field at a lower truncation than that used to fit the divergence and vorticity were identical to those shown in Tables 3 and 4.

2. v-VELOCITY AT 500 MB

2.1 Packing errors

The errors due to packing were investigated by comparing v grid point fields retrieved at triangular truncation T80 from coefficients which had been packed at differing densities with the grid point field retrieved at T80 from coefficients which had not been packed. (Again, the derived v grid point field is staggered relative to the original v field, so it is not possible to compare the new and old fields directly.)

Table 7. Worst errors of 500 mb v due to packing

Packing	Max abs error	Latitude	rms error	Latitude
4/word	2.5045 m/sec	88.125°N	1.1185 m/sec	88.125°N
3/word	0.779 m/sec	88.125°N	0.0366 m/sec	88.125°N
2/word	0.0001 m/sec	88.125°N	0.0000 m/sec	

Table 8. Errors of 500 mb v at equator due to packing

Packing	Max abs error	rms error
4/word	0.0104 m/sec	0.0032 m/sec
3/word	0.0003 m/sec	0.0001 m/sec
2/word	0.0000 m/sec	0.0000 m/sec

The significant errors with the 4/word packing are confined to a region close to the north pole. The maximum absolute error is 1.2 m/sec at 88.125°N, and less than .5 m/sec at all latitudes south of 80°N.

2.2 Truncation errors

The errors due to truncation were inspected for grid point fields retrieved at truncations T40/T80 and T40/T40 from coefficients which had not been packed.

Table 9. Worst errors in 500 mb v due to truncation

Truncation	Max abs error	Latitude	rms error	Latitude
T40/T80	4.5137 m/sec	50.625°N	1.2658 m/sec	88.125°N
T40/T40	5.0000 m/sec	50.625°N	1.5353 m/sec	88.125°N

Table 10. Errors in 500 mb v at equator due to truncation

Truncation	Max abs error	rms error
T40/T80	.7344 m/sec	.1954 m/sec
T40/T40	.7834 m/sec	.2217 m/sec

The errors for T40/T80 are almost everywhere slightly smaller than those for T40/T40 (which was not the case for u-velocities). Also the error due to truncating to T40 is everywhere greater than the error due to packing.

2.3 Divergence and vorticity

The errors due to packing were reduced by deriving the v grid point field from coefficients of divergence and vorticity.

Table 11. Worst errors due to packing in 500 mb v derived from divergence and vorticity

Packing	Max abs error	Latitude	rms error	Latitude
4/word	.0378 m/sec	75°N	.0141 m/sec	88.125°N
3/word	.0012 m/sec	78.75°N	.0005 m/sec	88.125°N
2/word	.0000 m/sec		.0000 m/sec	

Table 12. Errors at equator due to packing in 500 mb v derived from vorticity and divergence

Packing	Max abs error	rms error
4/word	0.0081 m/sec	0.0018 m/sec
3/word	0.0003 m/sec	0.0001 m/sec
2/word	0.0000 m/sec	0.0000 m/sec

The errors due to truncation were identical to those for grid point fields derived from v coefficients.

3. VERTICAL VELOCITY AT 500 MB

3.1 Packing errors

Fields retrieved at triangular truncation T80 from coefficients packed at different densities were compared with the original vertical velocity grid point field. There was no detectable effect due to packing.

Table 13. Worst errors of vertical velocity due to packing

Packing	Max abs error	Latitude	rms error	Latitude
4/word 3/word 2/word 1/word	.0656 pascal/sec	88.125°S	.0382 pascal/sec	88.125°S

Table 14. Errors of vertical velocity at equator

Packing	Max abs error	rms error
4/word 3/word 2/word 1/word	.0069 pascal/sec	.0021 pascal/sec

3.2 Truncation errors

Fields retrieved at truncations T80/T80, T40/T80 and T40/T40 were compared with the original vertical velocity field.

Table 15. Worst errors in 500 mb ω due to truncation

Truncation	Max abs error	Latitude	rms error	Latitude
T80/T80	.0656 pascal/sec	88.125°S	.0382 pascal/sec	88.125°S
T40/T80	.8077 pascal/sec	39.375°N	.1216 pascal/sec	39.375°N
T40/T40	.8077 pascal/sec	39.375°N	.1216 pascal/sec	39.375°N

Table 16. Errors of 500 mb ω at equator due to truncation

Truncation	Max abs error	rms error
T80/T80	.0069 pascal/sec	.0021 pascal/sec
T40/T80	.0978 pascal/sec	.0213 pascal/sec
T40/T40	.0978 pascal/sec	.0213 pascal/sec

The field derived at truncation T40/T80 is identical to that derived at T40/T40. The errors in the vertical velocity field are due only to truncation, not packing.

4. TEMPERATURE AT 500 MB

4.1 Packing errors

Fields retrieved at triangular truncation T80 from coefficients packed at different densities were compared with the original temperature grid point field.

Table 17. Worst errors in 500 mb T due to packing

Packing	Max abs error	Latitude	rms error	Latitude
4/word	.8086°K	88.125°S	.2829°K	88.125°S
3/word	.8341°K	88.125°S	.2943°K	88.125°S
2/word	.8324°K	88.125°S	.2936°K	88.125°S
1/word	.8324°K	88.125°S	.2936°K	88.125°S

Table 18. Errors of 500 mb T at equator due to packing

Packing	Max abs error	rms error
4/word	.2097°K	.0595°K
3/word	.1796°K	.0531°K
2/word	.1794°K	.0531°K
1/word	.1794°K	.0531°K

4.2 Truncation errors

Fields retrieved at truncations T80/T80, T40/T80 and T40/T40 were compared with the original temperature grid point field.

Table 19. Worst errors in 500 mb T due to truncation

Truncation	Max abs error	Latitude	rms error	Latitude
T80/T80	.8324°K	88.125°S	.2936°K	88.125°S
T40/T80	1.5736°K	73.125°S	.6653°K	86.25°S
T40/T40	1.5736°K	73.125°S	.6653°K	86.25°S

Table 20. Errors of 500 mb T at equator due to truncation

Truncation	Max abs error	rms error
T80/T80	.1794°K	.0531°K
T40/T80	.5586°K	.1376°K
T40/T40	.5586°K	.1376°K

The errors in the temperature field could be reduced by packing the coefficients 3/word instead of 4/word, but the error due to packing is significantly less than that due to truncating to T40.

5. GEOPOTENTIAL AT 500 MB

5.1 Packing errors

Fields retrieved at triangular truncation T80 from coefficients packed at different densitities were compared with the original geopotential field. In the tables below, the geopotential is given in units of metres * g, where g = acceleration due to gravity.

Table 21. Worst errors of 500 mb geopotential due to packing

Packing	Max abs error	Latitude	rms error	Latitude
4/word	172.6140 m*g	82.5°N	72.5795 m*g	88.125°N
3/word	36.2455 m*g	67.5°N	7.7479 m*g	88.125°S
2/word	36.3362 m*g	67.5°N	7.8060 m*g	88.125°S
1/word	36.3364 m*g	67.5°N	7.8061 m*g	88.125°S

Table 22. Errors of 500 mb geopotential at equator due to packing

Packing	Max abs error	rms error
4/word	28.5584 m*g	6.4710 m*g
3/word	5.9288 m*g	1.7407 m*g
2/word	5.9802 m*g	1.7194 m*g
1/word	5.9800 m*g	1.7194 m*g

5.2 Truncation errors

Fields retrieved at T80/T80, T40/T80 and T40/T40 from coefficients which had not been packed were compared with the original grid point field.

Table 23. Worst errors in 500 mb geopotential due to truncation

Truncation	Max abs error	Latitude	rms error	Latitude
T80/T80	36.3364 m*g	67.5°N	7.8061 m*g	88.125°S
T40/T80	130.0720 m*g	39.375°N	34.7853 m*g	88.125°N
T40/T80	130.0720 m*g	39.375°N	34.7853 m*g	88.125°N

Table 24. Errors of 500 mb geopotential at equator due to truncation

Truncation	Max abs error	rms error
T80/T80	5.9800 m*g	1.7194 m*g
T40/T80 } T40/T40	18.5789*g	6.7430 m*g

Obviously there is a serious error due to packing for the geopotential fields, which would be removed if the coefficients were packed 3/word instead of 4/word.

It was found that, apart from the region to the north of 80°N, the errors for the geopotential field retrieved at T40 from coefficients which had been packed 4/word were not significantly greater than those for the field retrieved at T40 from coefficients which had not been packed. Also, the field retrieved at T40 from coefficients packed 4/word had smaller errors than the field retrieved at T80 throughout the northern hemisphere. (This was not true for the southern hemisphere, where perhaps the sign changes in the Legendre polynomials prevented the error due to packing from accumulating).

6. HUMIDITY MIXING RATIO AT 850 MB

6.1 Packing errors

There were no detectable errors due to packing for humidity mixing ratio.

6.2 Truncation errors

Fields retrieved at T80/T80 and T40/T80 were compared with the original humidity mixing ratio field.

Table 25. Errors of 850 mb hmr due to truncation

Truncation	Max abs error	Latitude	rms error	Latitude
T80/T80	.0025 g/g	9.375°S	.0003 g/g	(many)
T40/T80	.0142 g/g	13.12°S	.0052 g/g	20.625°N

Table 26. Errors at equator of 850 mb hmr due to truncation

Truncation	Max abs error	rms error
T80/T80	.0018 g/g	.0002 g/g
T40/T80	.0085 g/g	.0032 g/g

7. RELATIVE HUMIDITY AT 850 MB

7.1 Packing errors

Fields retrieved at T80 from coefficients packed at different densities were compared with the original relative humidity grid point field.

Table 27. Worst errors of 850 mb relative humidity due to packing

Packing	Max abs error	Latitude	rms error	Latitude
4/word	22.3699	73.125°S	4.4828	73.125°S
3/word	22.3639	73.125°S	4.4858	73.125°S
2/word	22.3645	73.125°S	4.4860	73.125°S
1/word	22.3645	73.125°S	4.4860	73.125°S

Table 28. Errors of 850 mb relative humidity at equator due to packing

Packing	Max abs error	rms error
4/word	8.2150	1.1590
3/word	8.2176	1.1573
2/word	8.2171	1.1573
1/word	8.2171	1.1573

7.2 Truncation errors

Fields retrieved at T80/T80 and T40/T80 were compared with the original relative humidity field.

Table 29. Worst errors of 850 mb relative humidity due to truncation

Truncation	Max abs error	Latitude	rms error	Latitude
T80/T80	22.3645	73.125°S	4.4860	73.125°S
T40/T80	45.5892	50.625°N	12.2903	69.375°S

Table 30. Errors of 850 mb relative humidity at equator due to truncation

Truncation	Max abs error	rms error
T80/T80	8.2171	1.1573
T40/T80	25.6338	4.2226

8. MEAN SEA LEVEL PRESSURE

8.1 Packing errors

Fields retrieved at T80 from coefficients packed at different densities were compared with the original mean sea level pressure field.

Table 31. Worst errors in msl pressure due to packing

Packing	Max abs error	Latitude	rms error	Latitude
4/word	229.8675 pascals	88.125°S	84.8228 pascals	88.125°S
3/word	230.8989 pascals	88.125°S	85.1267 pascals	88.125°S
2/word	230.9115 pascals	88.125°S	85.1321 pascals	88.125°S
1/word	230.9115 pascals	88.125°S	85.1322 pascals	88.125°S

Table 32. Errors in msl pressure at equator due to packing

Packing	Max abs error	rms error
4/word	24.1993 pascals	6.0168 pascals
3/word	22.3495 pascals	5.6390 pascals
2/word	22.4240 pascals	5.6356 pascals
1/word	22.4239 pascals	5.6356 pascals

8.2 Truncation errors

Fields retrieved at T80/T80 and T40/T80 were compared with the original mean sea level pressure field.

Table 33. Worst errors in mean sea level pressure due to truncation

Truncation	Max abs error	Latitude	rms error	Latitude
T80/T80	230.9115 pascals	88.125°S	85.1322 pascals	88.125°S
T40/T80	727.3495 pascals	76.875°S	210.1279 pascals	86.25°S

Table 34. Errors in msl pressure at equator due to truncation

Truncation	Max abs error	rms error
T80/T80	22.4239 pascals	5.6356 pascals
T40/T80	56.9384 pascals	16.9844 pascals

9. MAPS

Figs. 1 to 6 show plots of 1000 mb geopotential and 850 mb temperature for the original fields, T80-smoothed fields and T40-smoothed fields. Over most of the northern hemisphere, all 3 plots were very similar, however T40 underestimated the depth of the low at 40N 150E, and greatly smoothed the features over Greenland. By removing the small scale noise, the T40 fit seems to improve the look of the temperature field.

10. CONCLUSIONS

Packing the spherical harmonic coefficients 4/word leads to significant errors for the geopotential and u-velocity fields. These errors however can be greatly reduced by packing the coefficients 3/word. Deriving the u-velocity from packed vorticity and divergence coefficients further reduces errors.

In general, the errors due to truncating the coefficients to T40, or lower truncations, are much larger than those due to packing the coefficients. However, to archive forecast fields at T80 rather than T40 would require almost four times more coefficients to be stored.

At present, the forecast fields are fitted with spherical harmonics at T80 truncation, but only the T40 subset of coefficients is archived. For all but velocity fields, the T40 fitted field and the field derived from the T40 subset of T80-fitted coefficients are identical. For u-velocities a significant error is introduced in polar regions by retrieving the fields at a truncation other than the one originally fitted. For T40 archives, it would be better to fit the

u-velocity at T40 in the first place. (It would *not* be simple to make the post-processing fit some fields at T40 and others at T80).

If the archives held T80 truncation coefficients, the u-velocity problem would be solved. The user could then retrieve fields either at T80, or at a lower truncation, depending on the degree of smoothing and level of truncation error he is prepared to accept.

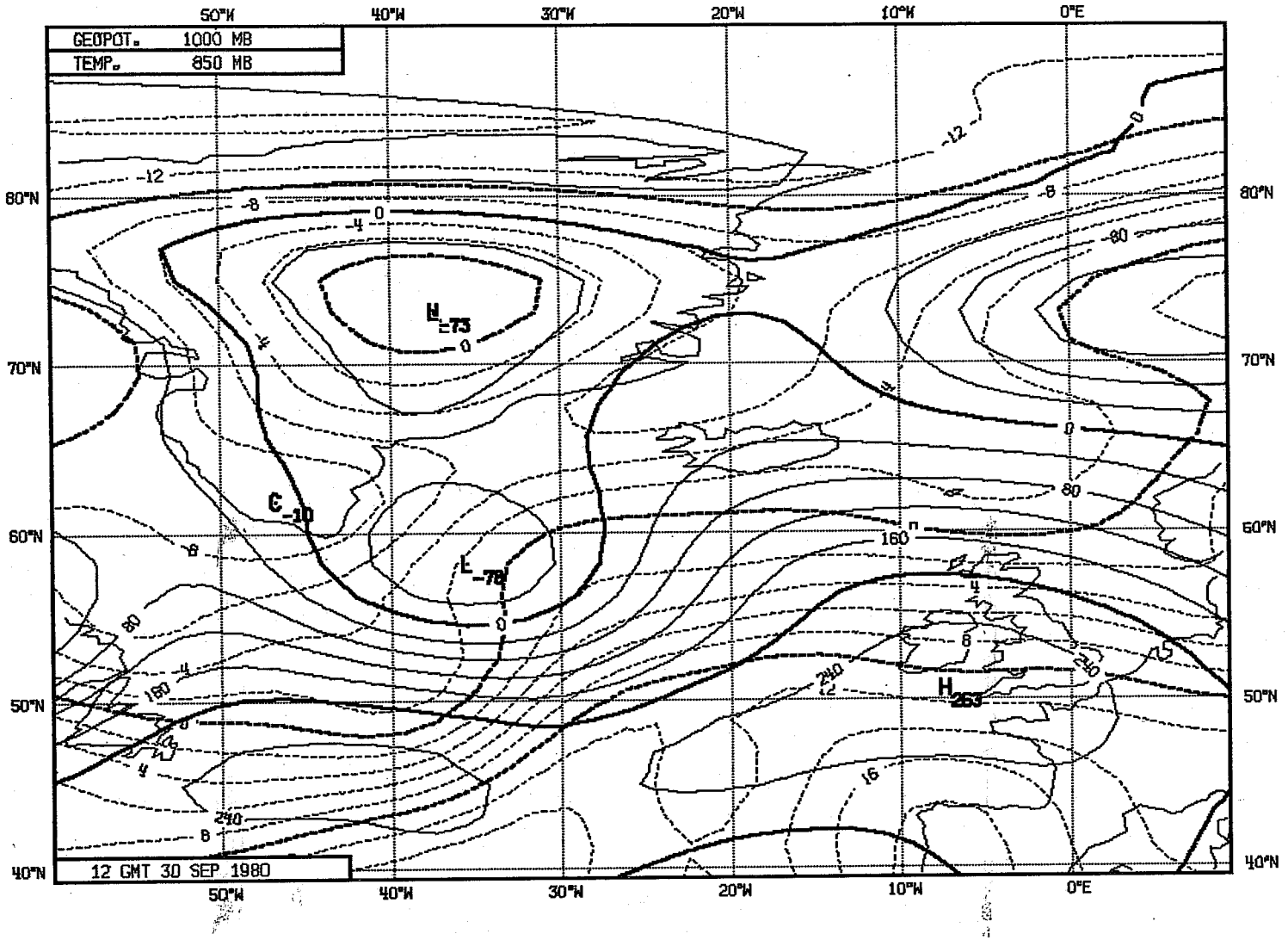

Fig. 1 1000 mb geopotential and 850 mb temperature smoothed to truncation T40

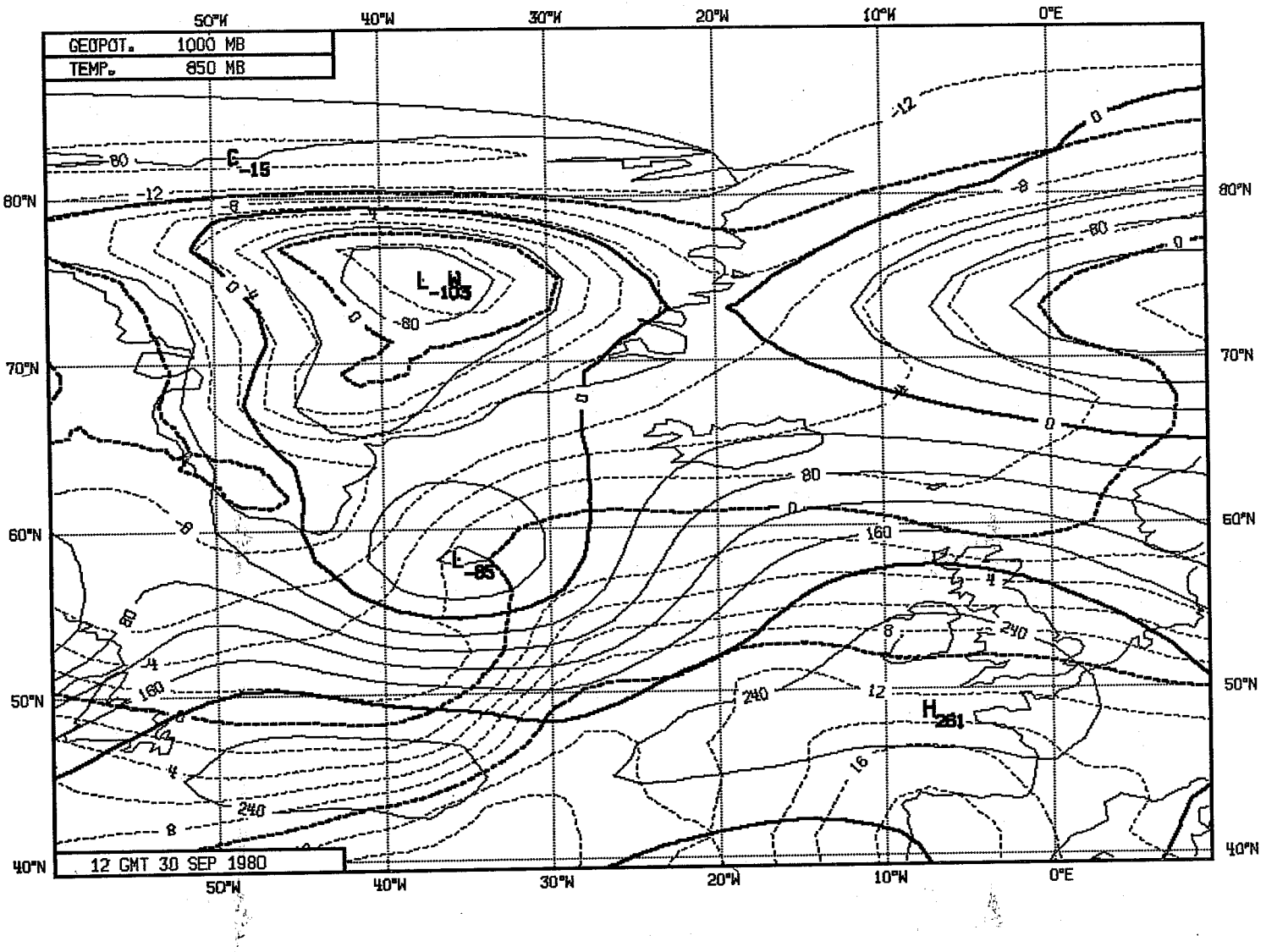

Fig. 2 1000 mb geopotential and 850 mb temperature smoothed to truncation T80

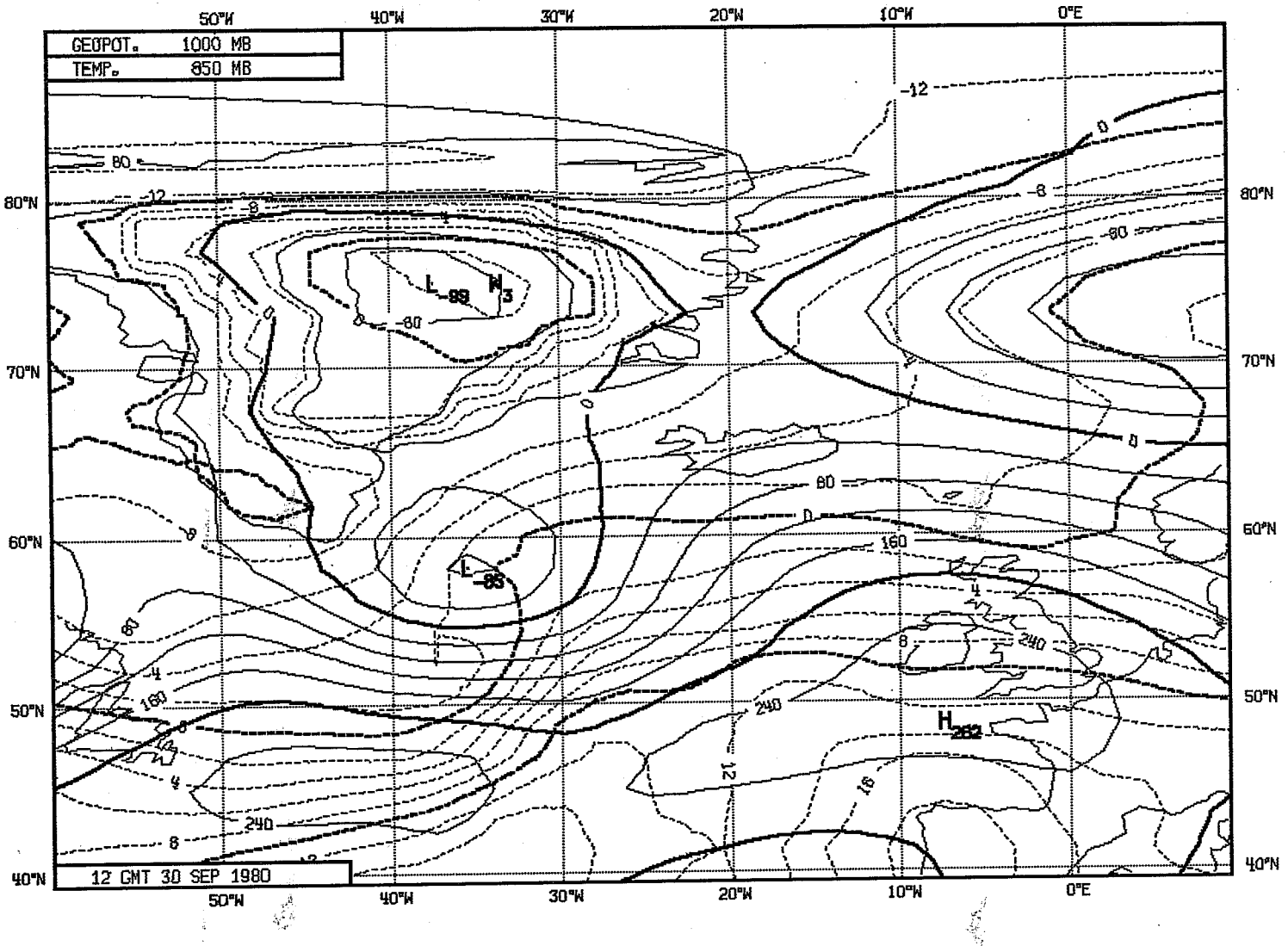

Fig. 3 Original unsmoothed 1000 mb geopotential and 850 mb temperature fields

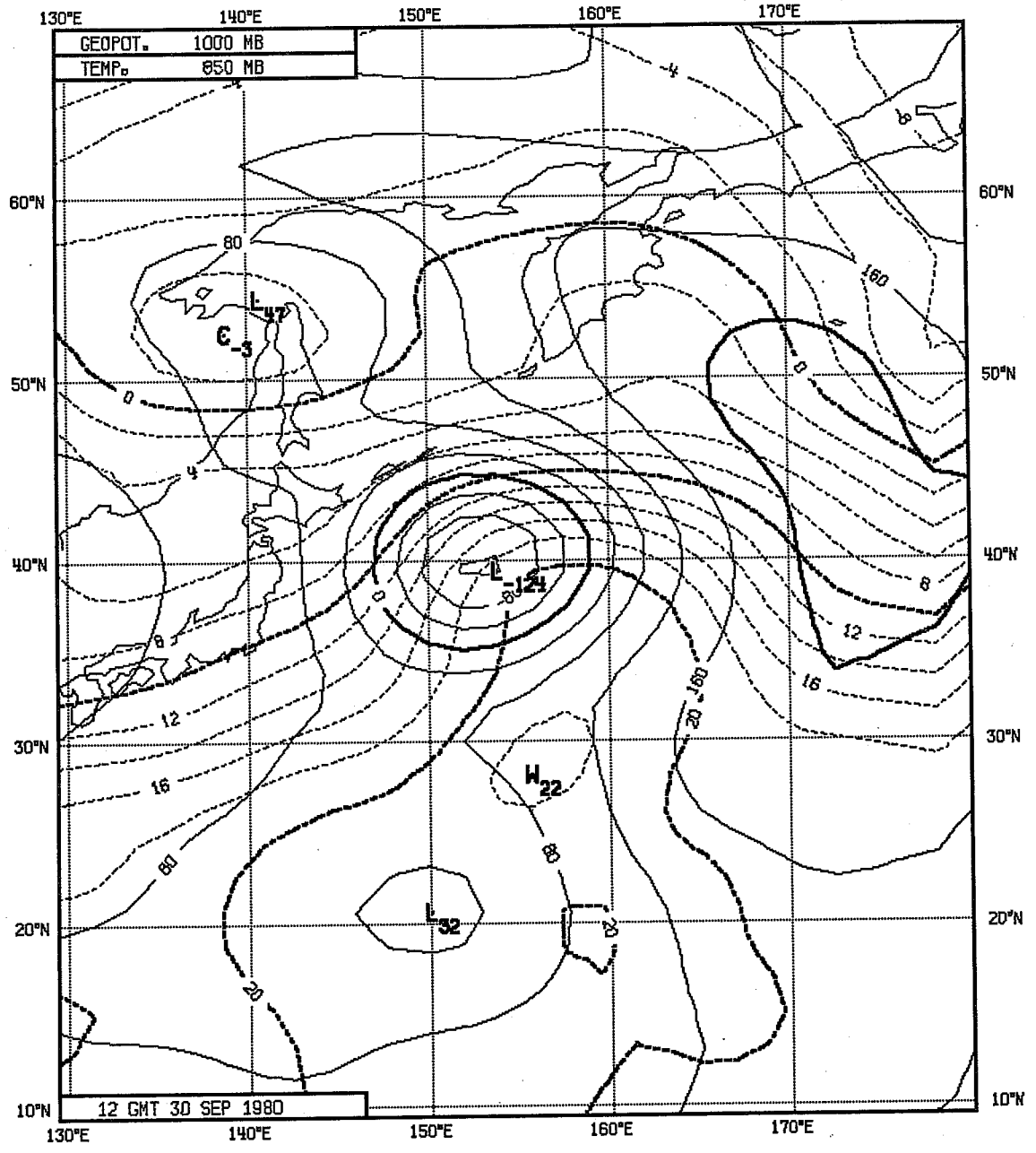

Fig. 4 1000 mb geopotential and 850 mb temperature smoothed to truncation T40

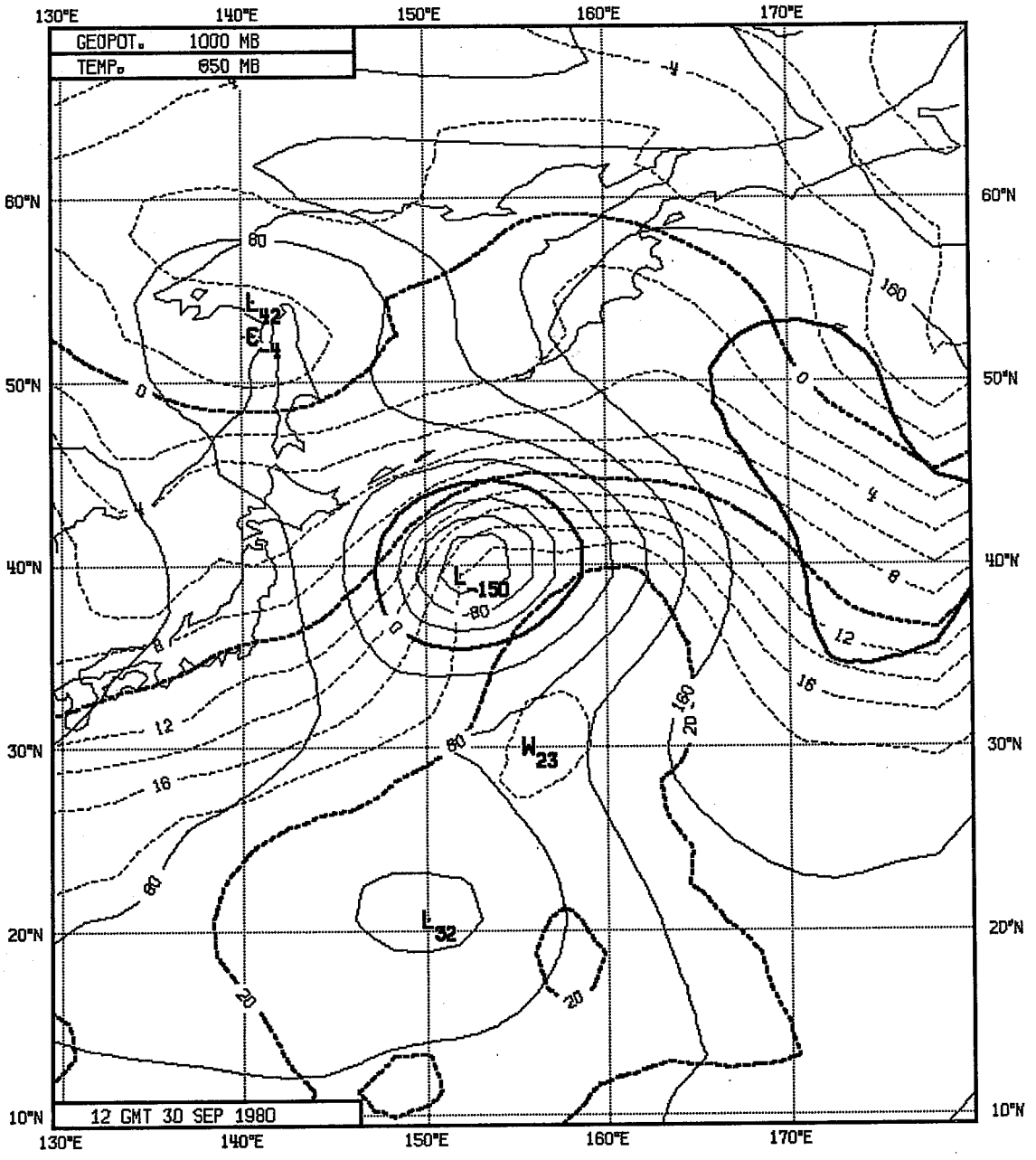

Fig. 5 1000 mb geopotential and 850 mb temperature smoothed to truncation T80

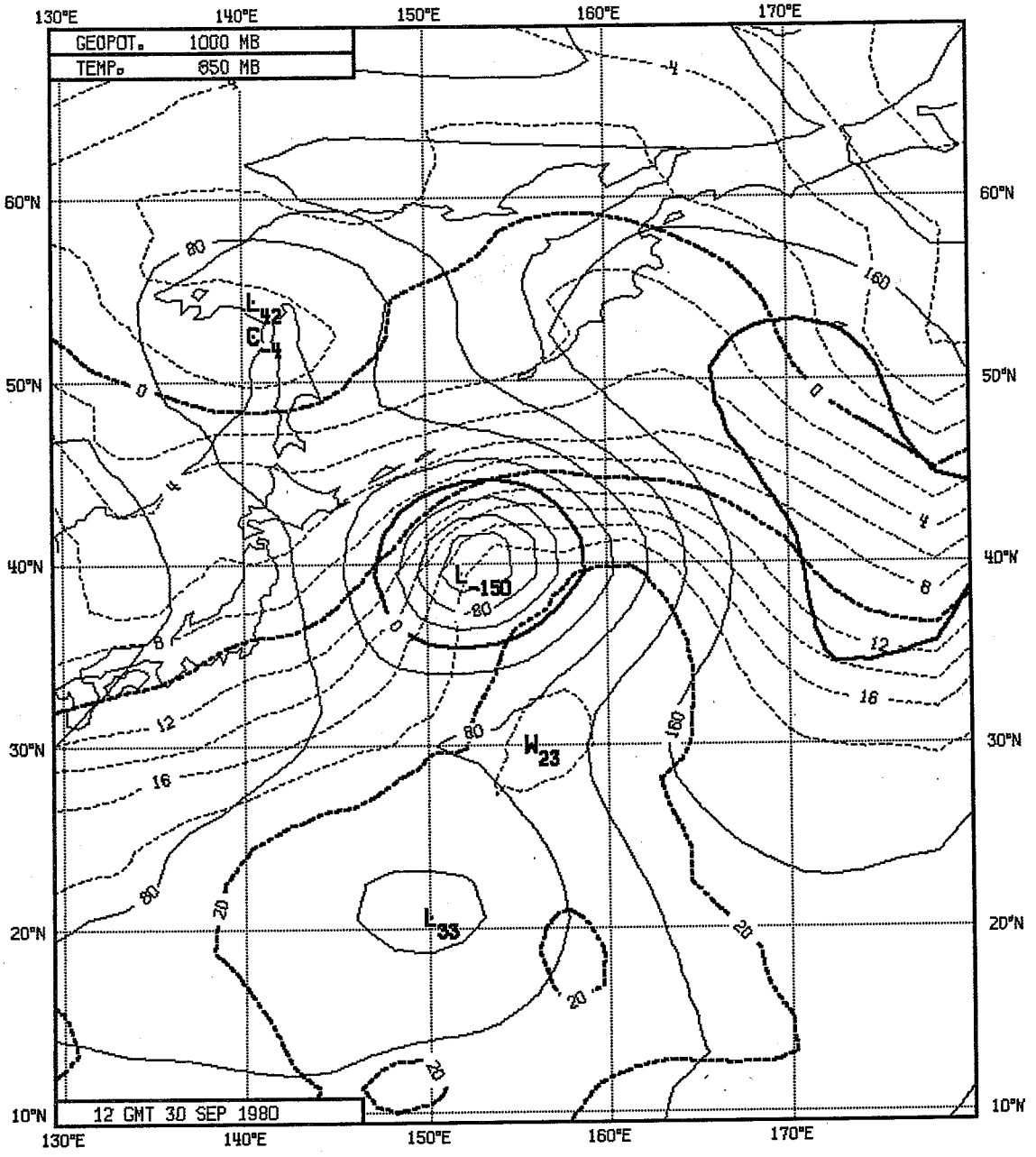

Fig. 6 Original unsmoothed 1000 mb geopotential and 850 mb temperature fields