PARAMETERIZATION SCHEMES FOR THE

PLANETARY BOUNDARY LAYER

A BRIEF REVIEW AND SOME GENERAL REMARKS

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1. INTRODUCTION

The Planetary Boundary Layer (PBL) is usually defined as the lowest part of the atmospheric flow where thermal and dynamical influences of the underlying surface (water or land) can be felt. The depth of the PBL varies by more than an order of magnitude between extreme conditions, its average value being around 1 km. The main physical feature of the PBL, which is also very often used as an alternative definition, is that it is continuously turbulent, due to the many instabilities which are induced by shear or convection close to the surface. In this short review paper, we shall mainly concentrate on the turbulent PBL and on the parameterization of the turbulent fluxes which characterize it. In the conclusion, we shall only briefly give some indications about the other physical mechanisms which can also account for the interaction between the earth's surface and the atmospheric flow.

The large scale atmospheric flow determines, for a significant part, the properties of the PBL. For given surface conditions it is for example possible to describe the micro-structure of the PBL by using upper boundary conditions and pressure field deduced from larger scales. This kind of approach has been intensively studied during the last ten years. Driedonks and Tennekes (1981) call it "passive", in the sense that in such a method the feedback of the PBL on the large scale flow is not taken into account. It should nevertheless be said that this approach has given rise to many useful concepts and has lead to the development of many detailed models (see e.g. Zeman, 1981, for review). Although many of those are time-consuming, the present perspective for powerful computers makes it possible to consider the use of such models for parameterization purposes.

This parameterization problem is unfortunately very often considered from a very restrictive, and possibly inadequate, point of view : Given a large scale flow as described for example by a numerical model, what are the minimum PBL constraints which one has to introduce in order to further integrate the large scale model ? Since the overall energetic budget must be balanced, one would then be inclined to answer that a crude description of sources (sensible and latent heat fluxes) and sinks (dissipation of kinetic energy through drag and momentum fluxes) may be enough. Although this approach may be called "interactive" (Driedonks and Tennekes, 1981), it is clear that it does not always lead to satisfactory answers.

This latter method has been used in the past mainly for two reasons : time-saving and the idea that global energetic balance would be enough to insure proper time evolution. This last concept must be studied more carefully. If it is in fact reasonable to think that PBL sources and sinks are really important only on the long term, it should be recognized that this can only be true for the larger scales of motion, above 2000 - 5000 km, where baroclinic instability feeds in most of the kinetic energy. Smaller scales are much more dependent on the detailed structure of the PBL, specially in the meso-scale range ($\sim$10-1000 km) where it is now well known that a detailed representation of PBL processes is required even for short term evolution. The present development of numerical prediction methods toward medium range, and also now-cast, by using improved spatial resolution makes it necessary to reconsider the parameterization problem along these lines. This will very likely reduce the gap between the techniques presently available for PBL modelling and those used for parameterization purposes.

The large scale flow as well as the PBL have the same lower boundary, i.e. the earth's surface. Part of the parameterization problem is then to control and/or predict the surface characteristics like its temperature and humidity as well as its roughness. We shall more or less assume here that these properties can be known, for example from the solution a surface energy budget (see section 2 for some details). The general idea on which are based the parameterization techniques is that it is possible to deduce the turbulent fluxes at the surface (momentum and heat) from the knowledge of the surface characteristics and flow properties, i.e. that the turbulence is approximately in stationary equilibrium. It should be noted here that it does not make any difference if one has to evaluate the surface fluxes for use in either a large scale model or a PBL model. The only essential feature is the lowest altitude at which the flow properties are known (or, more exactely, computed !). Depending upon this altitude, various techniques can be used, which will be presented in Section 2.

Once the surface fluxes have been determined, it still remains necessary to gain some information about the way they will be vertically distributed. The meteorological variables respond indeed to the flux divergence and not to the flux itself. The altitude chosen for the lowest levels in the large-scale model is then obviously of prime importance and, depending upon it, different solutions will be possible to vertically distribute the turbulent drag and forcing (see Section 3).

The above strategy to solve the parameterization problem has one merit : It can be put into action ! It nevertheless remains to ask one very important question : Is it possible to really infer the influence of the PBL from the knowledge of only large scale external properties ? This question can be addressed from the "spatial" point of view : Given a distribution of surface and atmospheric properties inside a 300x300 km^2 (i.e. the grid size), are the turbulent fluxes averaged over this area equal to those which are determined directly from the averaged external properties ? The same kind of question can be asked from the "temporal" point of view : Given the 24 hour variation of the PBL structure, can its time-averaged effect on the large scale flow be determined only from the time-averaged PBL structure ? There are presently no satisfactory and general answers to these questions. We shall nevertheless see below that it is very unlikely that the parameterization problem is "linear", and this should be kept in mind when trying to assess the validity of a parameterization technique for a given spatial and temporal scale.

2. DETERMINATION OF THE SURFACE FLUXES

As stated above it is first necessary to know the surface characteristics before one can determine the surface fluxes. Among those, the roughness length z_o is very often supposed to be a physical property of the underlying surface. It is for example possible to find in the litterature tables giving values of z_o depending on the type of surface. A well known exception concerns the sea where the roughness length depends on the wind velocity. In such a case the Charnock's (1955) formula

$$z_o = M \quad u_*^2/g \qquad ; \qquad M \sim 0.035 - 0.050 \, , \tag{1}$$

where $u_* = (-\overline{u'w'}_{sfc})^{1/2}$ is the friction velocity, is of rather general use. It should nevertheless be underlined that the roughness length is not a truly physical quantity, but just a mathematical constant introduced when on integrates vertically the universal law of the wall

$$\frac{\partial \bar{u}}{\partial z} = \frac{u_*}{kz} \qquad ; \qquad \bar{u}(z) = \frac{u_*}{k} \, Ln\left(\frac{z}{z_o}\right) \tag{2}$$

where $\bar{u}$ is the mean velocity and k the von Karman's constant. Nothing states than Z_o is not a function of external parameters like u_* and we shall indeed show below (see appendix) that z_o is a rather arbitrary parameter. Nobody should care too much to use very large values of z_o if this turns out to be necessary for a particular large scale model, even if this has to be done at the expense of physical insight, like in the case of z_o exceeding a few meters.

2.1. Surface temperature and humidity

When the underlying surface is the sea, it is most of the time possible to prescribe its temperature T_s and the humidity just above as

$$q_s = q_{sat}(T_s) \tag{3}$$

Such a procedure does not allow obviously for coupling of the atmosphere with the ocean, but this could be of little importance as long as one is not interested with climatic studies.

The land surface temperature and humidity are more difficult to determine. One usually relies on the solution of the surface energy balance

$$R_N = H_S + H_L + H_{GO} \tag{4}$$

where R_N is the net radiative flux reaching the surface and H_S, H_L and H_{GO} are respectively the sensible and latent heat flux in the atmosphere and H_{GO} the heat flux penetrating in the ground. The surface temperature T_S enters the radiative flux, through the long-wave emission $\varepsilon_g \sigma T_S^4$ leaving the surface, and the atmospheric heat fluxes H_S and H_L, see below. If the other terms in (4) are known, it is then possible to compute T_S. Such a method is nevertheless difficult to apply, mainly because the ground heat flux H_{GO} has to be determined through the (expensive) solution of the conductivity equation

$$\rho_G c_G \frac{\partial T_G}{\partial t}(z,t) = -\frac{\partial}{\partial z} H_G(z,t) \tag{5a}$$

$$H_G(z,t) = -\mu_G \frac{\partial T_G}{\partial z}(z,t) \tag{5b}$$

where the heat capacity $\rho_G c_G$ and heat diffusivity μ_G of the soil have to be known in order to compute the temperature T_G and heat flux H_G in the ground in term of which H_{GO} is given by

$$H_{G0}(t) = H_G(0,t) = -\mu_G \frac{\partial T_G}{\partial z}(0,t) \tag{6}$$

It appears to be necessary to consider at least 6 levels in the ground to obtain an accurate solution of (5).

In order to avoid such complexities, Bhumralkar (1975) proposed to integrate vertically (5a) across a small thickness δ just below the surface and to use for H_G (δ, t) the exact solution corresponding to the 24 hour oscillation. When δ goes to zero it comes

$$\frac{\partial T_s}{\partial t} = -\frac{2}{\rho_G c_G}\left(\frac{\pi}{\nu_G \tau_1}\right)^{1/2}\left[R_N - H_s - H_L\right] - \frac{2\pi}{\tau_1}\left(T_s - \overline{T}_G\right) \tag{7}$$

where $\nu_G = \mu_G/\rho_G c_G$, $\tau_1 = 24\,h$, and where $\overline{T}_G$ is the mean soil temperature equal, for example, to the ground temperature averaged over the previous 24 hour.

The humidity at the surface may be determined by simply assuming it is in a constant ratio with $q_{sat}(T_s)$ or by solving the water balance in one or more layers of soil (e.g. Deardorff, 1977). We shall not go into further details for this problem since the hydrological budget is still a matter of intense research. The reader is nevertheless referred to recent reviews by De Moor et al. (1979), Carson (1981) and Dooge (1981).

From now on we shall assume that the surface properties are known and we shall turn to the problem of computing the turbulent surface fluxes.

2.2. The Bulk transfer coefficients

The turbulent surface flux ($F_x = \overline{w'x'}_{sfc}$) of a conservative quantity x can be determined from the difference between $\overline{x}$ (z_1) a given level z_1 (the lowest level available in the large scale model) and $\overline{x}_{sfc}$ at the surface, by using a "drag" or "bulk transfer" coefficient C_x, i.e.

$$F_x = \overline{w'x'}_{sfc} = -C_x \,|\underline{V}|(z_\ell)\,[\bar{x}(z_\ell) - \bar{x}_{sfc}] \tag{8}$$

where $|\underline{V}|$ (z_1) is the wind velocity at the lowest level z_1. Eq. (8) is no more than a mathematical definition of C_x, although it bears resemblances with the well-known Boussinesq or Prandtl gradient-diffusion-type approximation. The question is now of course to estimate C_x, possibly as a function of z_1, atmospheric stability, roughness length. It is useful to specialize Eq. (8) to the three quantities of interest for meteorological purposes

$$\overline{V'w'}_{sfc} = -u_*^2 = -C_D \, V^2(z_\ell) \tag{9}$$

$$\overline{w'\theta'}_{sfc} = \frac{H_s}{\rho C_p} = Q_o = -u_*\theta_* = -C_H\,|\underline{V}|(z_\ell)\,[\bar{\theta}(z_\ell) - T_s] \tag{10}$$

$$\overline{w'q'}_{sfc} = \frac{H_L}{\rho L} = E_o = -u_* q_* = -C_q\,|\underline{V}|(z_\ell)\,[\bar{q}(z_\ell) - q_s] \tag{11}$$

where ρ is the mean density of air at the surface, C_p its heat capacity at constant pressure and L the latent heat of condensation. Eq. (9) does not give any information about the direction of the surface stress. It must thus be completed by the statement that $\overline{V'w'}_{sfc}$ is oriented along the wind direction close to the surface, i.e. in the present case at level z_1.

The simplest way to use (9) - (11) is to prescribe the value of the bulk transfer coefficients C_D, C_H and C_q. Many choices can be made, some of which are reviewed by Carson (1981). Let's simply say that the crudest, but still satisfactory, choice seems to be

$$C_D = C_H = C_Q \tag{12}$$

with C_D of the order of 3-5 10^{-3} over land and 1-2 10^{-3} over sea. Some attempts have been made to include the effect of surface topography (Gates et al., 1971), or atmospheric stability (Arakawa, 1972) on a rather empirical basis. We shall not comment these proposals but turn now to the similarity theories of the PBL, which provide an adequate framework to deal with these problems.

Two possibilities have to be considered, depending on the altitude of the lowest level z_1. If z_1 is of the order of a few tens of meter, it can be assumed to be within the so-called "surface", or "logarithmic", or "constant flux" layer, in which case the Monin-Obukhov's (1954) similarity theory can be applied. If z_1 is instead much larger, for example of the order of 1 km, one has to turn to the so-called Rossby similarity theory which is supposedly valid for the entire PBL.

2.3. The Monin-Obukhov similarity theory

The simplest example of the Monin-Obukhov similarity theory has already been given at the beginning of this section. Above an horizontal surface, the flow obeys for dimensional reasons the so-called logarithmic profile given by Eq. (2). The only "external" parameter which governs the flow is indeed then the friction velocity u_*, square-root of the downward surface momentum fluw $\overline{u'w'}_{sfc}$. In this case a drag coefficient can be deduced from Eqs. (2) and (9), i.e.

$$C_D = \left[\frac{k}{\mathrm{Ln}(z_\ell / z_0)} \right]^2 \qquad (13)$$

Such a formulation has already been used (see Manabe, 1969) in large scale models where the lowest level is at $z_1 \sim 100$ m and where above land one has $z_0 \sim 1$ cm. With such values (and $k = 0.4$), Eq. (13) leads to a value of approximately $2\,10^{-3}$ for C_D, in good agreement with the above quoted values.

Obviously Eq. (13) does not take any stability effect into account. Those can be dealt with by considering the surface buoyancy flux B_{sfc}

$$B_{sfc} = Q_0 + 0.608\ T_s\ E_0 \qquad (14)$$

as a new governing parameter for the similarity theory of the surface layer. The Monin-Obukhov similarity theory predicts then (see e.g. Businger, 1973)

$$\frac{\partial \bar{u}}{\partial z} = \frac{u_*}{k z}\ \varphi_u (z/L) \qquad (15)$$

$$\frac{\partial \bar{\theta}}{\partial z} = - \frac{Q_0}{u_* k z}\ \varphi_\theta (z/L) \qquad (16)$$

$$\frac{\partial \bar{q}}{\partial z} = - \frac{E_0}{u_* k z}\ \varphi_q (z/L) \qquad (17)$$

Where L is the Monin-Obukhov length

$$L = - \frac{u_*^3}{k \beta B_{sfc}} \qquad (18)$$

and the three "universal" functions φ_u, φ_θ and φ_q which describe stability corrections to the logarithmic profiles (2) have to be determined from experiment. These are now fairly satisfactorily known, with the possible exception of very stable situations, and have been critically reviewed by Yaglom (1977). As an example, the functions given by Businger et al. (1971) from the Kansas experiment are

$$\varphi_u(\zeta) = \begin{cases} (1-15\zeta)^{-1/4} & \text{for } \zeta \leq 0 \quad (\text{i.e. } B_{sfc} \geq 0) \\ (1+4.7\zeta) & \text{for } \zeta \geq 0 \quad (\text{i.e. } B_{sfc} \leq 0) \end{cases} \tag{19}$$

$$\varphi_\theta(\zeta) = \varphi_q(\zeta) = \begin{cases} 0.74\,(1-9\zeta)^{-1/2} & \text{for } \zeta \leq 0 \quad (\text{i.e. } B_{sfc} \geq 0) \\ 0.74 + 4.7\zeta & \text{for } \zeta \geq 0 \quad (\text{i.e. } B_{sfc} \leq 0) \end{cases} \tag{20}$$

By integrating $\varphi_{\bullet}(z/_L)$ with respect to z between z_o and z_1, it is possible to derive expressions relating the differences $\bar{u}(z_1)$, $\bar{\theta}(z_\ell)-T_s$, $\bar{q}(z_\ell)-q_s$ to the turbulent surface fluxes u_*, Q_o, E_o or equivalently to the bulk transfer coefficients C_D, C_H and C_q [see Eqs.(9) - (11)]. Such expressions can be solved for $C_{\bullet}$ by iterative methods (e.g. Clarke, 1970). More efficient methods have been recently proposed by Itier (1980), who suggests to simplify the calculation of the stability correction, and by Louis (1979), who gives a diagnostic formulation of $C_{\bullet}$ in term of the gradient Richardson number R_i corresponding to the layer between the surface and the first level z_1

$$R_i = \frac{\beta z_\ell\,[\bar{\theta}(z_\ell)-T_s]}{[\bar{u}(z_\ell)]^2} \tag{21}$$

Such a formulation is shown in Fig. 1 for C_D. It can be seen that stability decreases very efficiently the turbulent drag and completely inhibits it for Richardson numbers greater than 0.4.

The Monin-Obukhov similarity theory provides then a convenient framework to determine the influence of stability on bulk transfer coefficients in the case where the lowest level z_1 of the large scale model can be supposed to be within the surface layer, i.e. $z_1 \lesssim 100$ m. This method has already been applied quite successfully in some models.

A possible shortcoming is that the roughness length, which has to be prescribed in order to compute the bulk transfer coefficients [see e.g. Eq.(13)], is a quantity which is determined not only by surface characteristics as it should be, but also by the altitude z_1 of the first model level (see appendix). This could lead to roughness values adapted to a particular model and which would have to be changed when changing the vertical resolution ! In the case of fairly low first level, this would lead to unexpectedly large z_o values, as already stated at the beginning of this section.

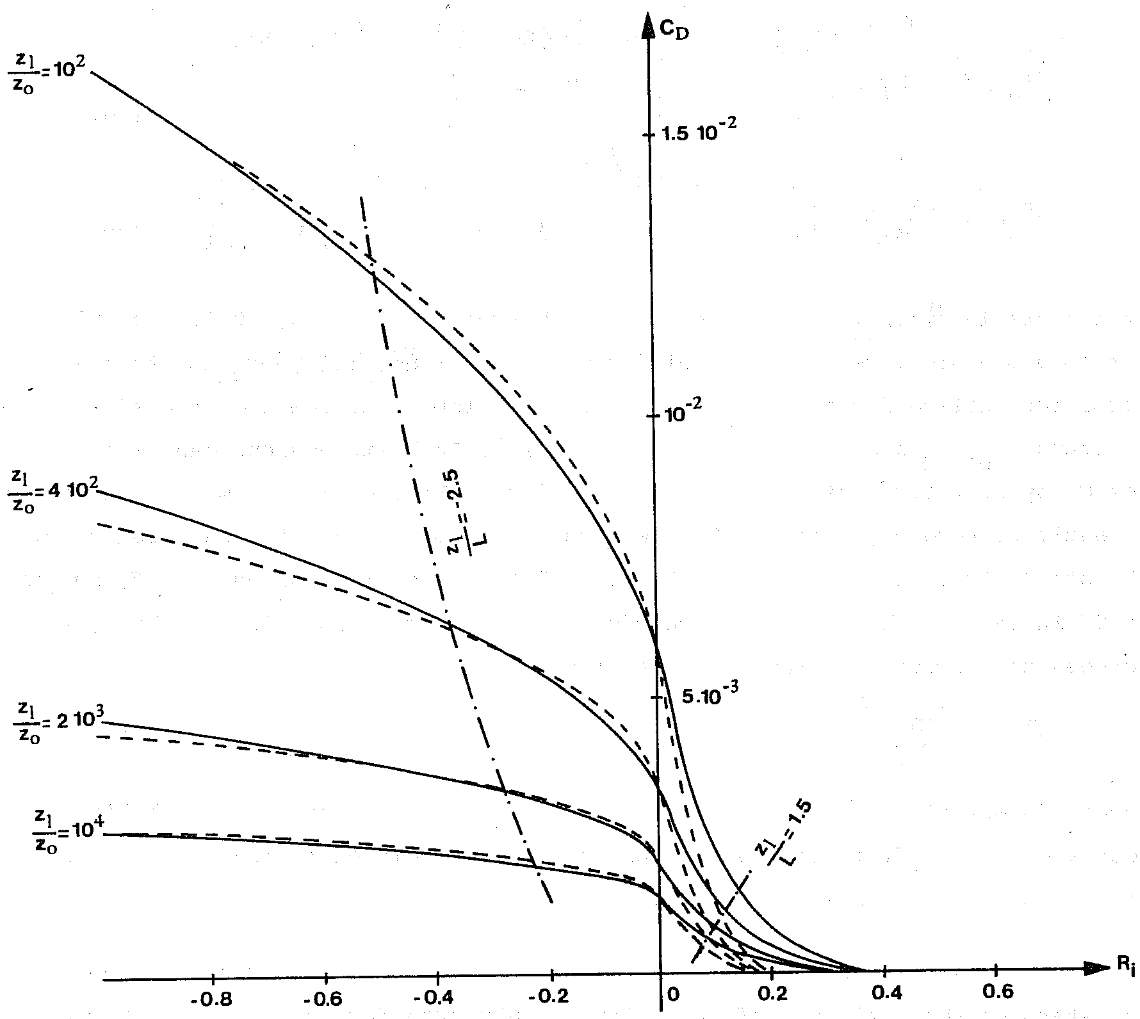

Figure 1: Drag coefficient C_D as a function of the Richardson number R_i in the surface boundary layer following Louis (1979) and De Moor et al. (1979)

2.4. The Rossby similarity theory

Application of the Rossby similarity theory to the whole PBL has been proposed by Kazanski and Monin (1960). Without going into too much detail, we shall simply say that it is based on the matching between a Monin-Obukhov similarity approach for the shallower surface layer and a similarity theory for the difference between actual values of wind, temperature and humidity at level z

inside the PBL and at level h at the top of the PBL. This lead to the well-known "universal" A, B, C, D functions

$$\frac{\bar{v}(h)}{u_*} = -\frac{\mathrm{Sgn}(f)}{k} A(\mu_E) \tag{22}$$

$$\frac{\bar{u}(h)}{u_*} = -\frac{1}{k}\left\{\mathrm{Ln}\left(\frac{k z_0}{L_E}\right) + B(\mu_E)\right\} \tag{23}$$

$$\frac{\bar{\theta}(h) - T_s}{\theta_*} = -\frac{\varphi_\theta(0)}{k\varphi_u(0)}\left\{\mathrm{Ln}\left(\frac{k z_0}{L_E}\right) + C(\mu_E)\right\} \tag{24}$$

$$\frac{\bar{q}(h) - q_s}{q_*} = -\frac{\varphi_q(0)}{k\varphi_u(0)}\left\{\mathrm{Ln}\left(\frac{k z_0}{L_E}\right) + D(\mu_E)\right\} \tag{25}$$

where $\bar{u}(h)$, $\bar{v}(h)$ are the wind components at the top of the PBL (often taken as the geostrophic wind components at this altitude) expressed in the coordinate system relative to the surface wind, f is the Coriolis parameter and μ_E is the stratification parameter

$$\mu_E = \frac{L_E}{L} \quad ; \quad L_E = k\,u_*/|f| \tag{26}$$

The A, B, C, D functions are not very precisely known from experiment due to many reasons we shall discuss later. A possible choice has been documented by Arya (1975)

$$A(\mu_E) = \begin{cases} 1.38 & \text{for } \mu_E < -50 \\ 5.14 + 0.142\,\mu_E + 1.17\,10^{-3}\mu_E^2 - 3.3\,10^{-6}\mu_E^3 & \text{for } \mu_E > -50 \end{cases} \tag{27}$$

$$B(\mu_E) = \begin{cases} 3.69 & \text{for } \mu_E < -50 \\ 1.01 - 0.105\,\mu_E - 9.9\,10^{-4}\mu_E^2 + 8.1\,10^{-7}\mu_E^3 & \text{for } \mu_E > -50 \end{cases} \tag{28}$$

$$C(\mu_E) = \begin{cases} 7.01 & \text{for } \mu_E < -25 \\ 1.86 - 0.377\,\mu_E - 5.39\,10^{-3}\mu_E^2 + 5.72\,10^{-5}\mu_E^3 & \text{for } \mu_E > -25 \end{cases} \tag{29}$$

One usually takes

$$D(\mu_E) = C(\mu_E) \tag{30}$$

although there exists other possibilities (e.g. Brutsaert and Chan, 1978).

If one assumes that the height z_1 of the lowest level in the large scale model is approximately equal to h, the depth of the PBL, Eqs.(22) - (30) relate the surface fluxes u_*^2, Q_o, E_o and the cross-isobaric angle α_o

$$tg\,\alpha_o = \frac{\bar{v}(h)}{\bar{u}(h)} \tag{31}$$

to the differences $\bar{u}(z_1)$, $\bar{v}(z_1)$, $\bar{\theta}(z_1) - T_s$ and $\bar{q}(z_1) - q_s$ or, equivalently, the bulk transfer coefficients C_D, C_H, C_q and the angle α_o to a surface Rossby number R_o

$$R_o = \frac{\left[\bar{u}^2(z_\ell) + \bar{v}^2(z_\ell)\right]^{1/2}}{z_o\,|f|} \tag{32}$$

and an external stratification parameter S

$$S = \frac{\beta\left[\bar{\theta}(z_\ell) - T_s\right]}{|f|\left[\bar{u}^2(z_\ell) + \bar{v}^2(z_\ell)\right]^{1/2}} \tag{33}$$

These relations are implicit and need to be solved iteratively, although approximate analytical solutions can be found (e.g. De Moor et al., 1979). The corresponding variation of C_D, C_H and α_o with respect to R_o and S are shown in Fig. 2.

The main difficulties in using the Kazanski-Monin theory to derive the surface heat fluxes are :

(i) the theory is not sufficiently justified by atmospheric data (see e.g. Arya, 1975, and Billard et al., 1982). There exists a large experimental scatter in the determination of the universal functions A, B and C, which can be reduced only slightly by improved experimental procedures (Yamada, 1976) ;

(ii) many important parameters are left out of the theory. For example stationarity and barotropy are assumed, which is obviously too restrictive. Inclusion of unstationary effect by replacing μ_E by μ_h (Zilitinkevich and Deardorff, 1974)

$$\mu_h = \frac{h}{L} \tag{34}$$

i.e. by taking explicitely into account the actual depth h of the PBL instead of its asymptotic depth given by the Ekman length L_E, does not lead to significant improvment (Billard et al. 1982). Inclusion of simple baroclinic effects by linearly developing A and B functions using a constant thermal wind is also a frustrating experiment, in the sense that the correlation between improved theory and experiment increases only marginally (Billard et al., 1982) ;

(iii) the theory cannot be extended to equatorial latitudes ($f \to 0$), nor does it provide a convenient framework to include cloud processes which are

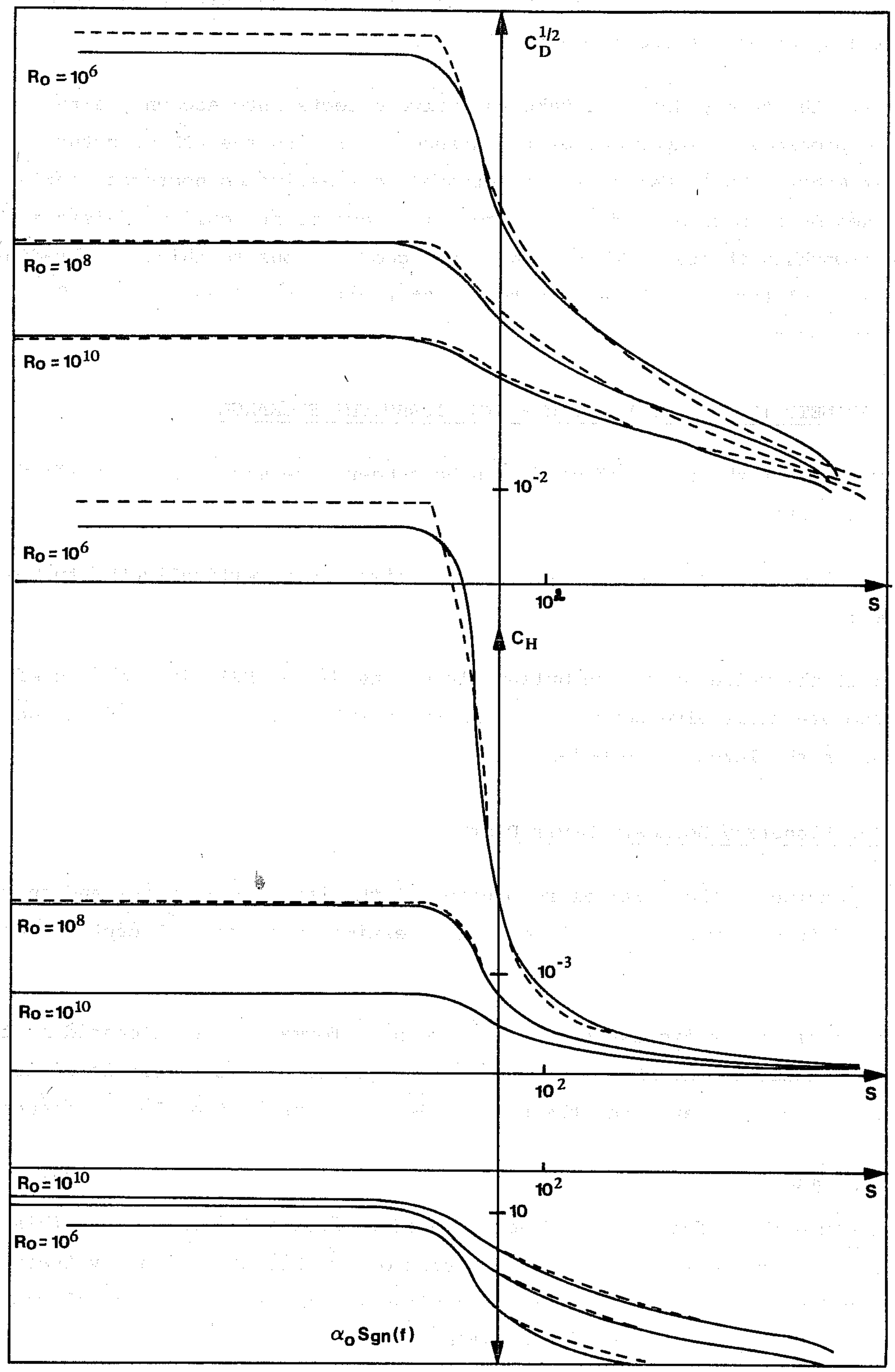

Figure 2: Bulk transfer coefficients for momentum C_D and heat C_H and cross-angle α_0 as functions of surface Rossby number R_0 and stratification parameter S in the planetary boundary layer following Arya (1975) and De Moor et al. (1979)

probably of prime importance for tropical and mid-latitude situations (see the discussion by Driedonks and Tennekes, 1981) ;

(iv) the theory does not take radiative effects into account, although those are probably as important as turbulence in driving the PBL structure (André et Mahrt, 1981). Using it, one probably overestimates nocturnal surface heat fluxes by as much as a factor of two. The same point could be raised against the Monin-Obukhov theory, although there are good reasons to think that radiative effects are relatively much less in this latter case (André and Mahrt, 1981 ; Garratt and Brost, 1981).

3. PARAMETERIZATION OF BOUNDARY-LAYER TURBULENT EXCHANGE

Once the turbulent surface fluxes have been determined, it is still necessary to estimate :

(i) the depth of the PBL, in order to distribute vertically the turbulent exchange ;

(ii) the value of the turbulent fluxes inside the PBL, in order to effectively compute their divergence, which is the quantity to be included in the rate equations in the large scale model.

3.1. The Planetary Boundary Layer Depth

Depending on the vertical resolution of the large scale model and on the altitude of it(s) lowest level(s), different estimates of the PBL depth h can be used.

We shall first discuss the case where no information is allocated by the large scale model to the PBL, i.e. when the height z_1 of the lowest level is of the order of, or greater than, the unknown depth h. One is then led to assume that

$$h = z_1 \tag{34}$$

and to include the effect of the divergence of turbulent fluxes only at this first level. In the case one uses the "unstationary" Rossby similarity theory by Zilitinkevich and Deardorff (1974) to retrieve the turbulent surface fluxes, Eq.(34) must be replaced by a rate equation for h (see below).

When the large scale model carries two or more levels in what should be the PBL, a choice must be made for h. It is either possible to prescribe it or to compute it. In the first case, h can be chosen either as a given height smaller than the vertical extent of the large scale model (like in the "A2-model" by Miyakoda and Sirutis, 1977) or on the contrary as its total vertical extent (e.g. Pearce, 1974).

The alternative solution is to compute h according to a rate equation, which implicitely implies that the large scale model takes into account the diurnal variation of the PBL. This equation has been extensively studied for dry convective situations in daytime and many forms have been proposed (e.g. Tennekes and Driedonks, 1981), although some of the most sophisticated equations appear unjustified when considering the poor accuracy of the data against which they can be tested (Artaz and André, 1980). Influence of cloud processes can be accounted for in some simplified situations (Benoit, 1976). Unfortunately the nocturnal evolution of h is much less known, may be simply because h is not always very clearly defined (Mahrt et al., 1981). It should also be said that the early evening collapse of h is not completely understood (e.g. Mahrt, 1981), with of course some unfavourable consequences on the further prediction of its nocturnal growth rate. Despite all these difficulties, rate equations for h have been proposed for, and used in, large scale models. Among those, the one by Deardorff (1972) is the most well known

$$\begin{cases} \left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}+\bar{v}\frac{\partial}{\partial y}\right)h=\bar{w}(h)+1.2\,\frac{Q_0}{\gamma h}+\frac{\partial}{\partial x}\left(K\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial h}{\partial y}\right) & \text{for unstable stratification} \\ h=\left(\frac{1}{30L}+\frac{|f|}{0.35\,u_*}+\frac{1}{z_t}\right)^{-1} & \text{for stable stratification} \end{cases} \qquad (35)$$

where γ is the lapse rate above the convective PBL, K an eddy coefficient accounting for subgrid-scale lateral diffusion and z_t the altitude of the tropopause. Eq.(35) has been extended by Deardorff (1976) to the case of cloud-topped PBL.

Turning finally to a more general point of view, it can be argued that such rate equations for h are defined on a much too small scale. For grid-size averages, there is probably no abrupt lid at the PBL top, but insted a much smoother transition extending up to the height of the deepest turbulent element present in the grid square, i.e. probably up to the tropopause (see e.g. Mac Bean et al., 1981).

3.2. The eddy diffusivities

Vertical turbulent fluxes $\overline{w'x'}$ in the PBL above the earth's surface are usually parameterized using the old Boussinesq assumption which states that they are proportional to the vertical derivative of the corresponding mean quantity $\overline{x}$:

$$\overline{w'x'} = -K_x\,\frac{\partial\overline{x}}{\partial z} \qquad (36)$$

where eddy diffusivities K_x are by many orders of magnitude greater than molecular diffusivities.

The simplest possible choice is of course to take constant values for $K_{\cdot}$, for example of the order of $10^2 - 10^3\ m^2\ s^{-1}$. This is however rather crude and the next step of increased complexity and accuracy is to prescribe a vertical profile $K_{\cdot}(z)$. Along those lines, O'Brien (1970) proposed to fit a cubic profile for $K_u = K_o$ between the value $K_{\cdot}(z_1)$, computed at the lowest level z_1 from turbulent surface fluxes and near-surface vertical gradients using Eq.(36), and a very small value around the PBL top at height h. These two methods have the same drawback of leading to eddy diffusivities which are almost independent of the structure and stability of the flow.

The simplest way to overcome this difficulty is to make use of the Prandtl's method, that is to express the eddy diffusivity as the product of an inverse turbulent time scale by the square of a mixing length

$$K \sim l^2 \left| \frac{\partial \underline{v}}{\partial z} \right| \tag{37}$$

In such a formulation the main question which remains to be solved concerns of course the choice of the mixing length l. From the early proposal by Blackadar (1962) it can assumed that l is of the order of a few tens of meters, i.e. of the size of efficient PBL eddies. A more precise formulation, which has the further advantage of being compatible with the neutral logarithmic law given by Eq.(2), is

$$l = \frac{kz}{1 + kz/l_\infty} \tag{38}$$

where l_∞ is an "asymptotic" mixing length which must be prescribed, very often of the order of 100 m. Eq.(37) and (38) do not allow for eddy diffusivities taking stability effect into account. As this is of prime importance since stability very efficiently inhibits vertical turbulent transfer, Eq.(37) has to be modified into

$$K_{\cdot} \sim l^2 \left| \frac{\partial \underline{v}}{\partial z} \right| f(R_i) \tag{39}$$

where $f(R_i)$ is a function of the local gradient Richardson number R_i evaluated between levels k and k+1

$$R_i = \frac{\beta\left[\bar{\theta}(z_{k+1}) - \bar{\theta}(z_k)\right]\left[z_{k+1} - z_k\right]}{\left[\bar{u}(z_{k+1}) - \bar{u}(z_k)\right]^2 + \left[\bar{v}(z_{k+1}) - \bar{v}(z_k)\right]^2} \tag{40}$$

This function has sometime been specified on purely empirical grounds (e.g. Deardorff, 1967) but is more appropriately parameterized either from simplified second-order models of turbulence (Mellor and Yamada, 1974) or from asymptotic arguments like in Louis (1979) where

$$f(R_i) = \begin{cases} 1 - \dfrac{b R_i}{1+c\,|R_i|^{1/2}} & \text{for unstable stratification } (R_i \leq 0) \\ \dfrac{1}{(1+b' R_i)^2} & \text{for stable stratification } (R_i \geq 0) \end{cases} \tag{41}$$

In Eq.(41) b, c and b' are dimensionless constants.

3.3. The eddy kinetic energy formulation and turbulence closure schemes

The methods described in the preceding subsections restrict turbulent mixing to take place only within the PBL and make the assumption that turbulence is then in stationary equilibrium with the mean flow. It is however known that the tropopause is another region where strong dissipation occurs, maybe as intensively as in the PBL. On the other hand the recent developments of PBL modelling indicate that prognostic equations for turbulence intensity are necessary if one wants to capture some of its essential features.

This two remarks have led to use an alternative formulation for $K_\bullet$, namely

$$K_\bullet = a_\bullet\, l\, \bar{e}^{1/2} \tag{42}$$

where $\bar{e}$ is the turbulent intensity

$$2\bar{e} = \overline{u'^2} + \overline{v'^2} + \overline{w'^2} \tag{43}$$

predicted from a properly parameterized form of its rate equation like

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x} + \bar{v}\frac{\partial}{\partial y} + \bar{w}\frac{\partial}{\partial z}\right)\bar{e} = \frac{\partial}{\partial z}\left(K_e \frac{\partial \bar{e}}{\partial z}\right) + K_u\left[\left(\frac{\partial \bar{u}}{\partial z}\right)^2 + \left(\frac{\partial \bar{v}}{\partial z}\right)^2\right] - \beta K_\theta \frac{\partial \bar{\theta}}{\partial z} - c_\varepsilon \frac{\bar{e}^{3/2}}{l} \tag{44}$$

In Eq.(44) the material derivative of $\bar{e}$ along the mean flow changes due to, from right to left, viscous dissipation, production by unstable stratification or destruction by stable stratification, mean shear generation and turbulent transport. The dimensionless constants c_ε, a_θ, a_u and a_e have to be determined from comparison of model results with some reference experiments. Eq.(44) together with the rate equations for the mean parameters form a closed system if the mixing length l is prescribed. It is usually chosen according to Eq.(38), but where l can either be prescribed as above or computed as the turbulence center of mass according to (Blackadar, 1962)

$$l_\infty = 0.1\, \frac{\int_0^\infty \bar{e}^{1/2} z\, dz}{\int_0^\infty \bar{e}^{1/2}\, dz} \tag{45}$$

Such a method has been used for preliminary tests by Miyakoda and Sirutis (1977), with apparently very promising results. Besides the fact it allows for a much better description of turbulent exchange, it presents the further advantage of offering various possibilities for rational improvement. Among those, Therry and Lacarrère (1981) proposed to distinguish between the mixing length responsible for vertical turbulent transfer, which appears in Eq.(42) and which is related to the vertical size of eddies, and the one responsible for viscous dissipation, which appears in the last term of Eq.(44) and which is more probably related to an overall size.

One can look for further progress by either improving the turbulence closure schemes or extending presently used methods to cloudy situations. In the first case it is well-known that PBL modelling offers now many methods which are much more sophisticated, including more and more rate equations for other turbulent quantities like fluxes and variances (e.g. Mellor and Yamada, 1974) and even triple correlations (André et al., 1978). Such a progressive complexification of the closure schemes brings in improvements for the description of turbulent exchange, but at a rate which tends to finally saturate ! On the other hand it can be thought that inclusion of cloud processes, and maybe falling rain, would be of prime importance for many meteorological situations. This can be done, at least in principle, by modifying the buoyancy term in Eq.(44) in order to account for the weight of cloud and/or rain drops (Bougeault, 1981 ; Redelsperger and Sommeria, 1982).

4. CONCLUSION

It is probably expected from such a review paper to give some conclusions about what could be called the "best" parameterization scheme presently available. In order to answer such a question, one should keep in mind that a parameterization scheme may be judged satisfactory from the theoretical point of view if

(i) it avoids as much as possible the use of ad-hoc assumptions ;
(ii) it is based on already tested theories ;
(iii) it includes the largest number of physical processes ;
(iv) it is formulated in a framework which will allows for further generalization without too much difficulties.

Considering these desired properties, it appears that retrieval of surfaces fluxes should be made by using the Monin-Obukhov similarity theory for the surface layer, and that PBL transfer should be parameterized with the aid of a rate equation for turbulent kinetic energy. This implies that the large scale model must carry its lowest level at an altitude of a few tens of meters at the most, and that it includes one more prognostic variable besides the usual ones.

It remains of course necessary to perform sensitivity and test experiments in order to judge by how much such a parameterization scheme really improves the forecast. This could be a very difficult question since the information gained by using a better PBL scheme can be lost if the parameterization schemes for other processes are comparatively too crude and do not allow for its efficient use. In this respect, it is certainly of prime importance to simultaneously improve the parameterization of radiative fluxes. This is necessary to correctly take into account either nocturnal or daytime cloudy situations, which altogether represent probably more than 75% of the cases ! This also underlines the necessity of giving good care to the hydrological processes, from water storage in the ground, to evaporation, cloud formation and dissipation, and precipitation.

APPENDIX : Dependence of the roughness length on the height of the lowest level.

This appendix presents preliminary results of a study which will be published elsewhere (André, 1982).

Let us assume that a grid square consists of a patchwork of individual surfaces with local roughness length $z_o(x, y)$. The averaged wind for the grid square is given at the lowest level z_l and will be noted U_l. In the case of neutral stratification, the universal law of the wall applies between $z_o(x, y)$ and z_l, i.e.

$$U_\ell = \frac{u_*(x,y)}{k} \, \mathrm{Ln} \, \frac{z_\ell}{z_o(x,y)} \tag{A1}$$

Eq.(A1) only states that it must exist a relationship between the local friction velocity and roughness length if one wants the logarithmic law to apply at every point of the grid square.

At lower altitudes ($z_o(x,y) < z < z_\ell$) the wind profile obeys

$$u(x,y,z) = \frac{u_*(x,y)}{k} \, \mathrm{Ln} \, \frac{z}{z_o(x,y)} \tag{A2}$$

or, by eliminating $u_*(x,y)$ from (A1)

$$u(x,y,z) = U_\ell \, \frac{\mathrm{Ln}\, z - \mathrm{Ln}\, z_o(x,y)}{\mathrm{Ln}\, z_\ell - \mathrm{Ln}\, z_o(x,y)} \tag{A3}$$

The grid-averaged wind $\bar{u}(z)$ defined by

$$\bar{u}(z) = \frac{1}{\Delta} \iint_\Delta u(x,y,z) \, dx \, dy \tag{A4}$$

is then given by

$$\bar{u}(z) = U_\ell \left\{ \frac{1}{\Delta} \iint_\Delta \frac{dx\,dy}{\mathrm{Ln}\, z_\ell - \mathrm{Ln}\, z_o(x,y)} \cdot \mathrm{Ln}\, z - \frac{1}{\Delta} \iint_\Delta \frac{\mathrm{Ln}\, z_o(x,y)}{\mathrm{Ln}\, z_\ell - \mathrm{Ln}\, z_o(x,y)} \, dx\, dy \right\} \tag{A5}$$

It can be seen immediately from (A5) that the grid-averaged wind obeys also a logarithmic law

$$\bar{u}(z) = \frac{u_*^{eff}}{k} \operatorname{Ln} \frac{z}{z_o^{eff}} \tag{A6}$$

with an "effective" friction velocity u_*^{eff}

$$u_*^{eff} = \frac{k U_\ell}{\Delta} \iint_\Delta \frac{dx\,dy}{\operatorname{Ln} z_\ell - \operatorname{Ln} z_o(x,y)} = \frac{1}{\Delta} \iint_\Delta u_*(x,y)\,dx\,dy \tag{A6}$$

which is simply the surface-averaged friction velocity, and with an "effective" roughness length z_o^{eff} given by

$$\operatorname{Ln} z_o^{eff} = \frac{\iint_\Delta \frac{\operatorname{Ln} z_o(x,y)}{\operatorname{Ln} z_\ell - \operatorname{Ln} z_o(x,y)}\,dx\,dy}{\iint_\Delta \frac{dx\,dy}{\operatorname{Ln} z_\ell - \operatorname{Ln} z_o(x,y)}} \tag{A7}$$

Eq.(A7) can easily be transformed into

$$\operatorname{Ln} \frac{z_o^{eff}}{z_\ell} = \frac{1}{\frac{1}{\Delta}\iint \frac{dx\,dy}{\operatorname{Ln} z_o(x,y) - \operatorname{Ln} z_\ell}} \tag{A8}$$

which shows that the "effective" roughness length is fairly different from the logarithmically-averaged roughness length. Eq.(A8) also indicates that the height z_1 of the lowest level in the large scale model may significantly influence the effective roughness length, specially when it is close to the highest roughness length to be experienced within the grid square. One can further show that

$$\frac{\partial z_o^{eff}}{\partial z_\ell} = \frac{z_o^{eff}}{z_\ell} \left\{ 1 - \frac{\Delta \iint \frac{dx\,dy}{\operatorname{Ln}^2 [z_o(x,y)/z_\ell]}}{\left(\iint \frac{dx\,dy}{\operatorname{Ln}[z_o(x,y)/z_\ell]} \right)^2} \right\} \tag{A9}$$

Using Schwarz' inequality $[(\int f(x)dx)^2 \leq \int f^2(x)dx]$, it comes from (A9)

$$\frac{\partial z_o^{eff}}{\partial z_\ell} \leq 0 \tag{A10}$$

Eqs.(A7), (A8) and (A10) can easily be interpreted :

(i) small values of $z_o(x,y)$ are averaged with a "small" weighting factor while large values of z_o are averaged with a "large" one. As a consequence, the effective roughness length is determined mostly by the large roughness elements present within the grid square ;

(ii) the above feature is more pronounced when the first level is lower (decreasing z_1) ;

(iii) for a given distribution of local roughness lengths $z_o(x,y)$, the effective roughness length increases for decreasing z_1. The increase in the effective roughness length becomes very sharp when the height z_1 of the lowest level in the large scale model decreases to values very close to the largest roughness length present in the grid square.

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