DYNAMICS OF CLOUD-MEAN FLOW INTERACTIONS IN THE TROPICAL ATMOSPHERE

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Summary: A balance condition is derived for atmospheric flow in tropical disturbances by examination of the dynamics of cloud-mean flow interaction. This balance condition together with the thermodynamic equation and the first order vorticity equation proposed by Cho et al. (1982) gives a set of first order equations for tropical disturbances.

1. INTRODUCTION

The theory for synoptic scale weather disturbances in the midlatitudes is considerably simplified by the existence of the geostrophic balance. In these weather systems, the velocity field is basically a rotational field with its vorticity one order larger in magnitude than the divergence. Consequently, there is an approximate balance between the centrifugal acceleration of the rotational flow and the Coriolis and the pressure gradient forces.

It is not yet known whether a similar condition exists for atmospheric flow in tropical disturbances. The dynamics of these disturbances is complicated by the presence of large numbers of cumulus clouds. Cumulus clouds affect the synoptic scale flow in several different ways. First, the latent heat released in these clouds represents a major energy source. Second, the vertical circulations induced by cumulus clouds redistribute the horizontal momentum in the vertical direction. It has long been recognized that the problem of cloud mean flow interactions is central to our understanding of tropical disturbances.

The purpose of this paper is to demonstrate, by examining a certain aspect of the dynamics of cloud mean flow interactions, that a balance condition quite similar to that of the midlatitude weather systems also exists for air flow in tropical disturbances. The discussion will proceed as follows. In section 2, certain features of the vertical circulation in a tropical disturbance will be illustrated by purely thermodynamic considerations. A particular method of decomposition of the wind field and the pressure field suitable for the analysis will then be presented in sections 3 and 4. Based on the results of sections 2-4, a balance condition will be derived in section 5 for atmospheric flow in tropical disturbances. The paper will then end with a few concluding remarks.

2. VERTICAL CIRCULATION IN A SYNOPTIC SCALE TROPICAL DISTURBANCE

Some features of the vertical circulation in a tropical disturbance may be derived from thermodynamic considerations. Using the potential temperature θ, the first law of thermodynamics for the mean flow can be written as

$$\frac{\partial\bar{\theta}}{\partial t} + \bar{\underset{\sim}{v}}\cdot\underset{\sim}{\nabla}\bar{\theta} + \bar{\omega}\frac{\partial\bar{\theta}}{\partial p} = - M_c \frac{\partial\bar{\theta}}{\partial p} \quad . \tag{1}$$

The overbar denotes the horizontal areal average which is used to separate the mean flow from the cloud-scale motion. The cloud effect is represented on the right-hand side of the equation in terms of the total cloud mass flux M_c. This is the representation first proposed by Ooyama (1971) and Arakawa and Schubert (1974) and later refined by Cho (1977). In Cho's formulation, cloud areas are defined as regions where condensation and evaporation processes take place. Consequently, M_c includes both the condensation-induced upward motion and evaporation-induced cloud-scale downward motion. If one defines the mean vertical motion in the cloud environment as

$$\tilde{\omega} = \bar{\omega} + M_c \tag{2}$$

equation (1) can also be written as

$$\frac{\partial\bar{\theta}}{\partial t} + \bar{\underset{\sim}{v}}\cdot\underset{\sim}{\nabla}\bar{\theta} + \tilde{\omega}\frac{\partial\bar{\theta}}{\partial p} = 0 \quad . \tag{3}$$

It indicates that in addition to the horizontal thermal advection, the mean temperature field is influenced by the vertical motion in the cloud environment. The portion of latent heat released inside cumulus clouds that may be imparted to the mean flow depends on the ability of the clouds to influence the vertical motion field in the cloud environment.

It is well known from observations that the temperature variations in synoptic-scale tropical disturbances are very small. The horizontal total derivative of potential temperature is only of the order of $0.3^{o}C$ day^{-1}:

$$\frac{\partial\bar{\theta}}{\partial t} + \bar{\underset{\sim}{v}}\cdot\underset{\sim}{\nabla}\bar{\theta} \approx 0.3^{o}C\ day^{-1}. \tag{4}$$

From equation (3), the typical values of $\tilde{\omega}$ should be of the order $0.3^{o}C\ day^{-1}/(\partial\bar{\theta}/\partial p)$, which is very small.

The heating rate due to cumulus clouds in convectively active regions of these disturbances is considerably larger, typically of the order

$$- M_c \frac{\partial \bar{\theta}}{\partial p} \approx 10^oC \text{ day}^{-1} \quad . \tag{5}$$

As noted by Wallace (1971), this implies that the cloud heating is balanced mainly by the adiabatic cooling associated with the large-scale ascending motion. As a zeroth order approximation, (1) may be reduced to

$$\bar{\omega} \frac{\partial \bar{\theta}}{\partial p} = - M_c \frac{\partial \bar{\theta}}{\partial p} \quad , \tag{6}$$

or

$$\bar{\omega} = - M_c \quad . \tag{7}$$

The large-scale mean ascending motion in the core regions of these disturbances is essentially equal to the total upward mass flux induced inside cumulus clouds. Consequently, the mean vertical motion in the cloud environment $\tilde{\omega}$ is only a small difference between two large quantities $\bar{\omega}$ and M_c. These conclusions can also be derived from a scale analysis. Interested readers are referred to Cho et al.(1983) for details.

3. DECOMPOSITION OF THE WIND FIELD

In view of the vertical circulation features discussed in section 2, a particular method to decompose the horizontal wind field will be introduced which is convenient for later analysis.

In the study of the dynamics of synoptic scale flow, cumulus clouds are usually treated as eddies. The synoptic scale meteorological fields are defined by an areal averaging. The area of average A is defined as being only a small portion of the large-scale weather system, yet large enough to contain a large number of cumulus clouds. Such an area can be divided into two parts: the area occupied by clouds A_c, and the area occupied by the cloud environment $(A-A_c)$. If a_i is the area occupied by the i'th cloud in A, then

$$A_c = \sum_{i=1}^{N} a_i \tag{8}$$

where N is the total number of clouds in the area. The fractional cloud coverage is defined as

$$\Sigma = A_c/A \quad . \tag{9}$$

It is typically of the order of a few per cent. For any meteorological variable α, the synoptic scale mean values $\bar{\alpha}$ can be expressed as

$$\bar{\alpha} = \Sigma \, \bar{\alpha}_c + (1 - \Sigma)\tilde{\alpha} \tag{10}$$

where $\bar{\alpha}_c$ is the ensemble mean over the cloud areas and $\tilde{\alpha}$ the mean over the cloud environment.

Typically, in areas occupied by cumulus clouds there are strong vertical motions and large values of divergence and vorticity which can be two orders of magnitude larger than those of the mean environment. The total divergence field and the vertical relative vorticity field are usually decomposed into a mean field and an eddy field:

$$\delta = \bar{\delta} + \delta' \qquad \zeta = \bar{\zeta} + \zeta' \tag{11}$$

In such a decomposition, $\overline{\delta'} = 0$ and $\overline{\zeta'} = 0$. Although the values of δ' and ζ' are much larger than $\bar{\delta}$ and $\bar{\zeta}$ only in cloud areas, the areal integrated contributions of δ' and ζ' from the cloud environment are equally important. Alternatively, one may also decompose the total divergence and vorticity field as follows:

$$\delta = \tilde{\delta} + \delta'' \qquad \zeta = \tilde{\zeta} + \zeta'' \tag{12}$$

where $\tilde{\delta}$ and $\tilde{\zeta}$ are the mean values over the cloud environment. In such a decomposition, the values of δ'' and ζ'' vanish in the cloud environment. They have significant values only inside cloud areas. The mean values of δ'' and ζ'' are given by

$$\overline{\delta''} = \bar{\delta} - \tilde{\delta} \quad \text{and} \quad \overline{\zeta''} = \bar{\zeta} - \tilde{\zeta} \tag{13}$$

They are not identical to zero. If the subscript i is used to denote the values of a quantity inside the i'th cloud, then

$$\delta''_i = \delta_i - \tilde{\delta} \quad \text{and} \quad \zeta''_i = \zeta_i - \tilde{\zeta} \quad . \tag{14}$$

Using the decomposition given by equation (12), we shall define the velocity field $\overline{\overline{\underset{\sim}{V}}}$ to be such that

$$\underset{\sim}{\nabla} \cdot \overline{\overline{\underset{\sim}{V}}} \equiv \overline{\overline{\delta}} = \tilde{\delta} \quad \text{and} \quad \hat{\underset{\sim}{k}} \cdot \underset{\sim}{\nabla} \times \overline{\overline{\underset{\sim}{V}}} \equiv \overline{\overline{\delta}} = \tilde{\delta} \tag{15}$$

where $\hat{k}$ is the unit vector in the vertical direction. $\overline{\overline{\underset{\sim}{V}}}$ can be interpreted as what the wind field would be if the divergence and vorticity fields were everywhere equal to their mean cloud environmental values. It is a synoptic scale velocity field distinctly different from the mean wind field $\overline{\underset{\sim}{V}}$, and the mean cloud environmental wind field $\tilde{\underset{\sim}{V}}$.

Relative to $\overline{\overline{\underset{\sim}{V}}}$, one may now describe the cloud-scale velocity fluctuations in terms of $\underset{\sim}{V}''$ given by

$$\underset{\sim}{V} = \overline{\overline{\underset{\sim}{V}}} + \underset{\sim}{V}'' \quad . \tag{16}$$

Since

$$\overline{(\overline{\overline{\underset{\sim}{V}}})} = \overline{\overline{\underset{\sim}{V}}} \tag{17}$$

areal averaging of equation (16) gives

$$\overline{\underset{\sim}{V}''} = \overline{\underset{\sim}{V}} - \overline{\overline{\underset{\sim}{V}}} \quad . \tag{18}$$

Furthermore, because of the identity

$$\underset{\sim}{V} = \overline{\underset{\sim}{V}} + \underset{\sim}{V}' = \overline{\overline{\underset{\sim}{V}}} + \underset{\sim}{V}'' \tag{19}$$

there is the following relationship between $\underset{\sim}{V}'$ and $\underset{\sim}{V}''$:

$$\underset{\sim}{V}' = \underset{\sim}{V}'' - \overline{\underset{\sim}{V}''} \quad . \tag{20}$$

The velocity fluctuations $\underset{\sim}{V}''$ is directly related to the cloud scale vorticity and divergence fluctuations ζ'' and δ''. From the definition of $\underset{\sim}{V}''$, one has

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{V}'' = \delta'' \tag{21}$$

and

$$\hat{k} \cdot \underset{\sim}{\nabla} \times \underset{\sim}{V}'' = \zeta'' \quad .$$

Area average of equation (21) gives

$$\begin{aligned} \underset{\sim}{\nabla} \cdot \overline{\underset{\sim}{V}''} &= \overline{\delta''} = \frac{1}{A} \sum_i \int_{a_i} \delta''_i \, da \\ \hat{k} \cdot \underset{\sim}{\nabla} \times \overline{\underset{\sim}{V}''} &= \overline{\zeta''} = \frac{1}{A} \sum_i \int_{a_i} \zeta''_i \, da \end{aligned} \tag{22}$$

The decomposition of $\overline{\mathbf{V}}$ into $\bar{\bar{\mathbf{V}}}$ and $\overline{\mathbf{V}''}$ according to equation (18) is not unique. One may assume, for example, that $\mathbf{V}''$ has zero deformation without loss of generality. This assumption will be made throughout this paper. Furthermore, $\overline{\mathbf{V}''}$ is also approximately irrotational. Cho et al. (1979) and Cho and Cheng (1980) showed that cloud vorticity, although it has very large point values, when averaged over the cloud cross sectional area, has a magnitude comparable to that of the large scale mean flow. Therefore, if the fractional cloud coverage Σ is small,

$$\bar{\zeta} \gg \hat{k}\cdot\nabla \times \overline{\mathbf{V}''} = \overline{\zeta''} \approx 0 \tag{23}$$

and

$$\bar{\zeta} = \bar{\bar{\zeta}} + \overline{\zeta''} \approx \bar{\bar{\zeta}} = \tilde{\zeta} \quad . \tag{24}$$

This approximation will be used later in this paper.

Another property of this wind decomposition is that $\bar{\bar{\mathbf{V}}}$ is approximately non-divergent. Results from section 2 indicate that

$$\bar{\bar{\delta}} = \tilde{\delta} \ll \bar{\delta} = \bar{\bar{\delta}} + \overline{\overline{\delta''}} \approx \overline{\overline{\delta''}} \ . \tag{25}$$

Therefore

$$\bar{\delta} \gg \bar{\bar{\delta}} = \nabla\cdot\bar{\bar{\mathbf{V}}} \approx 0 \quad . \tag{26}$$

This approximation will also be used later in section 5.

4. DECOMPOSITION OF THE PRESSURE FIELD

In order to study the dynamics of cloud-mean flow interactions using the wind decomposition defined in the previous section, a dynamically consistent decomposition of the pressure field is also needed. For the purposes of this analysis, the following form of the horizontal equation of motion will be used:

$$\frac{\partial}{\partial t} + \nabla(\tfrac{1}{2}\,\mathbf{V}\cdot\mathbf{V}) + \frac{1}{\rho}\nabla p + (f + \zeta)\hat{k} \times \mathbf{V} + w\frac{\partial \mathbf{V}}{\partial z} = 0 \quad . \tag{27}$$

Furthermore, the anelastic approximation will be made so that the density ρ will be replaced everywhere by its large scale mean $\bar{\rho}$ in calculating the inertia of air flow.

First, an average pressure field $\bar{\bar{p}}$ will be defined in such a way that

$$\frac{\partial \bar{\bar{\mathbf{V}}}}{\partial t} + \nabla\cdot(\tfrac{1}{2}\,\bar{\bar{\mathbf{V}}}\cdot\bar{\bar{\mathbf{V}}}) + \frac{1}{\bar{\rho}}\nabla\,\bar{\bar{p}} + (f + \bar{\bar{\zeta}})\ \hat{k} \times \bar{\bar{\mathbf{V}}} + \bar{\bar{w}}\,\frac{\partial \bar{\bar{\mathbf{V}}}}{\partial z} = 0 \quad . \tag{28}$$

The pressure fluctiation p" is then defined by

$$p = \overline{\overline{p}} + p'' \tag{29}$$

The equation governing $\underset{\sim}{V}''$ can be obtained by subtracting equation (28) from equation (27)

$$\frac{\partial \underset{\sim}{V}''}{\partial t} + \underset{\sim}{\nabla}(\overline{\overline{\underset{\sim}{V}}}\cdot\underset{\sim}{V}'' + \tfrac{1}{2}\underset{\sim}{V}''\cdot\underset{\sim}{V}'') + \frac{1}{\rho}\underset{\sim}{\nabla}\, p'' + \Big(\zeta''\,\hat{k}\times\overline{\overline{\underset{\sim}{V}}} + (f + \overline{\overline{\zeta}})\hat{k}\times\underset{\sim}{V}''$$
$$+ \zeta''\,\hat{k}\times\underset{\sim}{V}''\Big) + \Big(\overline{\overline{w}}\,\frac{\partial \underset{\sim}{V}''}{\partial z} + w''\,\frac{\partial \overline{\overline{\underset{\sim}{V}}}}{\partial z} + w''\,\frac{\partial \underset{\sim}{V}''}{\partial z}\Big) = 0 \quad . \tag{30}$$

At the cloud scale, the vertical velocity is governed by the vertical equation of motion:

$$\frac{\partial \overline{\rho} w''}{\partial t} + \underset{\sim}{\nabla}\cdot(\overline{\rho}\,\underset{\sim}{V}\,w'' + \overline{\rho}\,\underset{\sim}{V}''\,w'') + \frac{\partial \overline{\rho} w w''}{\partial z} + \frac{\partial \overline{\rho}\overline{\overline{w}} w''}{\partial z}$$
$$= -\frac{\partial p''}{\partial z} - \overline{\rho}\,g(\theta''/\overline{\overline{\theta}}) \quad . \tag{31}$$

Equations (30) and (31) together with the mass continuity equation

$$\underset{\sim}{\nabla}\cdot\overline{\rho}\,\underset{\sim}{V}'' + \frac{\partial \overline{\rho} w''}{\partial z} = 0 \tag{32}$$

then implies that the pressure field p" satisfy the following diagnostic equation

$$\nabla^2\, p'' + \frac{\partial^2 p''}{\partial z^2} + \underset{\sim}{\nabla}\cdot\,\overline{\rho}\,\nabla(\overline{\overline{\underset{\sim}{V}}}\cdot\underset{\sim}{V}'' + \tfrac{1}{2}\underset{\sim}{V}''\cdot\underset{\sim}{V}'') + \underset{\sim}{\nabla}\cdot\overline{\rho}\Big(\zeta''\,\hat{k}\times\overline{\overline{\underset{\sim}{V}}}'' + (f+\overline{\overline{\zeta}})\hat{k}\times\underset{\sim}{V}''$$
$$+ \zeta''\,\hat{k}\times\underset{\sim}{V}''\Big) + \underset{\sim}{\nabla}\cdot\overline{\rho}\Big(\overline{\overline{w}}\,\frac{\partial \underset{\sim}{V}''}{\partial z} + w''\,\frac{\partial \overline{\overline{\underset{\sim}{V}}}}{\partial z} + w''\,\frac{\partial \underset{\sim}{V}''}{\partial z}\Big)$$
$$+ \frac{\partial}{\partial z}\,\underset{\sim}{\nabla}\cdot(\overline{\rho}\,\underset{\sim}{V}\,w'' + \overline{\rho}\,\underset{\sim}{V}''\,\overline{\overline{w}}) + \frac{\partial^2}{\partial z^2}\,(\overline{\rho}\,w\,w'' + \overline{\rho}\,\overline{\overline{w}}\,w'')$$
$$+ \frac{\partial}{\partial z}\,(\overline{\rho}\,g\,\theta''/\overline{\overline{\theta}}) = 0 \tag{33}$$

This is the equation usually used to determine the pressure perturbations induced by cumulus clouds in numerical cloud models. Equation (33) together with equations (28) and (29) then completely sepcify the pressure decomposition which is consistent with the velocity decomposition introduced in the previous section.

5. THE BALANCE CONDITION

The thermodynamic considerations presented in section 2 indicate that the divergence field in the cloud environment is about one to two orders smaller in magnitude than the vorticity field. Therefore, the forces in the cloud environment have to be such that the cloud-environmental flow is approximately non-divergent. The implications of this condition on the distribution of the pressure field and its relationship to the momentum field will now be examined.

First a few notations will be introduced, referring to Figure 1 illustrating the area of averaging A. The outer boundary of this area will be denoted by C. The boundary of the i'th cloud will be denoted by C_i. $\hat{n}$ will be used to denote the unit normal vector pointing outward along a closed path.

By definition, $\delta'' = 0$, $\zeta'' = 0$, and $w'' = 0$ outside the clouds. The eddy momentum equation (30) may be reduced to

$$\frac{\partial \underset{\sim}{V}''}{\partial t} + \underset{\sim}{\nabla}(\underset{\sim}{\overline{\overline{V}}}\cdot\underset{\sim}{V}'' + \tfrac{1}{2}\,\underset{\sim}{V}''\cdot\underset{\sim}{V}'') + \frac{1}{\bar{\rho}}\,\underset{\sim}{\nabla}\, p'' + (f + \overline{\overline{\zeta}})\hat{k} \times \underset{\sim}{V}'' + \overline{\overline{w}}\,\frac{\partial \underset{\sim}{V}''}{\partial z} = 0 \qquad (34)$$

For simplicity, the notation $\underset{\sim}{F}''$ will be introduced:

$$\underset{\sim}{F}'' = \bar{\rho}\;\underset{\sim}{\nabla}(\underset{\sim}{\overline{\overline{V}}}\cdot\underset{\sim}{V}'' + \tfrac{1}{2}\,\underset{\sim}{V}''\cdot\underset{\sim}{V}'') + \underset{\sim}{\nabla} p'' + \bar{\rho}(f + \overline{\overline{\zeta}})\hat{k} \times \underset{\sim}{V}'' + \bar{\rho}\;\overline{\overline{w}}\,\frac{\partial \underset{\sim}{V}''}{\partial z} \qquad (35)$$

It will be referred to as the eddy forcing. Since $\underset{\sim}{V}''$ is non-divergent in the cloud environment, line integrations of equation (34) along C and C_i's give

$$\oint_C \underset{\sim}{F}''\cdot\hat{n}\; d\ell = \sum_i \oint_{C_i} \underset{\sim}{F}''\cdot\hat{n}\; d\ell \quad . \qquad (36)$$

The total normal eddy forcing across the outer boundary C of the area of averaging is equal to the sum of the normal eddy forcing across the outer boundaries of clouds located in the area.

The normal eddy forcing at the cloud boundaries may be evaluated from equation (33). Note that δ'', ζ'', and w'' all approach zero at the cloud boundaries. Integration of equation (33) over cloud areas gives

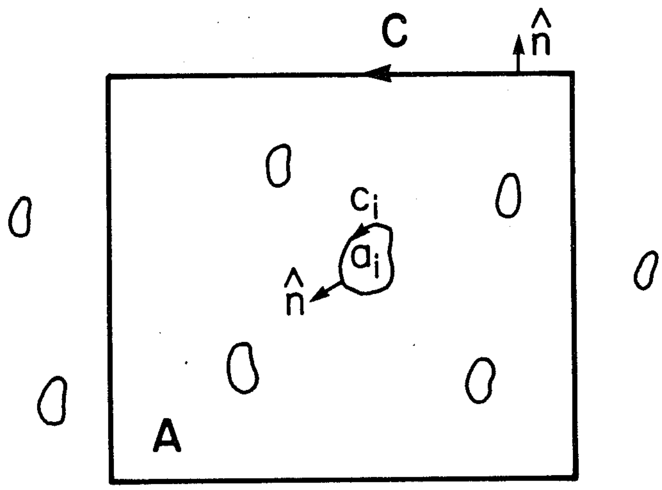

Figure 1. Area of averaging A. a_i is the cross-sectional area of the i'th cloud. C_i is the outer boundary of the i'th cloud and C is the boundary of the area of averaging. $\hat{n}$ is the unit normal vector pointing outward along a closed path.

$$F_c = \sum_i \oint_{C_i} \tilde{F}''\cdot\hat{n}\, d\ell = -\sum_i \int_{a_i} \Big(\frac{\partial^2 p''}{\partial z^2} + \frac{\partial}{\partial z}\bar{\rho}\, g\, \theta''/\bar{\bar{\theta}} + \frac{\partial}{\partial z}\nabla\cdot\bar{\rho}\, \tilde{V}''\, \bar{\bar{w}}$$
$$+ \frac{\partial^2}{\partial z^2}(\bar{\rho}\, w\, w'' + \bar{\rho}\, \bar{\bar{w}}\, V'')\Big)\, da \quad . \tag{37}$$

Here F_c is used to denote the total normal eddy forcing across the cloud boundaries. Formally, it is difficult to evaluate F_c using equation (37). It requires the solution of the poisson equation (33) for the pressure perturbation p". But a much simpler experession may be obtained by combining equation (37) with the vertical derivative of equation (31):

$$F_c = -\sum_i a_i \frac{\partial}{\partial t} \bar{\rho}\, \bar{\delta}_i \tag{38}$$

where $\bar{\delta}_i$ is the mean divergence over the cloud area. Following the derivation given by Cho (1977), it can be shown that

$$F_c = -A\, \bar{\rho}\, \frac{\Sigma}{\tau}\, (\delta_c(\tau) - \bar{\bar{\delta}}) \tag{39}$$

where τ is the mean cloud life-span, and $\delta_c(\tau)$ is the ensemble mean value of cloud divergence at the end of individual cloud life cycles.

The results derived so far are general. If the condition

$$\bar{\bar{\delta}} << \bar{\bar{\zeta}} \tag{40}$$

is introduced and the first order approximations are made that

$$\bar{\bar{w}} \simeq 0$$
$$\bar{\bar{\delta}} \simeq 0 \tag{41}$$

then $\bar{\bar{\tilde{V}}}$ is approximately a non-divergent wind field. Using

$$\bar{\bar{\tilde{F}}} = \bar{\rho}\, \tilde{\nabla}(\tfrac{1}{2}\, \bar{\bar{\tilde{V}}}\cdot\bar{\bar{\tilde{V}}}) + \tilde{\nabla}\, \bar{\bar{p}} + \bar{\rho}(f + \bar{\bar{\zeta}})\hat{k} \times \bar{\bar{\tilde{V}}} \tag{42}$$

to denote mean forcing, line integration of the mean momentum equation (28) along C gives

$$\oint_C \bar{\bar{\tilde{F}}}\cdot n\, d\ell = 0 \quad . \tag{43}$$

The total normal forcing across the boundary C can be obtained by combining equation (36) and (43):

$$\oint_C (\overline{\overline{\underset{\sim}{F}}} + \underset{\sim}{F}'')\cdot\hat{n}\ d\ell = -A\bar{\rho}\ \frac{\Sigma}{\tau}\ (\delta_c(\tau) - \overline{\overline{\delta}}) \quad . \tag{44}$$

Note that

$$\oint_C (\overline{\overline{\underset{\sim}{F}}} + \underset{\sim}{F}'')\cdot\hat{n}\ d\ell = \oint_C \Big[\bar{\rho}\underset{\sim}{\nabla}(\tfrac{1}{2}\ \overline{\overline{\underset{\sim}{V}}}\cdot\overline{\overline{\underset{\sim}{V}}} + \overline{\overline{\underset{\sim}{V}}}\cdot\underset{\sim}{V}'' + \tfrac{1}{2}\ \underset{\sim}{V}''\cdot\underset{\sim}{V}'')$$

$$+ \underset{\sim}{\nabla}(\overline{\overline{p}} + p'') + \bar{\rho}(f + \overline{\overline{\zeta}})\hat{k}x(\overline{\overline{\underset{\sim}{V}}} + \underset{\sim}{V}'')\Big]\cdot n\ d\ell$$

$$= A\Big[\nabla\cdot\bar{\rho}\ \underset{\sim}{\nabla}(\tfrac{1}{2}\ \overline{\overline{\underset{\sim}{V}}}\cdot\overline{\overline{\underset{\sim}{V}}} + \overline{\overline{\underset{\sim}{V}}}\cdot\overline{\underset{\sim}{V}''} + \tfrac{1}{2}\ \overline{\underset{\sim}{V}''\cdot\underset{\sim}{V}''}) + \nabla^2\ \bar{p} + \underset{\sim}{\nabla}\cdot\bar{\rho}(f + \overline{\overline{\zeta}})\hat{k}\ x\ \overline{\underset{\sim}{V}}\Big] . \tag{45}$$

The condition that $\overline{\overline{\delta}}$ is very small then gives us the following blance condition which is obtained by substituting equation (45) into equation (44):

$$\nabla^2\ \bar{p} + \underset{\sim}{\nabla}\cdot\Big[\bar{\rho}\ \nabla(\tfrac{1}{2}\ \overline{\overline{\underset{\sim}{V}}}\cdot\overline{\overline{\underset{\sim}{V}}} + \overline{\overline{\underset{\sim}{V}}}\cdot\overline{\underset{\sim}{V}''} + \tfrac{1}{2}\ \overline{\underset{\sim}{V}''\cdot\underset{\sim}{V}''})\Big] + \underset{\sim}{\nabla}\cdot\Big[\bar{\rho}(f + \overline{\overline{\zeta}})\hat{k}\ x\ \overline{\underset{\sim}{V}}\Big]$$

$$= -\bar{\rho}\ \frac{\Sigma}{\tau}\ (\delta_c(\tau) - \overline{\overline{\delta}}) \quad . \tag{46}$$

In the pressure coordinate system, the balance equation may be expressed in terms of the geopotential $\Phi = g\ z$:

$$\nabla^2\ \bar{\Phi} + \nabla^2(\tfrac{1}{2}\ \overline{\overline{\underset{\sim}{V}}}\cdot\overline{\overline{\underset{\sim}{V}}} + \overline{\overline{\underset{\sim}{V}}}\cdot\overline{\underset{\sim}{V}''} + \tfrac{1}{2}\ \overline{\underset{\sim}{V}''\cdot\underset{\sim}{V}''}) + \underset{\sim}{\nabla}\cdot\Big[(f + \overline{\overline{\zeta}})\hat{k}\ x\ \overline{\underset{\sim}{V}}\Big]$$

$$= -\frac{\Sigma}{\tau}\ (\delta_c(\tau) - \overline{\overline{\delta}}) \tag{47}$$

This balance condition may be written into a more convenient form by first making the approximation $\overline{\overline{\zeta}} \approx \overline{\zeta}$ given by equation (24), and then substituting the relation $\overline{\overline{V}} = \overline{V} - \overline{V''}$ given by equation (18):

$$\nabla^2\bar{\Phi} + \nabla^2(\tfrac{1}{2}\ \overline{V}\cdot\overline{V} - \tfrac{1}{2}\ \overline{V''}\cdot\overline{V''} + \tfrac{1}{2}\ \overline{\underset{\sim}{V}''\cdot\underset{\sim}{V}''}) + \underset{\sim}{\nabla}\cdot\Big((f + \overline{\zeta})\hat{k}\ x\ \overline{\underset{\sim}{V}}\Big) = -\frac{\Sigma}{\tau}\ (\delta_c(\tau) - \overline{\overline{\delta}}) \tag{48}$$

It was assumed when introducing $\overline{\underset{\sim}{V}''}$ in section 3 that it has zero deformation. Furthermore, from equations (23) and (25)

$$\nabla\cdot\overline{V''} \approx \bar{\delta}$$

and $\hat{k} \cdot \nabla \times \overline{V''} \simeq 0$.

Therefore, using $\overline{D}$ to denote the total deformation of the mean wind field $\overline{\underset{\sim}{V}}$, equation (48) can also be written as

$$\nabla^2\overline{\Phi} = \tfrac{1}{2}(f + \overline{\zeta})^2 - \frac{\overline{D}^2}{2} - \frac{f^2}{2} - \overline{\underset{\sim}{V}}\cdot\underset{\sim}{\nabla}f - \nabla^2 \tfrac{1}{2}\,\overline{\underset{\sim}{V}''\cdot\underset{\sim}{V}''} - \frac{\Sigma}{\tau}(\delta_c(\tau) - \overline{\overline{\delta}}) \quad . \qquad (49)$$

The last term in the balance equation (49) is probably negligible. It seems reasonable to assume that at the end of cloud life-cycles, $\delta_c(\tau) \sim \overline{\overline{\delta}} = \tilde{\delta} << \overline{\delta}$. Since $\Sigma/\tau \sim 1\ \text{day}^{-1}$ (Cho, 1977), comparable to the inverse of the time-scale of the synoptic scale flow, this term is considerably smaller than the other terms in the balance equation. Equation (49) can be further reduced to

$$\nabla^2\overline{\Phi} = \tfrac{1}{2}(f + \overline{\zeta})^2 - \frac{\overline{D}^2}{2} - \frac{f^2}{2} - \overline{\underset{\sim}{V}}\cdot\nabla\underset{\sim}{f} - \nabla^2 \tfrac{1}{2}\,\overline{\underset{\sim}{V}''\cdot\underset{\sim}{V}''} \qquad (50)$$

The geopotential field is related diagnostically to the centrifugal accelerations due to the mean rotation and deformation, and the mean eddy dynamic pressure due to cumulus clouds.

6. CONCLUDING REMARKS

We have shown in this paper that a balance condition exists for air flow in tropical disturbances, despite the fact that the mean divergence and vorticity are comparable in magnitude in these weather systems. The balance equation is derived from a condition deduced from thermodynamic considerations: the air flow in the clear area between cumulus clouds being approximately non-divergent. It relates the geopotential field diagnositcally to the centrifugal accelerations due to mean rotation and deformation, and the effy dynamic pressure due to cumulus clouds.

When applied to an axisymmetric vortex, the balance equation (50) becomes the nonlinear gradient wind balance equation often used in modelling hurricane development, provided that the effect of eddy dynamic pressure can be ignored, i.e.,

$$\nabla^2 \tfrac{1}{2}\,\overline{\underset{\sim}{V}''\cdot\underset{\sim}{V}''} \simeq 0 \ .$$

The existence of the balance condition also makes it possible to derive a set of first order equations for tropical disturbances. Using the scaling argument identical to that discussed in section 2 of this paper, Cho et al. (1982) derived a first order vorticity equation for tropical disturbances:

$$\frac{\partial(f + \overline{\zeta})}{\partial t} + \overline{\underset{\sim}{V}}\cdot\underset{\sim}{\nabla}(f + \overline{\zeta}) = \frac{\Sigma}{\tau}\,(\zeta_c(\tau) - \overline{\zeta}) \ . \qquad (51)$$

This equation, together with the balance condition derived in this paper

$$\nabla^2\bar{\Phi} = \tfrac{1}{2}(f + \bar{\zeta})^2 - \frac{\bar{D}^2}{2} - \frac{f^2}{2} - \bar{\underset{\sim}{V}}\cdot\underset{\sim}{\nabla} f - \nabla^2 \tfrac{1}{2}\, \overline{\underset{\sim}{V}''\cdot\underset{\sim}{V}'} \tag{52}$$

and the thermodynamic equation

$$\frac{\partial\bar{\theta}}{\partial t} + \bar{\underset{\sim}{V}}\cdot\underset{\sim}{\nabla}\bar{\theta} + \tilde{\omega}\,\frac{\partial\bar{\theta}}{\partial p} = 0 \tag{53}$$

forms a set of first order equations applicable to the tropical atmosphere.

According to these equations, the evolution of the mean vorticity equation can be determined to the first order from the simple vorticity equation (51), provided the amount of cloud activities is known. The mean geopotential and temperature fields must evolve with the vorticity field according to the balance equation (52). The thermodynamic equation can be then used to determine the mean vertical motion in the cloud environment $\tilde{\omega}$. Provided the amount of clouds is known, equations (51)-(53) give a complete first order description of the dynamics of tropical disturbances.

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