

INCLUSION OF CLOUD LIQUID WATER AS A PROGNOSTIC VARIABLE

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1. INTRODUCTION

In today's models for weather prediction and for studies of the atmosphere's general circulation, schemes based on simplified relations with mainly the relative humidity are used in the calculation of radiative fluxes. It is of course not necessarily wrong to utilize simplified descriptions of various processes. However, we should then know that the parameterization in question renders realistic features in the model circulation. That is, in this connection, realistic individual cloud features.

Our insight is inadequate as to how condensation processes should be described in models of the kind that we are concerned with here. That this is the situation is exhibited by the quality of the present parameterization schemes for condensation/cloudiness. A couple of examples may be given: model condensation and model cloudiness are obtained from independent descriptions, implying that there is not necessarily consistency between the two entities; furthermore, different constraints are applied in connection with cloudiness algorithms in order to suppress unrealistic features, such as too much cloudiness in the lowest layers of the atmosphere.

In order to improve on the situation hinted at above, I believe one should start out from more elaborate formulations of the various processes that are involved. Those formulations should form a physically and thermodynamically consistent picture of interplaying mechanisms. The set

of equations thus obtained are then integrated over time and space increments that are the smallest resolvable scales in a numerical treatment. In this way certain possibilities are given to obtain parametric relations between resolved and unresolved scales of motion.

A first step in the direction towards a more elaborate treatment of condensation, compared to current practice, is to consider cloud water content (without distinguishing between liquid water and ice) as a prognostic variable. This is a quantity for which we can formulate a continuity equation. The cloud water content has an influence on the resulting circulation via partly an effect on especially the shortwave radiative fluxes, partly a modulation of the hydrological cycle. From the point of view of model development, we furthermore note that cloud water content is a measurable quantity, implying that it is a useful parameter in connection with quantitative evaluation of condensation-cloud schemes.

In the following I will discuss what I consider to be the main components of a consistent parameterization of condensation including cloud water content as a prognostic variable.

2. BASIC COMPONENTS OF PARAMETERIZATION OF CONDENSATION

There are mainly three different scales, which are widely separated, to be considered in deducing parametric relations for condensation processes in models of the type we are concerned with here. Those scales are the microphysical scale, the macrophysical or cloudsize scale and the mesoscale.

The problems connected with the macroscale and the mesoscale motions are concerned with the fractional cloud cover, which has to be dealt with whether or not cloud water content is carried as a prognostic variable.

The microphysical processes are connected directly with the cloud proper, and thus also with the cloud water content. Nonetheless, I will dwell on the question of subgridscale appearance of condensation as well as on questions connected with a cloud water content. The three mentioned categories are not completely separable; in a consistent treatment of condensation processes, those three categories are coupled to each other when they are all considered.

2.1 Parameterization of microphysical processes

The growth of cloud particles to precipitating size and evaporation from rain or snow appear to be the two most important processes on the microphysical scale to be considered for parameterization. Strictly speaking, condensation also belongs to this category; the parametric description adopted without exception is that condensation occurs when the flux convergence of vapour is sufficient to maintain a relative humidity of 100%. The large amount of particles constituting the condensate thus formed will gradually grow through several different mechanisms to a precipitating size. Our parametric description of the various processes that are responsible for the droplet growth should thus yield a realistic partitioning of the condensate between cloud mass and precipitating mass.

It should be kept in mind that for the moment we are considering, say a unit volume, that is embedded in a region of condensation, i.e. in a cloud. Parameterization in this case implies that the microphysical processes inside that volume should be expressed in terms of volume-averaged condensation parameters, such as cloud water density and precipitating water density. In this context we may utilize the principles in, or even the specific details of, Kessler's (1969) work. He deduces for

example the rate of change of cloud water content due to coalescence and accretion processes; i.e. the rate of change of cloud water content as a function of cloud water density itself and of precipitating water density. In that expression, three parameters are introduced. Two of those are involved in the relation giving the rate of conversion to rain due to cloud water density alone (autoconversion). The third parameter describes the efficiency of rain formation due to collision between cloud droplets and raindrops (collection by rain).

Now, adopting the simplifying assumption that the precipitating water has an infinite fall velocity, we then have only cloud water content left as a three-dimensional prognostic quantity of condensation. This means that we have merely this quantity at our disposal for the parametric description of the conversion rate from cloud particles to precipitating particles. Hence, after integration of the growth rate equation for individual cloud droplets over all radii, according to an assumed size distribution, we get a prognostic equation for the cloud water content with the following form.

$$\frac{\partial m_c}{\partial t} = A_{mc} + Q_c - P_c \quad (2.1)$$

where m_c is the mixing ratio of the cloud water content, A_{mc} contains essentially the advective terms, Q_c is the net rate of condensation or production of condensate in mixing ratio equivalents and P_c is the rate of release of precipitation from the cloud. Note that equation (2.1) is applicable to scales that are substantially bigger than the diameter of individual droplets and smaller than or equal to the dimension of the cloud. For the release of precipitation due to cloud water density alone, Kessler (1969) assumes a linear function of m_c ; however if m_c is smaller than a prescribed threshold value the release of rain is zero. In the

cloud parameterization scheme that I proposed (Sundqvist, 1978) I employ a nonlinear function of m_c for P_c . We will consider that expression here partly because it is suitable for a specific demonstration later. Thus

$$P_c = c_0 m_c \left\{ 1 - \exp \left[- \left(\frac{m_c}{m_r} \right)^2 \right] \right\} \quad (2.2)$$

where m_r is a typical cloud water mixing ratio at which the release of precipitation is becoming efficient and c_0^{-1} is a typical time for conversion from cloud droplets to raindrops. Application of expression (2.2) implies that a relatively large portion of the condensate is used to make the cloud denser as long as m_c is much smaller than m_r and that consequently the rate of release of precipitation becomes more efficient the denser the cloud is.

The functional form (2.2) is used for stratiform as well as for convective condensation (Hammarstrand, 1983) in a scheme that is being tested at our department at present. This scheme comprises a consistent treatment of condensation in the two regimes. The two parameters c_0 and m_r have different values in stratiform and in convective condensation. The parameters in (2.2) could also be given functional forms; e.g. in deep convective clouds, m_r may be a function of height so that the cloud water content has a maximum at a realistic height above the cloud base. The conversion rate, c_0 , could be a function of cloud depth above the model level that is being considered at the moment; in this way the release of precipitation could be enhanced in a deep cloud as compared to a shallow one. Both parameters may be functions of temperature in order to simulate the specific situation of a cloud containing a mixture of water and ice crystals in certain strata. Results from studies of the type recently published by Ochs and Beard (1984), Willis (1984) and Heymsfield and Platt (1984) may be of valuable guidance in this context.

The release of precipitation from pure ice crystal clouds, mainly cirrus at temperatures below 233 K, has probably to be described by a function that is substantially different from (2.2). In this case, the growth of cloud particles to precipitating size takes place through diffusion and not coalescence and collection processes. Here we may capitalize on works by Heymsfield (1975).

In the scheme developed at our department, only either stratiform or convective condensation is allowed to be present in a gridbox at each moment. In the topmost layer of convection the condensation is treated as stratiform in order to allow for a possible development of cirrus. Thus, in that layer, the convergence of vapour is due to the flux from below accomplished by the convection.

2.2 Parameterization of macroscale and mesoscale motions

Since those aspects are not intended to be a main topic of this presentation, I included those parameterization problems essentially with the purpose of completing the discussion on parameterization of cloud water. The macroscale and the mesoscale condensation have the common feature that they both appear in merely a fraction of the gridbox volume. For simplicity we shall assume that the cloud typically fills the gridbox in the vertical. Hence, the subgridscale condensation appears as a fractional cloud cover in a horizontal cross-section. We note that condensation takes place though the grid point relative humidity is less than 100%, because the box is only partially occupied by cloud.

The macroscale motion is connected with the dynamics of individual clouds. For example a region may contain convective clouds (whether those are deep penetrative cumulonimbi or shallower convective clouds as we observe in cold air flow over a warm ocean surface or in the trade wind region) which

have both open and closed cell circulations. As a further example we shall take a region with stratocumulus; in this case, the fractional cloud cover is relatively large compared to the small areas of clear skies that are often observed in such cloud decks.

The mesoscale motion determines the size of the region of condensation, such as the dimensions of a stratocumulus deck or the size of a region containing moist convection. The mesoscale circulation may be connected with the synoptic scale situation, for example a frontal system (see Hobbs et al, 1983 wherein additional references may be found). The mesoscale motion may also be governed by external conditions, such as subgridscale features of the earth's surface, for example the topography.

There is a hierarchy of conditions for when condensation should appear in a grid box. Those conditions belong to two main categories, namely one for convectively unstable situations and one for absolutely stable stratification. The main conditions particular to the first mentioned category is of course the presence of potential instability. The main requirement usually adopted in stable stratification is that the relative humidity has to reach a certain value before condensation is allowed to appear. Although the detailed dynamics is substantially different in the two regimes, there is a common key question connected with the parameterization of the subgridscale processes. Namely, how the converging water vapour should be partitioned into condensation and a raising of the relative humidity in the box. We shall consider this aspect below. Let us first only formulate a set of equations that is applicable to GCM resolution.

We first imagine that we have variables related to condensation available with high resolution in a grid square, which contains condensation in

fraction b of the area. Letting α stand for any of the relevant variables, the representative gridpoint value of α is

$$\alpha = b \alpha_c + (1-b) \alpha_o \quad (2.3)$$

where subscript c stands for condensation and o represents the cloudfree region (for example, the relative humidity $U_c \equiv 1$ and $m_o \equiv 0$). Applying the averaging of (2.3), we obtain the following equations for temperature, specific humidity and cloud water content.

$$\frac{\partial T}{\partial t} = A_T + \frac{L}{C_p} (Q - E) \quad (2.4)$$

$$\frac{\partial q}{\partial t} = A_q - (Q_q - E_q) \quad (2.5)$$

$$\frac{\partial m}{\partial t} = A_m + Q - E - P \quad (2.6)$$

$$P = c_o m \left\{ 1 - \exp \left[- \left(\frac{m}{b m_k} \right)^2 \right] \right\} \quad (2.7)$$

The A -term has an analogous meaning to the one in equation (2.1); the temperature is denoted by T and the specific humidity by q . The evaporation, E , may result from precipitation and/or cloud particles. In (2.5) Q_q and E_q are marked because they are not necessarily the same as in (2.4) for convection at individual levels. We shall return to this later. The net production of condensate is $(Q-E)$ so this appears in both (2.4) and (2.6). We furthermore note that the averaging (2.3) applied to (2.2) leaves us with $m_c = m/b$ in the exponent of (2.7). This is also something that we would expect since m_r refers to typical values of individual clouds.

As we are going to consider the partitioning of water vapour for condensation and general moistening, it is expedient to rewrite equation (2.5) in terms of relative humidity, $U=q/q_s$.

$$q_s \frac{\partial U}{\partial t} + \frac{c_p}{L} U S_q \frac{\partial T}{\partial t} - \frac{q}{p_s} \frac{\partial p_s}{\partial t} = A_q - (q_q - E_q); S_q = \frac{\epsilon L^2 q_s}{R q T^2} \quad (2.8)$$

where S_q stems from the Clapeyron relation between saturation vapour pressure and temperature. It is assumed that we work in a sigma coordinate system so we get the tendency term for surface pressure, p_s . Eliminating the temperature tendency between (2.4) and (2.8) we obtain

$$q_s \frac{\partial U}{\partial t} + [q_q - E_q + U S_q (q - E)] - \left(A_q - \frac{c_p}{L} U S_q A_T \right) + \frac{q}{p_s} \frac{\partial p_s}{\partial t} = 0 \quad (2.9)$$

We now discuss stratiform and convective condensation separately and for simplicity we disregard the evaporation term, or assume that it is absorbed in the Q -term.

From relation (2.9) we see that we have three unknown quantities Q , Q_q and the tendency for the relative humidity. We thus need two more relations in order to close our system.

In the stratiform case, it is reasonable to assume that the rate of release of latent heat at a specific point results in a corresponding loss of vapour at the same point, i.e. $Q=Q_q$. Hence, in a stably stratified situation

$$Q = \frac{A_q - \frac{c_p}{L} U S_q A_T - q_s \frac{\partial U}{\partial t} - \frac{q}{p_s} \frac{\partial p_s}{\partial t}}{1 + U S_q} \quad (2.10)$$

To calculate the release of latent heat, we still need a relation that tells how the relative humidity changes as condensation goes on. A

conventional approach to obtaining the release of precipitation is to neglect that tendency term, implying that all the converging vapour is condensed and then immediately precipitated. But this is not a satisfying assumption for a scheme that is intended to contain a consistent handling of cloudiness. Without going into further details, we note from (2.3) that there is a relation between relative humidity and cloud cover.

We now turn to the convective case, in which we utilize the principle Kuo approach which is convenient for our demonstration. In this scheme we have

$$Q - E = \frac{C_p}{L} \overline{\epsilon} (T_c - T) \quad (2.11)$$

$$A_q - (Q_q - E_q) = \overline{\epsilon} M \quad (2.12)$$

$$M = q_c - q \quad (2.13)$$

where T_c is the temperature of the moist adiabat of the cloud and q_c is the saturation humidity with respect to T_c . The energy involved in the convection taking place in a grid column is provided by the fluxes through the boundaries of the convective column. Thus integrating (2.4) and (2.5) over such a column, the right hand sides then only contain the integral of the A-terms. Considering the energetics of a column we hence have (we use the \sim sign to denote the integral)

$$C_p \overline{\frac{\partial T}{\partial t}} + L \overline{\frac{\partial q}{\partial t}} = C_p \overline{A_T} + L \overline{A_q} = C_p \overline{A_T} + C_p \overline{\epsilon} (T_c - T) + L \overline{\epsilon} M \quad (2.14)$$

The last equality in (2.14) follows from (2.11-12). It is furthermore assumed that $\overline{\epsilon}$ is independent of height. Equation (2.14) yields the well known expression for $\overline{\epsilon}$ of the original Kuo scheme.

We now insert expressions (2.11-12) in relation (2.9) to investigate the heating/moistening question in the convective case. We then get

$$q_s \frac{\partial u}{\partial t} + \frac{c_p}{L} \left[\frac{c_p}{L} u s_f (T_c - T) - M \right] + \frac{c_p}{L} u s_f A_T + \frac{q}{p_s} \frac{\partial p_s}{\partial t} = 0 \quad (2.15)$$

In the original Kuo scheme, M is given by (2.13), implying that both heating and moistening are given by the scheme. Then relation (2.15) yields the resulting change of relative humidity in this approach. According to the Clapeyron relation, we get the following approximate expression for M in (2.15)

$$M = q_c - q = q \left\{ \exp \left[\frac{\epsilon L}{R} \left(\frac{1}{T} - \frac{1}{T_c} \right) \right] - 1 \right\} \approx u q_s \frac{\epsilon L}{RT^2} (T_c - T) = \frac{c_p}{L} u s_f (T_c - T) \quad (2.16)$$

Inserting (2.16) in (2.15), we thus find that the original Kuo scheme causes a change of relative humidity that is governed essentially by the gross temperature change (the surface pressure tendency term is relatively small in this context). In a convective column, A_T is negative and in a case of pronounced convection the magnitude may be of the order 5×10^{-4} K/s. With this value, the relative humidity of the lower troposphere would double its value in approximately five hours. This fast approach towards a fully saturated state has been a problem of concern to all those that have resorted to the Kuo scheme (see e.g. Kuo, 1974, Anthes, 1977).

Let us now go back and consider relation (2.15). If we abandon the original assumption about moistening (2.13) and instead say that the rate of moistening is governed by the net change of temperature, we find from (2.15) that the relative humidity will change only if the surface pressure

changes (or if we have some evaporation terms included). Our expression for M is then

$$\frac{dM}{dt} = \frac{c_p}{L} U S_g \left[\frac{d}{dt} (T_c - T) + A_T \right] = \frac{c_p}{L} U S_g \frac{\partial T}{\partial t} \quad (2.17)$$

Combining relations (2.14) and (2.17) we now get the following expression for $\frac{dM}{dt}$

$$\frac{dM}{dt} = \frac{L \tilde{A}_g - c_p U S_g A_T}{c_p (T_c - T) (1 + U S_g)} \quad (2.18)$$

The above demonstration thus shows that the question of partitioning vapour for condensation and moistening is analogous in the stably stratified situation and the convectively unstable situation.

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