PARAMETERIZATION OF STRATIFORM CONDENSATION

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1. INTRODUCTION

Many of today's models for weather prediction and general circulation simulation have become quite elaborate through inclusion, for exemple, of an advanced boundary layer treatment and of radiation calculations accounting for at least some kind of energy budget consideration at the earth's surface. However, treatment of condensation and especially associated cloudiness is not carried out to the same degree of refinement. Therefore it seems that this last mentioned problem area still needs much more attention than in the past in order to make it possible to achieve a substantial improvement in our prediction skill.

By stratiform condensation we mean the release of latent heat in stably stratified conditions. This energy input into the atmosphere takes place at a slow rate and it is generally not a primary driving force of circulation systems. But analyses of observations and results from model experiments show that this heat release renders distinct modifications to the circulation characteristics (e.g., additional intensity, sharpening of gradients, shrinking scale of the system). There is an indirect effect that may be equally - or perhaps even more - important. Namely, the stratiform cloudiness that results from this condensation is relatively extensive. So there is a substantial effect on the radiation conditions. This modulation of the radiation does not only concern medium range and long term evolutions but equally well developments on the time scale of a day. The latter aspect is specially applicable to models that contain an elaborate treatment of the boundary layer, the thermal stability of which is strongly dependent the intensity of the insolation. This statement is of course true for convective as well as for stratiform cloudiness. The parameterization of moist convection is covered in other lectures here, and we shall in this sequel focus our discussion on stratiform condensation and associated cloudiness. It is inevitable, however, that some overlapping will occur, because some of the processes are principally the same in the two stability regimes; it is merely a difference in intensity.

If we look at clouds in individual atmospheric layers of about 1 km thickness (which is about the same as a typical model layer), we find that it is rather a rule than an exception that the horizontal extension is smaller than usually employed grid sizes. So there are two main levels of parameterization of condensation to be considered; one for the microphysical processes and one for the subgrid scale features connected with the condensation. We shall furthermore note that it is not unlikely that the vertical variation of stratiform condensation also is of subgrid scale with regard to typical vertical model resolutions.

In the following, we shall discuss those parameterization aspects in relation to the degree of division into sub-processes.

The least complex, non-trivial way of taking condensation into account is to release latent heat according to a relative humidity criterion and to let an equvivalent amount of condensate immediately precipitate.

In models that contain radiative processes, it is necessary to have cloudiness included. In reality, clouds are formed because there is not an immediate fall out of the condensate that is produced. Hence, in order to treat clouds in connection with the release of latent heat, it's necessary to describe how the the condensate is partitioned between cloud water and precipitation. A logical first step in refinement of model condensation is to include cloud water content as a dependent variable and to still assume an immediate fall out of the precipitating water. Model cloud water is an important physical quantity since it may be verified, provided there are observational data. Data, at least of vertically integrated cloud water content is beginning to be available from satellite measurements. The distribution of the condensate on cloud and precipitation is connected with the microphysical processes, which hence must be parameterized in a model description. We will consider this matter in Section 2.

In Section 3, we will discuss the problem regarding parameterization of the subgrid scale features of condensation and associated clouds.

In both Section 2 and Section 3 we will consider specific parameterization approaches in order to allow a more concrete discussion.

Parametric descriptions of the type alluded to above, of course render a few parameters that must be given values and possible functional forms. We will pay some attention to those aspects in Section 4. In this section we will furthermore briefly discuss problems related to cirrus clouds.

2. PARAMETERIZATION OF MICROPHYSICAL PROCESSES

We are here concerned with the condensation proper and the subsequent growth of aggregate water molecules to cloud droplets, which may continue to grow to drops of precipitating size.

Empirically we know that atmospheric condensation takes place at very low supersaturations as a result of the abundance of condensation nuclei in the air. Thus, we assume that condensation takes place at 100% relative humidity, provided there is convergence of vapour into the volume regarded. To simplify the discussion, we will for the moment consider merely the liquid phase of the condensate.

The growth to droplet size is essentially accomplished through diffusion. Further growth to precipitating drops is dominated by the coalescence process: due to their random motion, cloud droplets now and then collide and fuse into a larger drop - Kessler (1969) calls this autoconversion by cloud; precipitating drops falling through a cloud may grow further by collecting some of the droplets that are situated in the cylinders swept by the individual raindrops - Kessler (1969) calls this process cloud collection by rain.

Our task is now to deduce mathematical formulations for the rate of condensation and for the partitioning of the resulting condensate on cloud water and precipitating water. As mentioned in the introduction, we shall adopt the simplifying approximation that precipitating matter falls out instantaneously. Then we merely have cloud water content as a dependent variable resulting from the condensation. As a consequence, we have disposed of the possibility to have an explicit relation for the process of cloud collection by rain mentioned above. Instead we have to parameterize the rate of release of precipitation in terms of the cloud water content alone.

In a volume where condensation is taking place, the rate of change of temperature, T, specific humidity, q, and mixing ratio of cloud water content, m, is given by

$$\frac{\partial \hat{T}}{\partial t} = A_{\hat{T}} + \frac{L}{c_p} \hat{Q} \tag{2.1}$$

$$\frac{\partial \hat{q}}{\partial t} = A_{\hat{q}} - \hat{Q} \tag{2.2}$$

$$\frac{\partial \hat{m}}{\partial t} = A_{\hat{m}} + \hat{Q} - \hat{P} \tag{2.3}$$

We denote the in-cloud values by a hat. The A-terms in (2.1), (2.2), (2.3) contains all other processes but the condensation process considered here. The latent heat of vaporization is denoted by L and the specific heat at constant pressure by c_p. The rate of release of latent heat (in mixing ratio equivalents) is $\hat{Q}$ and the rate of release of precipitation is $\hat{P}$.

Generally we have the relation (or definition)

$$q = U\, q_s(T) \tag{2.4}$$

where U is the relative humidity and $q_s(T)$ is the saturation specific humidity at temperature T. In differential form, relation (2.4) reads (employing the Clausius-Clapeyron relation)

$$\frac{1}{q}\frac{\partial q}{\partial t} = \frac{\varepsilon L}{RT^2}\frac{\partial T}{\partial t} + \frac{1}{U}\frac{\partial U}{\partial t} - \frac{1}{p}\frac{\partial p}{\partial t} \tag{2.5}$$

where ε is the ratio of the molecular weights of water to air, R is the specific gas constant for dry air and p is the pressure at the height in question. Now, we are considering a volume where condensation is taking place and thus

$$q = \hat{q} = q_s(\hat{T}) \;;\; U = \hat{U} = U_s \equiv 1 \tag{2.6a,b}$$

with (2.6a,b) applied to (2.5), we note that the rate of change of relative humidity, i.e. the second term on the right hand side, is identically equal to zero. Inserting this modified form of (2.5) in (2.2) and then eliminating the temperature tendency between the resulting equation and (2.1) we obtain

$$\hat{Q} = \left(A_{\hat{q}} - \frac{c_p}{L} S_q A_{\hat{T}} + \hat{q}\,\frac{1}{p}\frac{\partial p}{\partial t}\right)\left(1 + S_q\right)^{-1} \tag{2.7}$$

where

$$S_q = \frac{\varepsilon L^2}{R c_p} \cdot \frac{q_s}{T^2} \tag{2.8}$$

The rate of release of latent heat (or production of condensate) is thus governed by essentially the convergence of water vapour and by temperature changes, which are primarily due to adiabatic expansion, all modified by the factor $(1+S_q)^{-1}$, which is a reflection of an assumed wet bulb process implying that the converging vapour is in part used for release of heat and in part for maintaining saturation with regard to the changing temperature.

A parametric formulation for the rate of release of precipitation, P, is needed. For the present discussion, we adopt the following form from Sundqvist (1978)

$$\hat{P} = c_o \hat{m} \left[1 - e^{-\left(\frac{\hat{m}}{m_r}\right)^2} \right] \tag{2.9}$$

Two additional parameters, c_o and m_r, are introduced through relation (2.9). The parameter c_o^{-1} gives a characteristic time for conversion of cloud droplets into precipitating drops. The parameter m_r gives a typical cloud water content at which the release of precipitation begins to be efficient. Hence, the factor in the square brackets of (2.9) implies that $\hat{P}$ is small for relatively small values of $\hat{m}$. Thus, for relatively small values of m, the released condensate is mainly used to increase the cloud water content; it is not until this quantity has become large ($\hat{m} \gg m_r$) that a precipitation is released efficiently.

We shall include evaporation terms in the equations a little later, so the microphysical mechanisms involved will be considered now.

We assume that evaporation is due to diffusion of water molecules from a drop so we utilize appropriate growth rate formulas for individual drops situated in an environment of subsaturation. Due to their small size, cloud particles evaporate in a time that is smaller, or at most, comparable to the time step that is used in GCMs. Therefore, we assume that the evaporation of cloud water brought into clear air is instantaneous.

Regarding evaporation from precipitating drops, we asssume for the moment that we know the density or mixing ratio of precipitating matter in the air. We adopt the Marshall-Palmer (1948) drop size distribution. The rate equation for evaporation of a single drop can then be integrated over the whole size spectrum, thus rendering the rate of evaporation from the precipitating water mass. Since we do not have this quantity explicitly available we have to accept an approximate value deduced from the rate of precipitation at the altitude under consideration. Assuming a mean fall speed (V_r = 5 m/s) of rain, we obtain the following mixing ratio of precipitating water at a level p

$$\hat{N} = \tilde{\hat{P}}(p) / (\rho V_r) \; ; \quad \tilde{\hat{P}}(p) = \frac{1}{g} \int_{p_{cloud\,top}}^{p} \hat{P} \, dp \tag{2.10}$$

where ρ is air density and g the acceleration of gravity. Then applying the rate equations as described above, we get the following expression for the rate of evaporation

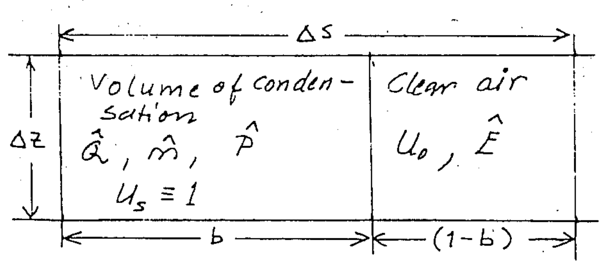
$$\hat{E}_r = k_E (u_s - u) \cdot (\tilde{\hat{P}})^{1/2} \tag{2.11}$$

We have introduced three parameters, c_o, m_r and k_E to describe the microphysical processes in terms of the macroscale quantities. We will discuss those parameters regarding their size and possible functional forms in Section 4.

3. PARAMETERIZATION OF SUBGRID SCALE PROCESSES

The convergence of water vapour, given by quantities on the resolvable scale, is the principle source for possible condensation in a gridbox. The occurrence of subgrid scale condensation implies that the model relative humidity still is less than 100% ($U < 1$). The basic question is therefore how the converging vapour is shared between a general increase of the relative humidity in the grid box on one hand and condensation on the other hand. The consequent question then is how the condensing vapour is partitioned between already existing clouds and formation of additional cloud volume. Note that those principle questions apply regardless of type of subgrid scale condensation, i.e., whether it is stratiform or convective.

For the remaining discussion, we will make the simplifying assumption that the cloud fills the gridbox in the vertical direction, so partial cloud cover implies subgrid scale in the horizontal direction. The figure below is a sketch of the condensation situation that we are considering.



The average gridbox values of the condensation quantities are

$$Q = b\hat{Q} \tag{3.1}$$

$$m = b\hat{m} \tag{3.2}$$

$$P = b\hat{P} = c_0 m\left[1 - e^{-\left(\frac{m}{b m_r}\right)^2}\right] \tag{3.3}$$

$$E_0 = (1-b)\hat{E} \tag{3.4a}$$

$$E_0 = E_r + E_{cloud} \tag{3.4b}$$

$$U = bU_s + (1-b)U_0 \tag{3.5a}$$

$$b = \frac{U-U_0}{U_s-U_0} \tag{3.5b}$$

A fundamental feature of convection is that it redistributes latent and released heat over the convectively unstable layer. This implies that the change of vapour content at an individual level does not necessarily correspond to the amount of released heat at the same level; it is only the vertically integrated amounts that have to be the same. In the case of stratiform condensation, the situation is fundamentally different, because it is then quite reasonable to assume that there is a one to one correspondance between the amount of condensed vapour and the amount of released heat in individual strata.

Following the averaging procedure demonstrated through for example (3.5a), the prognostic equations relevant for the present discussion become (with minor approximations)

$$\frac{\partial T}{\partial t} = A_T + \frac{L}{c_p}Q - \frac{L}{c_p}E_0 \tag{3.6}$$

$$\frac{\partial q}{\partial t} = A_q - Q + E_0 \tag{3.7}$$

$$\frac{\partial m}{\partial t} = A_m + Q - (P - E_h) - E_o \tag{3.8}$$

In accordance with the above discussion we notice that the terms connected with condensation in (3.6) and (3.7) are set identical to each other but of opposite sign, so the release of latent heat is assumed to take place at the location of vapour convergence.

In the stratiform case, it is natural to set conditions for the appearence of condensation in terms of relative humidity. From (3.5) we see that U_o is the threshold value (b=0) that the relative humidity has to reach before subgrid scale condensation will be allowed to appear. So we utilize relation (2.5) and rewrite (3.7). We then obtain a relation containing the tendencies for U and for T. We eliminate the latter with the aid of (3.6) and obtain (in analogy with (2.7))

$$\begin{aligned} Q &= \left(A_q - \frac{c_p}{L} U S_q A_T + U q_s \frac{1}{p}\frac{\partial p}{\partial t} - q_s \frac{\partial U}{\partial t}\right)(1+US_q)^{-1} + E_o = \\ &= \left(M - q_s \frac{\partial U}{\partial t}\right)(1+US_q)^{-1} + E_o \end{aligned} \tag{3.9}$$

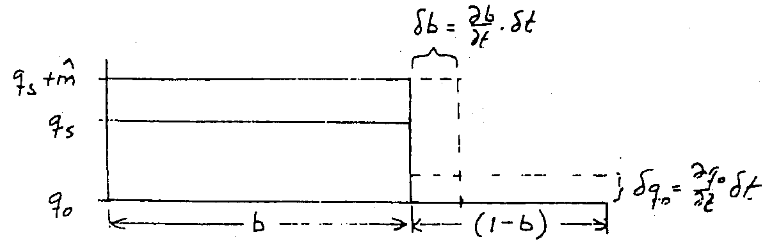
Thus, to close the system, we need an independent relation for the U-tendency. This is a concrete demonstration of the earlier mentioned problem as to how the converging vapour should be partitioned on moistening in general and on condensation.

To close the system, we adopt the following hypothesis

i) the portion of the converging vapour that goes to the already cloudy part of the box - i.e. bM - is used for continued condensation

ii) the remaining part of the vapour that converges into the grid box - i.e. (1-b)M - is used for
a) primarily maintaining the (prescribed) relative humidity in (1-b)
b) in the second place a change of the cloud cover in the box.

This hypothesis is visualised in the sketch below.



Thus, item ii) above takes the following mathematical form

$$(1-b)\frac{\partial q_o}{\partial t} + (q_s - q_o + \hat{m})\frac{\partial b}{\partial t} = (1-b)M + E_o \tag{3.10}$$

We furthermore assume that the threshold value U_o may be expressed as a function of b, F(b), instead of being an absolute constant. Hence,

$$U_o = U_{oo} + F(b) \tag{3.11}$$

The function F has to have the following end values

$$F(0) = 0 \quad ; \quad F(1) = U_s - U_{oo} \qquad 12)$$

With regard to (3.11), the q_o tendency is

$$\frac{\partial q_o}{\partial t} \simeq q_s \frac{\partial U_o}{\partial t} = q_s F' \frac{\partial b}{\partial t} \quad ; \quad F' = \frac{dF}{db} \tag{3.13}$$

When considering relations (3.5a), (3.11) and (3.13), we obtain the following U-tendency from (3.10)

$$q_s \frac{\partial U}{\partial t} = \frac{(U_s - U_{oo} - F + (1-b)F')}{U_s - U_{oo} - F + (1-b)F' + \frac{m}{bq_s}} \cdot \left[(1-b)M + E_o\right] \tag{3.14}$$

Inserting (3.14) into (3.9) we get the following expression for the release of latent heat

$$Q = \left[\left(bN + \frac{m}{bq_s}\right)\cdot M + E_o\left(1 + US_q\right)\left(N + \frac{m}{bq_s}\right) - NE_o\right] \times$$
$$\times \left[\left(1 + US_q\right)\left(N + \frac{m}{bq_s}\right)\right]^{-1} \tag{3.15}$$

where

$$N = U_s - U_{oo} - F + (1-b)F' \qquad (3.16)$$

The closed system that handles the parameterization of stratiform condensation consists of equations (3.6) - (3.8) and (3.15). The parametric description of the microphysical processes is given by expressions (3.3) - (3.4).

The partial cloud cover is obtained from the definition of the grid point value of relative humidity, (3.5a,b), provided that we know the relative humidity in the clear air of the box; one assumption about this is made through relations (3.11) - (3.12). Note that there is no simplifying assumption involved in obtaining the diagnostic relation (3.5a) or the inverted form (3.5b). So principally, we have an exact expression for the partial cloud cover. However, experience gained so far indicates that it is not possible to find a simple, (or perhaps any) generally valid relation between the relative humidity and the fractional cloud cover. Slingo (1980), for example, shows an expressive diagramme of this situation. In terms of this presentation, this means that U_o or U_{oo} are not fixed value parameters, but quantities that have to be related to, for instance, orography and synoptic situation. In order to describe the fractional cloud cover, we may eventually have to employ a parametric relation that contains not only relative humidity but other meteorological variables and orographic quantities as well.

4. PARAMETERS AND SPECIAL CONSIDERATIONS

4.1 Parameters

Below we give a summary of the introduced parameters and their expected magnitudes.

Parameter	magnitude
c_o	$\sim 10^{-4}$
m_r	$\sim 0.3 \cdot 10^{-3}$
k_E	$\sim 10^{-5}$
U_{oo}	~ 0.8

Optimum magnitudes for the above parameters have to be found through experimentation with the large scale model that is used. It is conceivable that the parameterization approach can be improved if we give functional forms to some of the parameters.

Thus, we may simulate the coalescence process by making c_o an increasing function (and perhaps m_r a decreasing function) with increasing precipitation intensity through an atmospheric layer in question. We may furthermore simulate an enhanced growth rate from cloud droplets to precipitation particles in clouds where ice crystals and water droplets coexist (the Bergeron-Findeisen process). In this case we let the two above mentioned parameters increase and decrease repsectively as the temperature decreases from about 268 K to about 245 K. For a further temperature drop, the parameters should maybe cease to decrease and to increase respectively as the probability for existence of exclusively ice particles becomes gradually larger for those low temperatures.

4.2 Special considerations

The experience gained so far from modelling work including parameterization (in many cases extremely simplified) of cloudiness has shown that cirrus cloud and stratocumulus cloud both require particularly elaborate treatment.

From the point of view of radiation, it is important that the cirrus cloud is given the right density and partial transparancy and the right duration. These very cold (ice crystal) clouds may require a particular description of the rate of release of precipitation since the crystals grow through diffusion and not through coalescence. This implies that the rate of release of precipitation cannot be described in terms of cloud water content. Instead, the rate of precipitation release is essentially governed by the vapour convergence in the grid box and the subgrid scale upwinds that carry the ice crystals. Those effects consequently also determine the typical water content of the cirrus cloud.

The partial cirrus cloud-cover may also need its own description with regard to the fact that the equilibrium vapour pressure over an ice surface is lower than over a water surface. It may be that, for condensation to take place, a high relative humidity is required, partly because of lack of effective freezing nuclei. Then, once a cirrus cloud is formed, it may be relatively resistant during advection since the humidity may be high enough to be near saturation with respect to ice.

5. REFERENCES

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