THE VERIFICATION OF OBJECTIVE ANALYSES:
DIAGNOSTICS OF ANALYSIS SYSTEM PERFORMANCE

A. Hollingsworth and P. Lönnberg
European Centre for Medium Range Weather Forecasts
Reading, RG2 9AX, U.K.

Abstract

We use the spatial coherence of the fit of analyses to observations as the basis for new diagnostics of the performance of any linear objective analysis system. The diagnostics provide answers to such questions as:

* Is the system using mass (or wind) observations efficiently?;

* Is data on mass/wind balance used efficiently?;

* What is the effective data density?

Up to now these questions could only be answered indirectly, by evaluation of the forecasts resulting from the analyses, or by study of the changes made by the initialisation. The lack of simple objective methods to address these questions has sometimes resulted in controversy about the interpretation of data impact studies.

Suppose, in a practical analysis system, that the observation errors are uniform and uncorrelated, and the observations are homogeneously distributed. We show (for any practical linear analysis system) that if the Observation-minus-Analysis (OmA) auto-correlations are positive when extrapolated to zero separation, then the analysis has certainly not extracted all the information from the observations, and does not fit the data to within the observational error. If however the extrapolated OmA correlations are negative, then the analysis system does fit the data to within observational accuracy. If, in addition, the weights given to the observations in the analysis of the observed values are known, one can derive useful estimates of the analysis error at the observation points. This last result leads to an estimate of the effective data density. Along with the new verifications of mass and wind analyses, methods are developed to estimate the effectiveness with which data on mass-wind balance have been used.

The results for practical analysis systems are used to interpret empirical determinations of the OmA auto-correlations in the ECMWF analysis system, for data with uncorrelated errors. We shall say that a practical analysis system is efficient if the data are fitted to within observational error, and is inefficient otherwise. The methods demonstrate that operational mid-tropospheric wind analyses over North America are quite efficient, but that the windshear and thickness analyses near the tropopause over North America are inefficient. The analysis of mass wind balance over North America is efficient for any single level, but the analysis of thermal wind balance for thin layers near the tropopause is inefficient, most notably on short horizontal scales.

The analysis of the wind shear near the tropical tropopause is somewhat better than over North America, probably because of the use of sharper vertical structure functions.

1. INTRODUCTION

The most expensive aspect of producing a weather forecast is the taking and transmission of observations. The annual cost of this essential prerequisite to the forecast process far exceeds the annual cost of running a forecast centre. It is necessary therefore to ensure that forecast centres make the best possible use of the observations. This paper is an attempt to quantify the extent to which an operational data assimilation system extracts all useful information from the radiosonde network.

Many investigations of the quality of objective analyses have used either bulk verification statistics or forecast verifications to evaluate analysis performance. These methods are valuable but they have important limitations. The forecasts are the acid test of analysis quality, but they are expensive, and they do not provide the guidance needed to identify analysis problems. On the other hand, bulk statistics on the mean and standard deviation of the Observation minus Analysis (OmA) differences are cheap to generate and have proved very useful in diagnosing 'zero-order' problems with the data or with the assimilation system (Hollingsworth et al, 1986, Uppala, 1987). Once the blatant problems have been eliminated, one needs more refined diagnostics to identify 'first-order' problems which must be overcome to bring a system to a high level of performance.

In this paper we show that the spatial coherence of the fit of the analyses to the data is a sensitive diagnostic of the accuracy of the analysis system, provided the observation errors are uncorrelated. The spatial correlations of the Observation minus Analysis (OmA) differences provide both qualitative and quantitative information on analysis accuracy, and are much more informative than a bulk measure of the fit.

We shall say that an analysis system which fits the data to within observational error is efficient, and that an analysis system which does not fit the data in this way is inefficient. The qualitative results on analysis accuracy provided by the spatial coherence are derived and discussed below, and are summarised here for convenience: If the (extrapolated) OmA spatial correlation is positive at zero station separation then the analyses do not fit the data to within the observational accuracy, and the analysis system is inefficient. If the (extrapolated) OmA spatial correlation is negative at zero station separation then the analyses do fit the data to within the observational accuracy, and the analysis is efficient.

A simple example may illustrate this result. Suppose an analysis system is presented with two series of equally accurate observations from a pair of closely spaced and unbiased stations. One expects the truth to lie somewhere between the two observations, and so a good analysis system will compromise between them. In such a case there will be a zero or negative correlation between the two time series of OmA differences at the stations. If however the analysed values at the two stations tend to lie to one side or the other of both observations, then the analysis system cannot be very accurate, and the correlation of the OmA differences will be positive.

The quantitative information on practical analysis accuracy is derived for analysis systems which are linear in the observations. Many operational analysis systems are linear in this way. For such linear systems one can always define the weight given to an observation in the analysis of the observed value. There is a simple quadratic relationship between the rms analysis error at the observation points, the average value of these weights at the observation points, and the value of the OmA correlation at zero separation (derived by extrapolation). If a linear analysis system is efficient, knowledge of the average of these weights, and of the OmA correlations, provides a useful quantitative estimate of the rms analysis error at the observation points.

Amongst linear analysis systems there is the theoretical, but well-defined, optimal interpolation analysis system (O/I) (Gandin, 1965; Phillips, 1982) which minimises the expected rms value of the analysis error at all points, including the observation points. For this theoretical optimal analysis system the quadratic relation between the weight, the OmA correlation at zero separation, and the rms analysis error, simplifies to a pair of equalities: both the weight given to the observation in the analysis of the observation, and the average squared analysis error at the observation points, are proportional to the value of the OmA spatial correlation extrapolated to zero separation. These results provide a means for judging how well an efficient analysis system performs relative to the theoretically optimum system. They also enable one to define a measure of the effective density of an observing network.

The reasoning which leads to these results is developed in Sections 2 and 3. We begin in Section 2 by reviewing the definition of the theoretical optimum interpolation (O/I) analysis method. Many operational analysis systems are approximations to this method. None of them are optimal, because of limitations in our knowledge of the necessary statistics, or because of limitations on the amount of data used in the calculation. We derive the analysis error covariance matrix at the observation points for the theoretical O/I analysis. We show that for the O/I analysis, the off-diagonal elements of the analysis error covariance matrix can be calculated directly from the corresponding terms of the OmA covariance matrix, provided the observation errors are uncorrelated. This result implies (for the theoretical O/I system), that one can get a good estimate of the diagonal terms of the analysis error covariance matrix by binning the off-diagonal terms of the OmA covariance matrix according to station separation, and then extrapolating them to zero separation.

This last result for the theoretical O/I analysis system suggests that it ought to be possible to estimate the diagonal terms of the analysis error covariance matrix of a practical system through extrapolation of the off-diagonal terms of its OmA covariance matrix to zero station separation.

For any practical analysis system, the OmA covariances may be readily calculated for a wide range of station separations. Earlier work on forecast errors, (Drozdov and Shepelevskii, 1946; Gandin, 1963; Rutherford, 1972; Hollett, 1975; Julian and Thiebaux, 1975; Hollingsworth and Lönnberg, 1986 (hereafter called HL); Lönnberg and Hollingsworth, 1986 (hereafter called LH); Thiebaux et al, 1986) provides useful guidance on methods of extrapolation.

In Section 3 we use the guidance of Section 2 to derive a simple quadratic relation (Eq 15) between the extrapolated OmA correlations, the weights used in the analysis, and the rms analysis error at the observation points; the relation is valid for any linear analysis system, provided the observation errors are uncorrelated and uniform, and the observation network is homogeneous. In the theoretical O/I system the quadratic relation reduces to a pair of simpler equations (Eq 20 and 21). The results of Section 3 show that in an efficient practical analysis system the extrapolated correlation should be negative. In the theoretical optimal system its value should be the negative of the weight used for the local observation in the analysis of the observed value. It follows that if the practical system performs as well as the theoretical system, then the weight actually used in the analysis calculation should turn out to be close to the weight determined a-posteriori from the analysis verification. In a sense this is a test of the practical analysis through back-substitution. If the practical analysis system is efficient, then one can determine the rms analysis error at the observation points. One can also make an estimate of the effective data density. This latter quantity indicates if there is a potential for increasing the resolution of the analyses.

The results of Section 3 can be used to examine the efficiency of operational analyses of geopotential, wind, thickness and wind shear. In Sections 4 and 5 we use them to show that ECMWF operational wind analyses in mid-troposphere are quite efficient over North America, but are inefficient near the tropopause. In Section 5 we extend the methods to provide estimates of the efficiency with which data on mass-wind balance have been used. The same methods can also be used for examination of the balance between wind-shear and thickness. We show that over North America the data on mass-wind balance is used reasonably well at single levels near the tropopause, but that data on the balance between wind shear and thickness is not used efficiently near the tropopause. Section 6 examines the quality of the upper-tropospheric wind analyses in the Tropics, and concludes that the analyses are reasonably good at observation points, insofar as their quality can be judged from our methods with a sparse network. After a summary of the main points, Section 7 discusses the implications of the results for future work.

All the present results are strictly valid only at observation points, because of the correlation between analysis error and observation error, whereas results from OmF differences could be assumed valid at any point within the domain of study. However an analysis system which performs badly at the observation points can hardly be expected to do well away from them. Although the present results are inherently limited by this restriction, they are nevertheless valuable diagnostics of analysis system performance.

The results presented in the paper are based on calculations with radiosonde data. Many observing system studies indicate that radiosondes are perhaps the most vital single component of the Global Observing System in the extra-tropics of the northern hemisphere. Given the simple nature of their error structure, their importance makes them a suitable subject for this study. The methods are general, and may be applied to mobile observing systems such as satellites, aircraft, and ships. We shall consider the degree to which the assimilation system extracts all useful information from these systems in a later study.

2. THE O/I ESTIMATE OF THE ANALYSIS ERROR COVARIANCE AT OBSERVATION POINTS

Many practical operational analysis systems have been developed as approximations to the theoretical optimum interpolation (O/I) analysis method developed by Gandin (1965). The multivariate form of the O/I analysis is discussed by Phillips (1982) who shows that the theoretical multivariate O/I analysis is complete in the sense that a variational analysis cannot extract (from the data) any further information on slow modes, provided the O/I analysis meets the following conditions: 1) the analysis is provided with a first guess containing only slow modes; 2) the first guess error covariances used are for slow mode errors only, and are accurately specified by a power spectrum of slow mode error; the observation error covariances must also be accurately known along with the cross-covariances of observation and first guess error; 3) all observations are used in the analysis for each grid-point variable.

No practical operational analysis system satisfies all the required conditions. Nevertheless an understanding of the properties of the theoretical O/I system provides useful guidance for an evaluation of the performance of any practical analysis system. In this section we derive an expression for the analysis error covariance matrix $\underline{\underline{A}}$ for the theoretical O/I system (Eq. 5), where $\underline{\underline{A}}$ is defined at observation points. We then show (Eq 6b) that the off-diagonal elements of $\underline{\underline{A}}$ are equal to the off-diagonal elements of the OmA covariance matrix (with sign changed), provided the observation errors are uncorrelated. This suggests that by binning the off-diagonal elements of the OmA covariance matrix of the O/I analysis according to station separation and extrapolating to zero separation, one can estimate the diagonal elements of $\underline{\underline{A}}$, i.e. the analysis error variances for the O/I analysis. Finally we show that for any linear analysis there is a simple relation (Eq. 7) between the weight given to an observation in the analysis of the observed value and the analysis error.

Consider the analyses made by the theoretical method of multivariate O/I using correct (climatological) statistics for first-guess and observation error, and using all of the observations in a single matrix calculation (Phillips 1982). Let $\underline{\underline{D}}$ represent the covariance matrix of the observation errors at the set of observation points, and let $\underline{\underline{P}}$ represent the covariance matrix of the prediction errors for the same observations; both scaled by the magnitudes of the prediction error in the usual way.

For any linear analysis system, the analysis equation for the analysed values at the observation points may be written in the form

$$\underline{\alpha} = \underline{\underline{W}}\,\underline{\delta} \qquad (1)$$

where $\underline{\delta}$ is the vector of differences between observation and first-guess at the observation points (the data increment);

$\underline{\alpha}$ is the vector of differences between analysis and first guess (the analysis increment);

[Wij] is the weight given to observation j in the analysis of observation i;

and both $\underline{\alpha}$ and $\underline{\delta}$ are normalised by the rms first-guess error.

For an optimum interpolation (O/I) analysis the weight matrix $\underline{\underline{W}}$ is given by

$$\underline{\underline{W}} = \underline{\underline{P}} \cdot (\underline{\underline{P}} + \underline{\underline{D}})^{-1} \qquad (2)$$

where $\underline{\underline{P}}$ is the correlation matrix of forecast error at the observation points for the observed variables, and $\underline{\underline{D}}$ is the corresponding scaled observation error covariance matrix.

Eq (1) may be rewritten in the form

$$\underline{a} = \underline{p} + \underline{\underline{W}} \cdot (\underline{d} - \underline{p}) \qquad (3)$$

where $\underline{a}$ is the vector of analysis errors at observation points, $\underline{p}$ is the vector of first guess errors at the observation points, and $\underline{d}$ is the vector of observation errors at the observation points.

Post-multiplying each side of Eq (3) by its transpose and then taking expectations gives

$$\langle \underline{a} \cdot \underline{a}^t \rangle = \underline{\underline{P}} - \underline{\underline{W}} \cdot \underline{\underline{P}} - \underline{\underline{P}} \cdot \underline{\underline{W}}^t - \underline{\underline{W}} \cdot (\underline{\underline{P}} + \underline{\underline{D}}) \cdot \underline{\underline{W}}^t \qquad (4)$$

where < > denotes expectation, and we have used the fact that

$$\langle \underline{p} \cdot \underline{p}^t \rangle = \underline{\underline{P}} \text{ and } \langle \underline{d} \cdot \underline{d}^t \rangle = \underline{\underline{D}}.$$

For an O/I system this becomes

$$\langle \underline{a} \cdot \underline{a}^t \rangle = \underline{\underline{P}} - 2 \; \underline{\underline{P}} \cdot (\underline{\underline{P}}+\underline{\underline{D}})^{-1} \cdot \underline{\underline{P}} + \underline{\underline{P}} \cdot (\underline{\underline{P}}+\underline{\underline{D}})^{-1} \cdot (\underline{\underline{P}}+\underline{\underline{D}}) \cdot (\underline{\underline{P}}+\underline{\underline{D}})^{-1} \cdot \underline{\underline{P}},$$

or

$$\underline{\underline{A}} = \underline{\underline{P}} - \underline{\underline{P}} \cdot (\underline{\underline{P}}+\underline{\underline{D}})^{-1} \cdot \underline{\underline{P}} \qquad (5)$$

where $\underline{\underline{A}}$ represents the covariance matrix of the analysis errors at the observation points, scaled by the prediction errors.

The analysis error covariance matrix $\underline{\underline{A}}$ contains all the information needed on analysis errors at the observation points. This matrix is difficult to calculate either directly, since the analysis errors are unknown, or through the analytical expression (Eq 4). However the off-diagonal elements of $\underline{\underline{A}}$ can be determined from the corresponding elements of the covariance matrix of the OmA differences, provided the observation errors are uncorrelated.

An expression for the covariance of the difference between the observed and analysed values can be derived as follows. For any linear analysis algorithm, it follows from Eq (3) that

$$(\underline{d} - \underline{a}) = (\underline{\underline{I}}-\underline{\underline{W}}).(\underline{d} - \underline{p})$$

and

$$<(\underline{d} - \underline{a}).(\underline{d} - \underline{a})^{t}> = (\underline{\underline{I}}-\underline{\underline{W}}).<(\underline{d} - \underline{p}).(\underline{d} - \underline{p})^{t}>.(\underline{\underline{I}}-\underline{\underline{W}}^{t})$$

$$=(\underline{\underline{I}}-\underline{\underline{W}}).(\underline{\underline{P}}+\underline{\underline{D}}).(\underline{\underline{I}}-\underline{\underline{W}}^{t})$$

$$=(\underline{\underline{P}}+\underline{\underline{D}})-(\underline{\underline{P}}+\underline{\underline{D}}).\underline{\underline{W}}^{t}-\underline{\underline{W}}.(\underline{\underline{P}}+\underline{\underline{D}})+\underline{\underline{W}}.(\underline{\underline{P}}+\underline{\underline{D}}).\underline{\underline{W}}^{t} \qquad (6a)$$

For an O/I analysis, where $\underline{\underline{W}}=\underline{\underline{P}}.(\underline{\underline{P}}+\underline{\underline{D}})^{-1}$, one finds

$$<(\underline{d}-\underline{a}).(\underline{d}-\underline{a})^{t}>=\underline{\underline{D}} -(\underline{\underline{P}}-\underline{\underline{P}}.(\underline{\underline{P}}+\underline{\underline{D}})^{-1}.\underline{\underline{P}}) = \underline{\underline{D}}-\underline{\underline{A}} \qquad (6b)$$

It is shown in the Appendix that $\underline{\underline{D}} - \underline{\underline{A}}$ is positive definite.

Eq (6b) implies that the off-diagonal terms of the analysis error covariance matrix $\underline{\underline{A}}$ are given by the corresponding terms of the OmA covariance matrix, provided the observation error covariance matrix $\underline{\underline{D}}$ is diagonal, or equivalently that the observation errors are mutually uncorrelated. This condition is satisfied for certain important observing systems. Since the off-diagonal terms of $\underline{\underline{A}}$ are therefore readily available for a range of station separations, it is natural to suppose that the diagonal terms of $\underline{\underline{A}}$ can be estimated by extrapolation of the off-diagonal terms to zero station separation, when the matrix $\underline{\underline{D}}$ is diagonal. This idea is made explicit for the O/I analysis system in section 3.3 (Eq. 20) and is explored for practical analysis systems in the rest of the paper.

For later discussions of the correlation of observation error and analysis error in practical analysis systems, it is convenient to derive an expression for the diagonal terms of the weight matrix of any linear analysis system. If one post-multiplies Eq (3) by $\underline{d}^t$, and then takes expectations, one finds that

$$\langle \underline{a}.\underline{d}^t \rangle = \underline{\underline{W}}.\underline{\underline{D}}$$

For uncorrelated observation errors this implies that at each observation point

$$(\sigma_a/\sigma_d)_i \; R_{ii} = W_{ii} \qquad (7)$$

where σ_a and σ_d are the rms values of analysis error and observation error at the point i, (each normalised by the rms prediction error) and R_{ii} is the correlation coefficient between the observation error at observation point i and the analysis error at observation point i. Eq (7) says that the weight given to an observation in the analysis of the observed variable at the observation point is proportional to the correlation between observation error and analysis error at the observation point. Since this correlation is positive in the theoretical O/I system (see Eq. 18), one expects it to be positive in a practical system also, so the weights W_{ii} too will be positive, provided always that the observation errors are uncorrelated.

3. ESTIMATION OF ANALYSIS ERROR AT OBSERVATION POINTS FOR PRACTICAL SYSTEMS

In practice, one cannot use all observational data in a single analysis calculation for the globe, because of the sheer volume of data to be considered. Moreover one can never have perfect knowledge of the statistical properties of prediction or observation error. Practical compromises have to be made in every operational system. The results of Section 2 for an O/I analysis cannot be strictly valid for any working system, but they do offer certain ideas on how to estimate analysis error in a practical analysis system.

In this section we explore the suggestions of the last section for a simple approach to the problem of estimating analysis error. The main focus is the spatial correlation structure of the OmA differences. This correlation provides useful information on the analysis errors, and on the efficiency of the analysis algorithm, provided one can assume that the observation errors are spatially uncorrelated. The more difficult case of spatially correlated observation error will be addressed in a later study.

In the calculations presented in the rest of the paper, we discuss analyses of a set of data which are homogeneously distributed over the area of interest, with the data being of uniform quality. In such a case the weight given to the local observation in the analysis of the observed variable will also tend to be homogeneous. In section 3.1 the OmA covariance is separated into spatially correlated and spatially un-correlated parts. A method is proposed to extrapolate the spatially correlated part of the OmA covariance to zero station separation; we shall call the extrapolated value the zero intercept, or simply the intercept. A simple argument shows that if the intercept is negative then the analysis fits the data to within observational error (i.e. is efficient), the analysis system is inefficient if the intercept is positive.

The rest of the section is concerned with the question 'how negative should the intercept be?' in an efficient system. The answer to this question provides indications of where there is scope for improvement in an efficient analysis system. In Section 3.2 we derive an equation (Eq 15) which relates the rms analysis error at the observation points (called simply the analysis error) to the intercept, and to the weight used in the analysis of the observed value at the observation point (called simply the weight). One can delineate a rather small area on the weight-intercept plane within which the weight and intercept of any efficient practical analysis system can be found. In section 3.3 we show that the weight and intercept for the theoretical O/I analysis must lie along a line in the weight-intercept plane which roughly bisects the area in which an efficient practical system must be found. This suggests that efficient practical systems which lie far from this line have scope for further improvement, but that systems which lie close to this line probably have little scope for further improvement. In section 3.4 we interpret the analysis error estimates derived in section 3.2 to provide a definition of effective data density. This can indicate if the observations would support an increase in the resolution of a practical analysis system.

3.1 The spatially correlated component of the OmA differences

Given a homogeneously distributed set of observations of uniform quality, and the corresponding set of analyses based on them, one can express the spatial variation of the covariance of the OmA differences as

$$\langle (d-a)_i, (d-a)_j \rangle = \sigma_d^2 \, \delta_{ij} - 2 \, \sigma_d \, \sigma_a \, R_{a/o}(r) + \sigma_a^2 . R_{a/a}(r)$$

where

r is the separation of points i and j,

d=observation error at a single point (assumed unbiased and with zero spatial correlation),

a=analysis error at a single point,

$R_{a/o}(r)$=correlation of observation and analysis error,

$R_{a/a}(r)$=auto-correlation of analysis error, and

δ_{ij}= Kronecker delta.

The function b(r) defined by

$$b(r) = -[2.\sigma_d \, \sigma_a . R_{a/o}(r) - \sigma_a^2 . R_{a/a}(r)] \qquad (8)$$

is the spatially correlated part of the OmA covariance.

The OmA covariance is then the sum of the spatially correlated part b(r) and the spatially uncorrelated part due to the observation error:

$$\langle (d-a)_i, (d-a)_j \rangle = \sigma_d^2 \, \delta_{ij} + b(r) \qquad (9)$$

The function b(r) is well defined at observation points. We extend the definition to all values of r in the following way. By analogy with the methods used to fit the spatial correlations of forecast errors we define the function b(r) by finding a truncated series of Bessel functions which provides a least squares fit to the empirical OmA correlation data at the observation points and which satisfies conditions of boundedness at infinity and zero slope at the origin. Clearly, b(r) should tend to zero at large separations (HL/LH). The truncated series expansion may be extrapolated to the origin to give an intercept b(0), which provides a key indicator of analysis performance, for one then has the relation

$$\sigma_d^2 = \langle (d-a)^2 \rangle - b(0) \qquad (10)$$

where < > denotes expected value.

One might object that the analogy between OmF and OmA statistics is not good enough to justify the approach used here. Forecast error covariances are usually assumed independent of the positions of the current set of observations. Such an assumption is not likely to be as valid for the analysis error. If there are enough station pairs within each distance bin used, then one may expect that the dependence on observation position will be averaged enough to product a statistically reliable result. Large volumes of data are used for our calculations, as shown in section 4.

Eq (10) is the first important result of this paper. If the intercept b(0) is negative or zero then the analysis system fits the data to within observational accuracy and is efficient. If b(0) is positive then the analysis system is inefficient.

Eq (10) also provides a new method of estimating the observation error to be used in the analysis calculation and is a useful check on the more usual method using forecast error statistics.

3.2 The weight-intercept relation for a practical analysis system

In the rest of this section we address the question 'how negative should b(0) be?'. We derive an important relation Eq (15) between the analysis error, the weight and the intercept, and use simple arguments to show that an efficient practical analysis should only operate in a restricted region of the weight-intercept plane, so that the analysis error can satisfy certain bounds.

From Eq (8) one has the relation

$$\sigma_a^2 - 2.\sigma_a \sigma_d.R_{o/a}(0) - b(0) = 0. \qquad (11)$$

which is the analogue of Eq (6b) for a practical analysis system. The second term in Eq (11) has a close relation to the average weight given to the observations as may be seen from Eq (7). A simpler derivation of Eq (7) is as follows:

Suppose the observing network is fixed and the analysis algorithm is linear with fixed parameters, so that the weight given to an observation in the analysis of any variable at any point is also fixed. Let w denote the weight given to the collocated observation in the analysis of the observed quantity at a typical observation point. Then the analysis error, a, at the observation point is given by

$$a = w \cdot d + \text{other terms}$$

where d is the observation error at the point. Multiplying by d and taking expectations it follows that

$$\langle a.d\rangle = w \langle d.d\rangle \qquad (12)$$

if all observation errors are uncorrelated. Since the left hand side of Eq (12) may be rewritten as

$$\langle a.d\rangle = \sigma_a \sigma_d.R_{o/a}(0),$$

and the right hand side of Eq (12) may be rewritten as

$$w \cdot \langle d.d\rangle = w \cdot \sigma_d^2 ,$$

Eq (12) may be recast as

$$w = \frac{\sigma_a}{\sigma_d} \cdot R_{o/a}(0) \tag{13}$$

which is equivalent to Eq (7). This demonstrates the close relation between the second term of Eq (11) and the weight.

Using the assumption that the observing network is homogeneous, and that the data are of uniform quality, the weight given to an observation in the analysis of the observed variable at the observation point will tend to be homogeneous. It follows from Eqs (11) to (13) that the average analysis error at the observation points can be estimated if the average weight w and the intercept b(0) are known.

As noted in the discussion of Eq (7), the correlation $R_{o/a}(0)$ should be positive in a linear analysis, and so one expects that the weight w should be positive in a linear analysis. To see the implications of Eq (11) it is convenient to rewrite the equation in simpler notation in the form

$$\left(\frac{\sigma_a}{\sigma_d}\right)^2 = 2\left(\frac{\sigma_a}{\sigma_d}\right).R + B \tag{14}$$

where $R=R_{o/a}(0)$ and $B=b(0)/(\sigma_d^2)$. Using Eq (13) this may be rewritten as

$$\left(\frac{\sigma_a}{\sigma_d}\right)^2 = 2.w + B. \tag{15}$$

Eq (15) is the second important result of this paper and implies that isolines of σ_a/σ_d are straight lines in the w-B plane.

The solutions of Eq (14) may be written in the form

$$\sigma_a/\sigma_d = R \pm (R^2 + B)^{\frac{1}{2}} \tag{16}$$

Substituting for w from Eq (13), Eq (16) becomes

$$w = R^2 \pm R\,(R^2 + B)^{\frac{1}{2}} \tag{17}$$

Eq (17) implies that isolines of R are parabolas in the w-B plane. Fig 1 shows isolines of σ_a/σ_d and R as functions of w and B; the isolines of σ_a/σ_d are straight lines, while the isolines of R form a family of parabolas which are tangent to each other at the origin. The correlation R must lie in the range [-1, 1], so the isolines of the correlation are only drawn for values in this range. The branch of the solution is chosen so that w and R have the same sign, as required by Eq (13) since σ_d and σ_a are positive.

In principle therefore one might expect an analysis system to operate within the area on the w-B plane bounded by the analysis error isoline $(\sigma_a/\sigma_o)^2 = 1 = 2w+B$, and the parabolic isolines given by R=1 and R=-1. The following arguments show that an efficient practical analysis system should operate in a very much smaller area of the w-B plane:

(i) w positive

As discussed already, the weight given to an observation in the analysis of the observed variable at the observation point should be positive, so the analysis system should operate in the right half of the w-B plane.

(ii) B negative

We have defined an efficient analysis system as one which fits the data to within observational error. For an efficient analysis system Eq (10) implies that the intercept B should be negative.

(iii) Upper bound on analysis error

If only one observation is present, and the ratio of observation error to background error is known accurately, Eq (4) shows that, for a practical system, the analysis error at the observation point is given by

$$\left(\frac{\sigma_a}{\sigma_d}\right)^2 = \frac{1}{1 + \sigma_d^2}$$

The availability of additional observations ought to reduce the analysis error. The analysis system should therefore operate in the lower right quadrant of the w-B plane, below the line $\left(\frac{\sigma_a}{\sigma_d}\right)^2 = 1/(1+\sigma_d^2)$.

(iv) Upper bound on correlation implies lower bound on analysis error

Again, if only one observation is present and the ratio of observation error to background error is known accurately, Eq (13) shows that for a practical system the correlation between analysis and observation error at the observation point is given by

$$R = \frac{\sigma_a}{\sigma_d} = \frac{1}{(1+\sigma_d^2)^{\frac{1}{2}}}$$

The availability of additional observations would reduce the correlation between analysis error and the error of any one observation. The value $R_{max}=1/(1+\sigma_d^2)^{\frac{1}{2}}$ therefore provides an upper bound for R. The corresponding parabolic isoline on the w-B plane is a lower bound to the area of the plane where a practical analysis system may be expected to operate. Combining this result with the conclusion of item (iii) above implies that for fixed values of σ_d and B the following bounds are valid for σ_a/σ_d:

$$0 \leqslant \left(\frac{1}{1+\sigma_d^2}\right)^{\frac{1}{2}} - \left(\frac{1}{1+\sigma_d^2} + B\right)^{\frac{1}{2}} \leqslant \frac{\sigma_a}{\sigma_d} \leqslant \left(\frac{1}{1+\sigma_d^2}\right)^{\frac{1}{2}} \qquad (18)$$

To illustrate these bounds, Fig 2 shows the region of the w-B plane (hatched) which satisfy the bounds for two values of σ_d: $\sigma_d=1$ in Fig 2a and $\sigma_d=0.5$ in Fig 2b. In both figures the region where one expects an effective analysis system to operate is considerably smaller than the area outlined in Fig 1. Note too that the range of permissible weights depends strongly on the accuracy of the observations.

3.3 The weight-intercept relation for an O/I analysis system

Consider now the weight-intercept relation for a theoretical O/I system. For such a system we know from Eq (6b) that for any two points i and j,

$$\langle (d-a)_i, (d-a)_j \rangle = \sigma_d^2\, \delta_{ij} - \sigma_a^2\, R_{a/a}(r)$$

while we have just seen that for any linear analysis system

$$<(d-a)_i,(d-a)_j> = \sigma_d^2 - 2\,\sigma_d\,\sigma_a \cdot R_{a/o}(r)+\sigma_a^2 \cdot R_{a/a}(r)$$

These two relations can only be compatible at zero separation if in the theoretical O/I system $R\left(=R_{o/a}(0)\right)$ satisfies

$$R = \frac{\sigma_a}{\sigma_d}\ , \tag{19}$$

a result which is due to the optimal choice of weights. For the O/I system Eq (14) then takes the simple form

$$\left(\frac{\sigma_a}{\sigma_d}\right)^2 = -\,B\ , \tag{20}$$

while Eq (13) implies

$$w = -\,B \tag{21}$$

in an O/I system.

Eq (20) implies that for an O/I analysis the intercept -B determines the rms analysis error at observation points, as expected from the discussion of Section 2. It also follows that as more data is provided to an optimal system, the weight-intercept relation for the system should trace out the straight line w=-B, with the normalised intercept B tending to zero. This line is indicated on Figs 1 and 2a,b. We note that the line along which the theoretical O/I analysis must operate roughly bisects the area within which an efficient practical analysis system should operate. This implies that the theoretical analysis is efficient, a result which is proved directly in Eq (A.5) of the Appendix.

Eq (21) suggests a check of the performance of a practical analysis system through a form of back-substitution. The (a-priori) weight w used in the practical analysis is known, while the (a-posteriori) weight -B can be calculated from the OmA statistics. Equality of the a-priori and a-posteriori weights is guaranteed for the theoretical O/I analysis. Since the statistics and data selection used in a practical analysis calculation will normally have

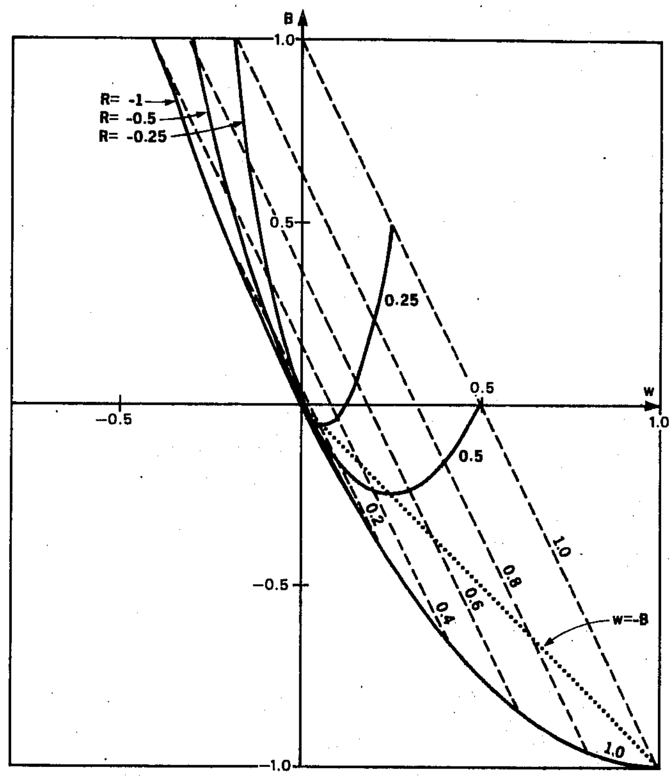

Fig 1 Isolines (solid) of the correlation R defined in the text (for $-1<R<1$, in steps of 0.25), and isolines (dashed) of σ_a/σ_o (in the range 0 to 1 in steps of 0.2), plotted as functions of the weight w (horizontal axis), and the normalised intercept B (vertical axis).

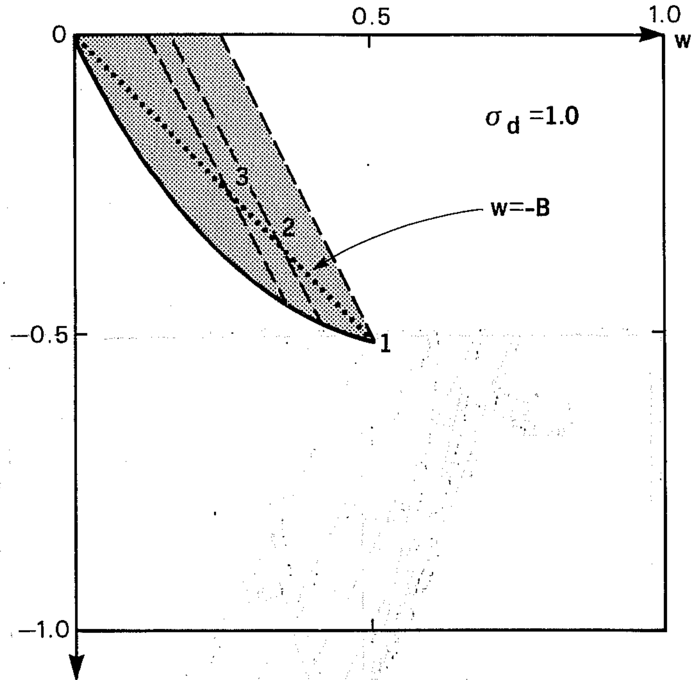

Fig 2a The shading denotes the area of the w-B plane where an efficient analysis system may be expected to operate if the normalised observation error is 1. The solid line denotes the upper bound of the correlation R, and the dashed lines indicate isolines of analysis error, and are labelled 1,2,3, etc, according to the corresponding effective data density, as discussed in the text.

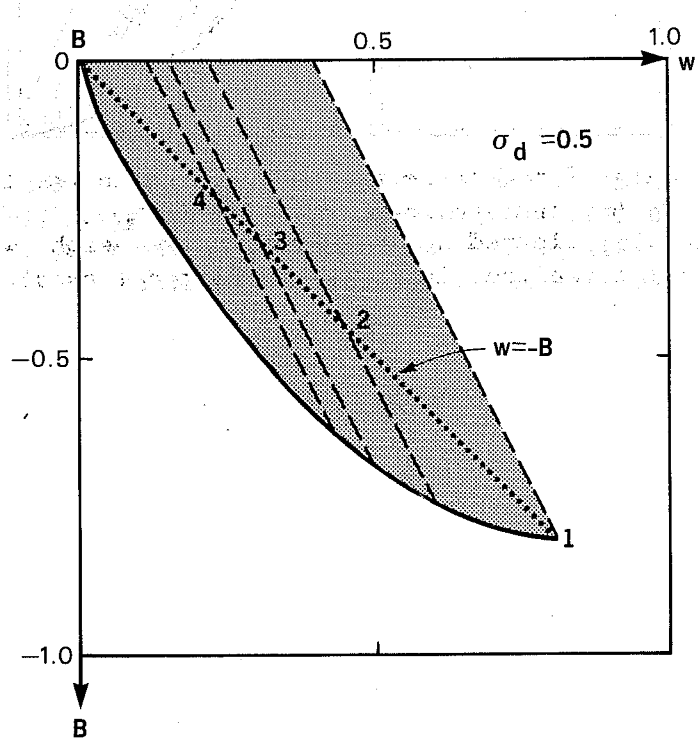

Fig 2b As Fig 2a when the normalised observation error is 0.5.

limitations, one cannot expect that any practical system will satisfy w = -B, but it should approximately satisfy this relation. If an analysis system is efficient, it is reasonable to suppose that a large difference between w and -B is an indication that there is considerable scope for improvement. Approximate equality of w and -B for a practical system suggests that there is little scope for further improvement.

3.4 An estimate of the effective data density

The precise point along the w=-B line where a theoretical O/I analysis will be found depends on the analysis error, and therefore on the accuracy of the first-guess, on the accuracy of the observations, and on the effective data density. Similarly, the precise point within the permissible range where one will find the B and w parameters for an efficient practical analysis system will depend on the formulation of the algorithm, on the accuracy of the first-guess, on the accuracy of the observations, and on the effective data density. Quantification of the effective data density is our next topic.

The isolines of σ_a on the w-B plane provide a method to estimate the effective data density, if w, σ_d and B are known for a given analysis system. The estimate is derived by calculating how many co-incident observations of the same quality would be needed at an isolated observation point to give the same analysis error at the point.

Suppose that N observations of uniform rms error σ_d are coincident, that the observation errors are uncorrelated, and that all the coincident observations are used in an O/I analysis. It can be shown rigorously that the O/I algorithm has the effect of averaging the observations into a 'super-observation' (Lorenc, 1981) with corresponding rms observation error σ_s given by

$$\sigma_s^2 = \sigma_d^2/N \ .$$

If the coincident observations are the only ones available, the analysis error for the observed variable at the observation point is given by

$$\sigma_a^2 = \sigma_s^2/(1+ \sigma_s^2)$$

so that

$$\left(\frac{\sigma_a}{\sigma_d}\right)^2 = \frac{1}{N+\sigma_d^2} \tag{22}$$

Knowing w and B for a practical analysis system, one can deduce σ_a/σ_d from Eq (15), and so one can solve Eq (22) for N, which we define as the effective data density. It is the number of coincident observations one would need to have at an isolated point to achieve the same level of analysis error at the isolated point. On Figures 2a,b we have drawn the isolines $\left(\frac{\sigma_a}{\sigma_d}\right)^2 = \frac{1}{N+\sigma_d^2}$ for N=1,2,3, and labelled them accordingly. This gives us a convenient set of nomograms to estimate the effective data density.

A high effective data density in a practical analysis system has implications for the resolution of the analysis system. In the theoretical O/I system, observational error is defined to include the sampling error due to unresolved scales. In some practical analysis systems (e.g. the ECMWF system) the spectral representation of the prediction error auto-correlations is truncated relative to empirical determinations of the OmF correlations. If the specified spectra of the prediction error auto-correlations are widened to resolve finer structure, (so that the length scales of the prediction error auto-correlations are reduced relative to the observation spacing), then the observation error must be correspondingly reduced. Higher resolution analyses will then be generated, and the implicit data redundancy will be reduced. An example for the North American network is discussed in the next section.

3.5 Discussion

In the last two sections we have considered the estimation of rms analysis error at observation points for both theoretical and practical analysis systems. For any efficient analysis system, the spatial correlation of the observation minus analysis differences should be negative at short separations and zero at large separations. The extrapolated value of the correlation at zero separation then provides an estimate of the observation error, Eq (10), and bounds on the analysis error Eq (18). Use of the normalised intercept B in conjunction with the average weight w provides an estimate of the analysis error at the observation point Eq (15).

For any analysis system, the occurrence of a positive intercept in the OmA extrapolation implies that the analysis algorithm does not extract all available information from the observations.

For a theoretical O/I analysis system, the normalised intercept B is proportional to the analysis error at the observation points Eq (20), and to the weight given to the observation in the analysis of the observed variable at the observation point Eq (21). For a practical analysis system, comparison of the normalised intercept B with the weight actually used provides a valuable check on how much scope is available for improving the system.

The normalised intercept also provides an estimate of the effective data density at each observation point. If this is substantially larger than one, then it means there is a possibility of reducing the apparent redundancy in the observations by increasing the resolution of the practical analysis system.

Care must be taken not to push the interpretation of the estimates of the analysis error too far. The calculation of the intercept b(0) is based on data at observation points, and so the estimates ONLY APPLY AT OBSERVATION POINTS. The estimates say nothing about the behaviour of the analysis error between observation points, where one expects an increase of analysis error to the level of the first guess error (roughly at the rate of the variation with distance of the square of the first guess error correlation). However, an analysis which is bad at the observation points will probably be bad between the observation points.

The above results apply to the spatial correlation of the OmA differences for a scalar quantity. The essential condition for the validity of Eqs (10), (11) was that observation error for adjacent stations be spatially uncorrelated. This is generally true for height observations in the horizontal, but not in the vertical (LH 1986). Since rawinsonde observational errors are independent for each wind component in the horizontal and in the vertical (HL 1986) and since the prediction errors for u and v are also uncorrelated at zero separation, it follows that the above arguments apply to the vector wind correlation <u,u>+<v,v>. For isotropic forecast errors the vector wind correlation is isotropic, even though the component correlations <u,u>, <v,v> are anisotropic.

4. DIAGNOSIS OF THE ECMWF ANALYSIS IN MID TROPOSPHERE OVER NORTH AMERICA

In this section we use the results of section 3 to test the performance of the ECMWF analysis system in mid-troposphere over North America; we show that the wind analyses are quite good there. Diagnostic results near the tropopause (Section 5) indicate that the analyses can be markedly improved in that region of large vertical shears.

4.1 The data

The data used in the study comprise North American data for 1200 GMT rawinsondes, for the corresponding analyses, and for the 6-hour forecasts used as background for the analyses, for a three month period from 1 December 1986 to 28 February 1987. Only observational data accepted and used by the analysis are considered in the calculations. For the correlation calculations, a minimum of 60 pairs of reports was required before a station pair was used. The correlations for station pairs were composited in 100 km bins and averaged using the Fisher z-transform before being plotted.

As background for the discussion, it is convenient to note the estimates of first guess error and observation error determined from the observation minus first-guess (OmF) differences for the period of interest, Dec 1986 to Feb 1987, for the North American rawinsondes. The methodology is discussed in HL/LH who showed results for the North American network for Jan-March 1983. Hollingsworth et al (1986) showed similar results for Jan-March 1984. Fig 3a,b shows the estimated observation and first-guess errors for height and wind, estimated from the 1986/87 data. For the heights the forecast errors are 2-3 m lower than those in HL/LH, while for the winds the forecast errors are about 1.5 m/s lower at the maximum near the tropopause.

Also shown on these plots are the vertical profiles of the rms OmA difference for height and wind. Since the rms observation errors and rms OmA differences were estimated from different data sources (OmF in one case and OmA in the other), it is encouraging that the OmA statistics show lower values than the

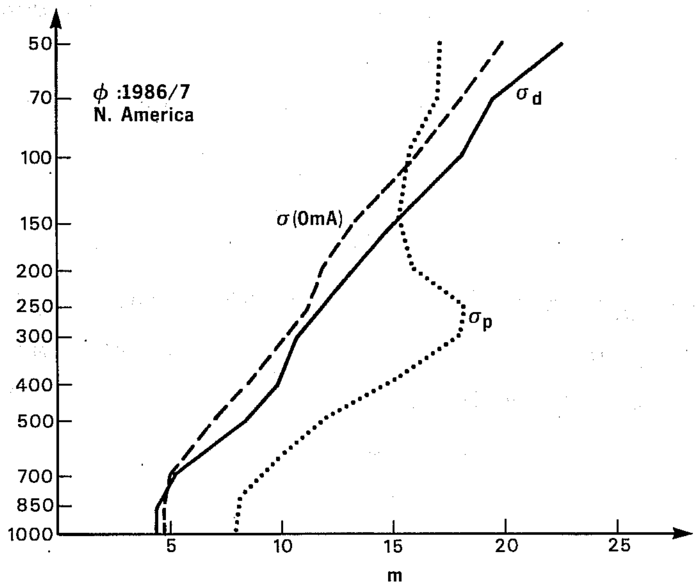

Fig 3 a) The height observation error (solid) and height forecast error (dotted), both estimated from the Observation minus 6-hour Forecast (OmF) statistics for North American radiosondes for Dec 1986 to Feb 1987. The dashed line shows the rms Observation minus Analysis (OmA) statistics for the same period.

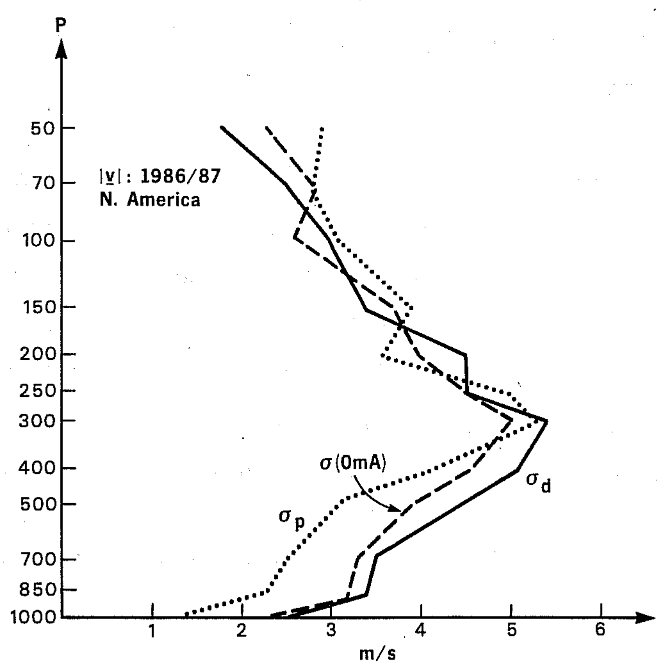

Fig 3 b) As a) for the wind statistics.

estimated observation errors, indicating that the analysis system is drawing slightly tighter to the data than the observation error, as it should. The main exceptions are the mass and wind analyses at 1000 and 850 mb, and the winds at 250, 150, 70, and 50 mb. As we shall see in Section 5, there are deficiencies in the analyses near the tropopause.

4.2 The weights and the intercepts for the 500mb wind field

Fig 4 shows the spatial correlation for the 500mb OmA statistics for the 500mb wind field. The number plotted beside each data point tells how many station pairs were used to generate the data point. For convenience the plots are normalised by the OmA covariance at zero separation.

For separations larger than 600km the correlation is essentially zero. Discounting the single pair of stations with smallest separation, the data between 250 and 600km suggest that the intercept would be about -0.25, giving a normalised B value of about -0.20. The rms value of the OmA differences for the vector wind is 3.95m/s, so the estimated observation error is 4.4 m/s based on the OmA statistics, and compares well with the value of 4.7 m/s determined from the OmF statistics in Fig 3. The intercept b would need to be -.41 (with a normalised value of $B = b(0)/\sigma_d^2$ =-(.41/1.41)=-.29) for the estimate from the OmA statistics to reach 4.7 m/s.

If the ECMWF analysis system behaved like the theoretical O/I system then the weight given to each of the wind components would be 0.2, based on the OmA statistics. The weights given to the zonal wind component at 500mb in a typical analysis of the 500mb wind field are shown in Fig 5. Over the areas with dense coverage the weight varies between 0.20 and 0.30, with values as large as 0.43 at isolated stations. The average value of the weights shown in Fig 5 is 0.275, which is in fair agreement with the value expected from the intercept: 0.20. The fact that the operational analysis system specifies a weak vertical correlation (0.18 between 700 and 500mb, and 0.34 between 500 and 400mb) for rawinsonde wind observation error may contribute to the differences between the average weight used (0.275) and the weight estimated from the intercept (0.20).

Assuming that the value for the weight in an O/I analysis of these data would lie between the two estimates from the practical analysis (say 0.25), and that

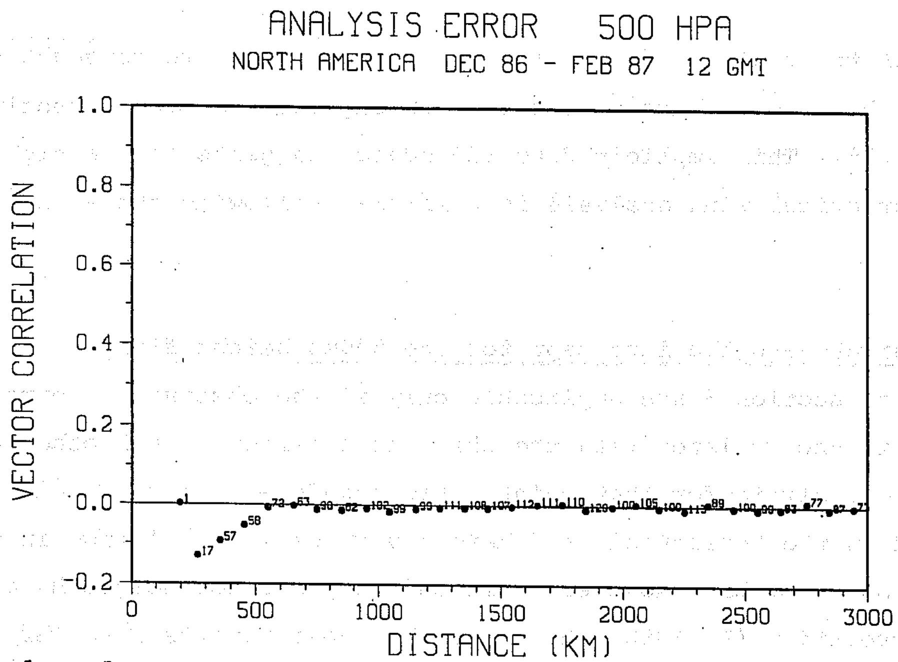

Fig 4 The dependence on station separation of the auto-correlation of the 1986/87 OmA North American differences for the 500mb vector wind. The numbers of station pairs contributing to each distance bin is indicated.

Since the wind observation errors have little spatial correlation, this auto-correlation should be zero for large separations and negative for short separations; the extrapolated value at zero separation (with sign changed) should equal the average of the weights shown in Fig 5.

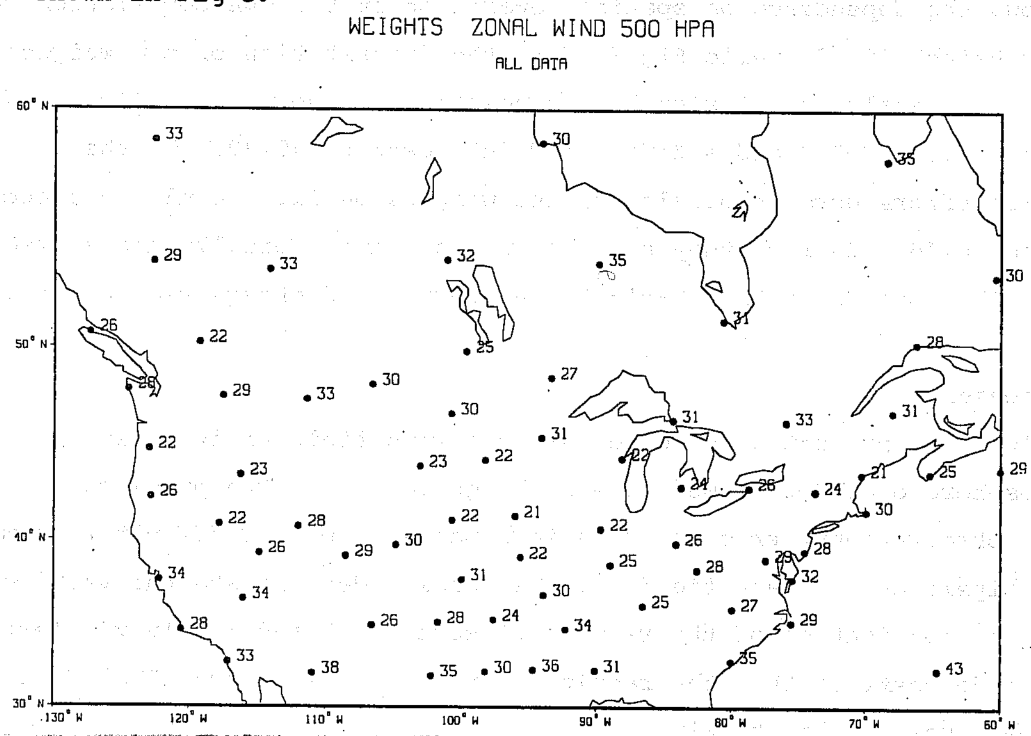

Fig 5 The weight given to the 500mb u-component observation in the analysis of that observation at the station, for a typical 12Z analysis in 1986/87.

The average of these weights may be compared with the estimate derived by extrapolating the data of Fig 4 to zero separation.

the square of the ratio of 500mb wind observation error to forecast error is 1.5 (Fig 3) then the methods of section 3.5 suggest that the effective data density is 1.75. This implicit data redundancy suggests that a higher resolution practical wind analysis is possible, following the arguments in section 3.5.

4.3 The weights and the intercept for the 500mb height field

The results of section 3 are applicable only if the observation error at a given point is uncorrelated with the observation error at all other points affecting the analysis for that point. Rawinsonde errors for wind are uncorrelated in the horizontal, and have a very weak correlation in the vertical as shown by HL. However rawinsonde reports for height have a marked vertical correlation (LH 1986) while the thickness reports have very little vertical correlation. In the current ECMWF system the vertical correlation of sonde height error at 500mb with 400mb is specified to be 0.67, and for 500mb with 700mb is specified to be 0.57. One should not expect then to find as good correspondence between the weights and intercept for the height field as was found for the wind field at 500mb.

Fig 6 shows the dependence on spatial separation of the OmA correlation for the 500mb height field, while Fig 7 shows the distribution of the weights in a single typical analysis. A visual extrapolation of the data in Fig 6 gives an intercept of about -0.2 and a normalised intercept of -0.18. If the observation errors were uncorrelated, the weights on Fig 7 would be expected to be about 0.18. In fact they are much larger, and typically lie between 0.5 and 0.7. This must be attributable to the correlated observation errors.

4.4 Discussion

The results just presented indicate that the wind-field analyses in mid-troposphere over North America must be quite good. The presence of important observational error correlations makes it more difficult to comment on the analysis of the mass field. These results demonstrate the value of quantitative application of the results of sections 2 and 3. In the next section we demonstrate that the results of these sections are also of value even if only used qualitatively.

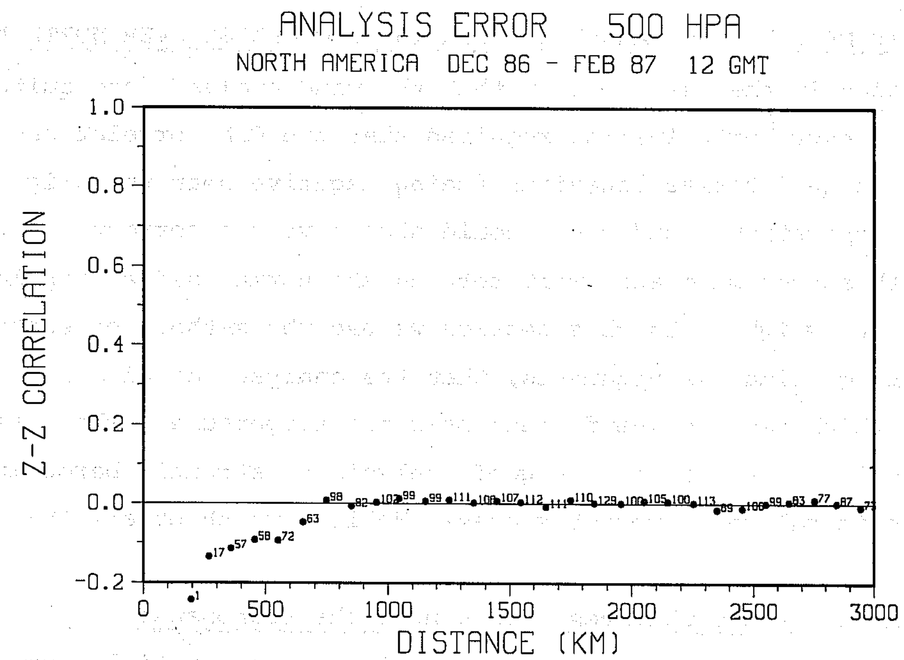

Fig 6 The dependence on station separation of the auto-correlation of the 1986/87 North American OmA differences for 500mb height. This may be converted to a plot of auto-covariance using the rms OmA statistics from Fig 3. The number of station pairs contributing to each distance bin is indicated.

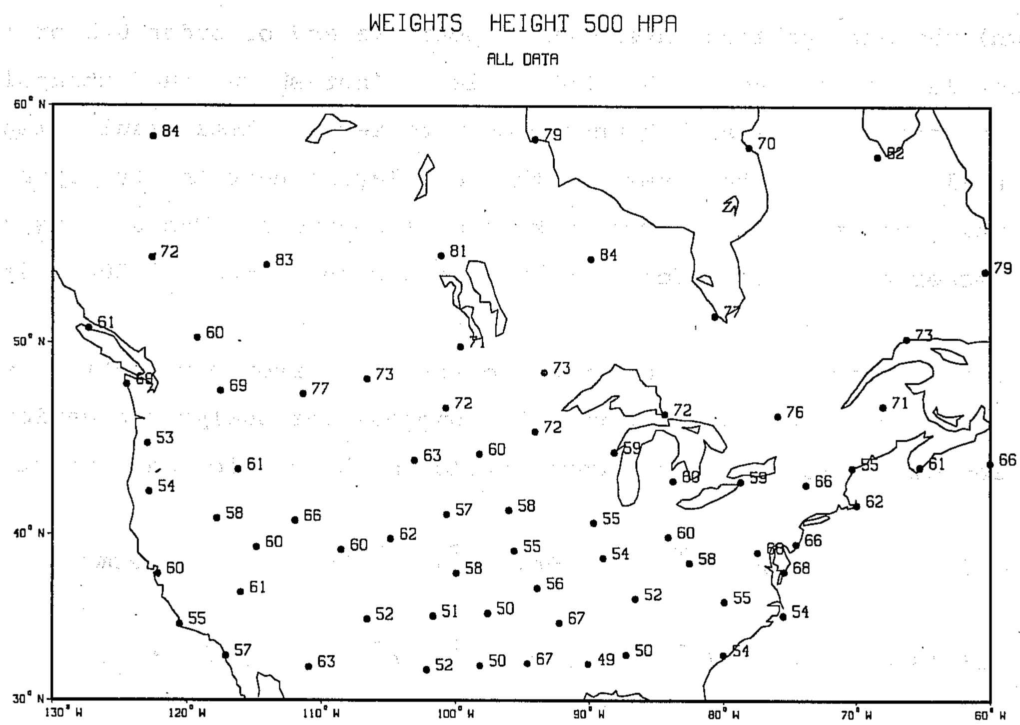

Fig. 7 The weight given to the 500mb geopotential observation in the analysis of that observation at the station, for a typical 12Z analysis in 1986/87.

The average of these weights does not correspond to the extrapolated correlation data in Fig 6 because of the vertical correlation of height observation error.

5. DIAGNOSIS OF THE ECMWF ANALYSIS NEAR THE TROPOPAUSE OVER NORTH AMERICA

The demonstration in the last section that the wind analyses are quite good in mid-troposphere over North America required that the OmA correlations should have the correct qualitative behaviour (being negative near the origin and zero at large separation), and they should also have the correct quantitative behaviour, with approximate agreement between the normalised extrapolated intercept and the weight. In this section we use the methods of section 3 to demonstrate, by qualitative arguments, that the analyses of wind and more especially of thickness are inefficient near the tropopause. These results are of practical importance since a good analysis of strongly baroclinic structure is critical for forecast success (Hollingsworth et al, 1985).

5.1 The analysis of the thickness field near the tropopause

Fig 8 shows the dependence on station separation of the spatial correlation of the OmA differences for the thickness of the 250-200mb layer. The result for the pair of stations with shortest separation does not always agree with the findings for the other stations, and so the results for this station pair may be discounted. Both for this layer, and the 300-250mb and 200-150mb layers (not shown) the extrapolated intercept is positive and of order 0.3 or more. For thicker layers such as the 300-200 mb layer (not shown) the extrapolated intercept is still positive, but much closer to zero. These results imply that the analyses of the thickness of the thin layers near the tropopause is inefficient, since all the intercepts should be negative. The same is true, but to a lesser degree, even for thicker layers such as the 300-200mb layer.

This situation probably arises because the forecast error correlation is specified to be too high in the vertical. Suppose the analysis equations for analysis increments a_1, a_2 at adjacent levels in the vertical are written

$$a_1 = A.d_1 + \alpha.d_2 + \ldots = A[\ (d^+) + (d^-)] + \alpha[\ (d^+) - (d^-)] + \ldots \text{ other terms}$$

$$a_2 = \beta.d_1 + B.d_2 + \ldots = \beta[\ (d^+) + (d^-)] + B[\ (d^+) - (d^-)] + \ldots \text{ other terms}$$

where a_1, a_2, are the analysed departures from the first guess, d_1, d_2 are the observational departures from the first guess, d^+ is the vertical average of d_1 and d_2, and d^- is $(d_1-d_2)/2$. If these equations are differenced, we find

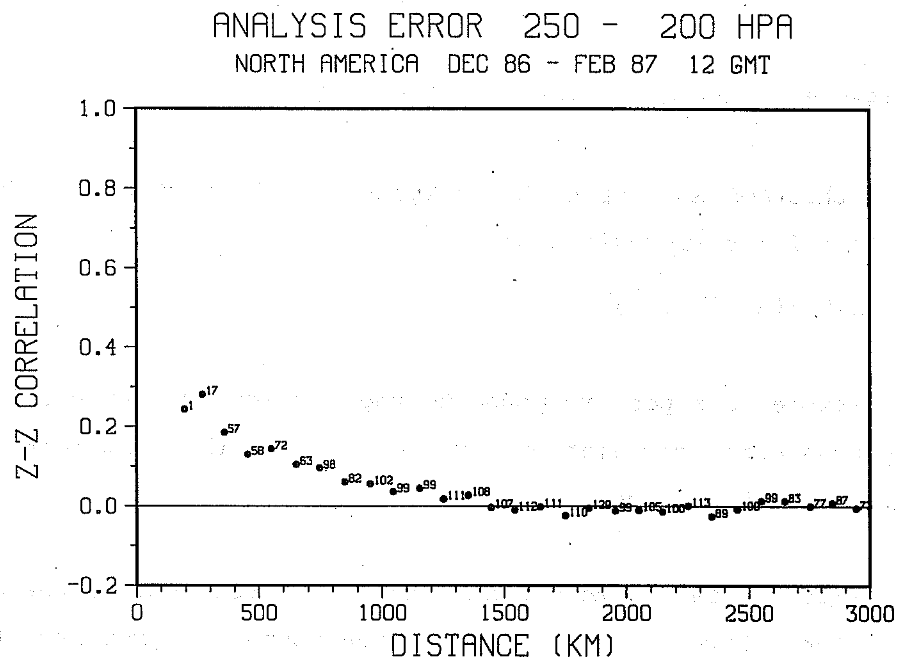

Fig 8 The dependence on station separation of the auto-correlation of the 1986/87 North American OmA differences for the 250 to 200mb thickness.

Since the thickness observation errors have little spatial correlation, this auto-correlation should be zero at large separations, and negative at short separations (500km or less).

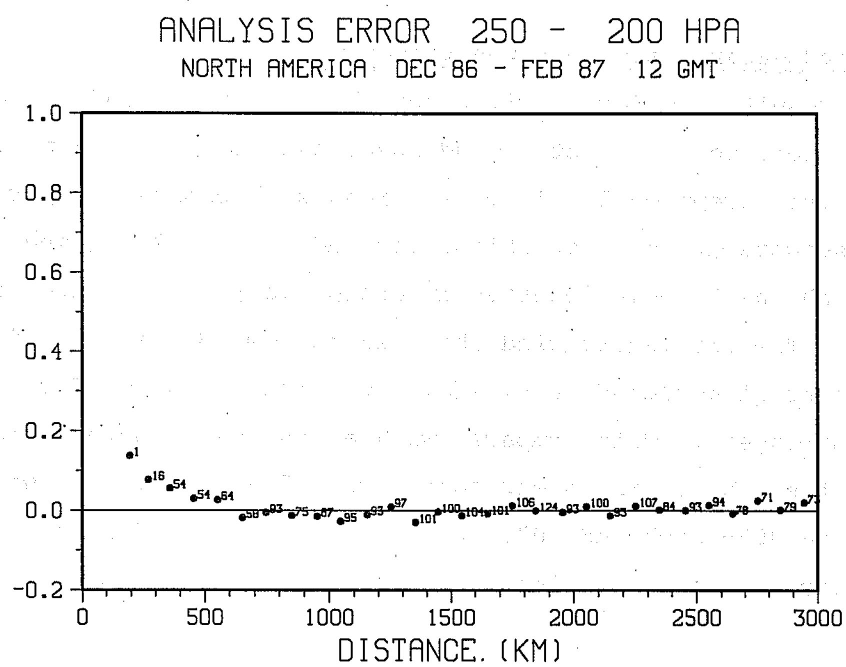

Fig 9 The dependence on station separation of the auto-correlation of the 1986/87 North American OmA differences for the 250 to 200mb vector wind difference.

Since the wind shear observation errors have little spatial correlation, this auto-correlation should be zero at large separations, and negative at short separations (500km or less).

that

$$a_1 - a_2 = [A-B + (\alpha-\beta)] \cdot (d^+) + [A+B - (\alpha+\beta)] \cdot (d^-)$$

One may expect that A~B and α~β so the response to the vertical difference in the data is, to a first approximation,

$$a_1 - a_2 = [A+B - (\alpha+\beta)] \cdot (d^-)$$

There will therefore be a poor response to the vertical difference in the data if the first-guess error correlations, represented by the term (α+β) in the last expression, are too large.

A further contributory factor to the inefficient performance of the analysis system must be the WMO reporting practice which rounds all reported heights to 10m above 500mb. As noted by LH, the rounding error in the thickness then has a triangular distribution with an rms value of 4m. For the 250-200mb layer the rms of the OmA thickness differences illustrated in Fig 8 is 8m, corresponding to an rms OmA difference of 1.2K, while the rounding error in the observation is 0.6K.

5.2 The wind shear field near the tropopause

Fig 9 shows results similar to Fig 8 for the spatial dependence of the OmA auto-correlations for the vector wind shear over the 250-200 mb layer. The results for the exceptional pair of stations with shortest separation may again be discounted. Both for this layer and the 300-250mb and 200-150mb layers (not shown) the extrapolated intercept is positive with values between 0.1 and 0.2. The rms vector wind shear in the OmA differences for the 250-200mb layer illustrated is 6.3m/s, with similar values for the 300-250mb and 200-150mb layers. This compares with an estimated observation error of 5.8 m/s for the vector wind shear across the 250-200mb layer, with similar values for the 300-250mb and 200-150mb layers. The wind shear analyses for these thin layers are inefficient.

For thicker layers, such as the 300-200mb layer, the extrapolated intercepts for the vector wind shears are smaller, but still positive. The extrapolated intercepts for the wind analyses at 300, 250, and 200mb (not shown) are essentially zero.

All these results indicate that the wind shear analyses near the tropopause are inefficient, but perhaps not as seriously so as the thickness analyses. Taken with the thickness results, one may conclude that improved analysis resolution is needed in the vertical near the tropopause. The need for improved vertical resolution may also imply a need for better horizontal resolution in order to maintain dynamical balance. The earlier discussion of the effective data density for wind at 500mb points in the same direction.

To see some of the history of the problem, we compare the 1986/87 analysis quality for the wind shear in the 250-200mb layer with that for Jan-March 1984, as shown in Fig 10. The comparison shows that there was a considerable improvement in the analysis of the wind shear over the intervening period. This was mainly due to a set of changes to the analysis algorithm in mid 1984 (Shaw et al, 1987) which were designed, inter alia, to improve the vertical resolution of the algorithm.

5.3 Geostrophic and thermal wind balance

The results above indicate that the thickness analysis and the wind analysis are inefficient near the tropopause. We may therefore expect the thermal wind analysis to be inefficient. To explore this question we discuss the <t,z> correlation for the OmA differences, where z is the height at one of a pair of stations and t is the velocity component transverse to the line between the stations, measured at the other station. HL/LH showed that the height/streamfunction correlation of forecast errors can be diagnosed from simple calculations based on the <t,z> correlation. Simple arguments may be used to put bounds on the OmA <t,z> covariance in an O/I system.

a) The <t,z> correlation for OmA differences in an O/I analysis

Consider an O/I analysis involving at least two observations, one of height at station 1, and one of transverse wind at station 2. The O/I analysis equations for the analysed values of the observed variables at the observation points are

$$t_2^a-t_2^p = w_1.(t_2^o-t_2^p) + w_2.(z_1^o-z_1^p) + \dots\text{other terms}$$

$$z_1^a-z_1^p = w_3.(z_1^o-z_1^p) + w_4.(t_2^o-t_2^p) + \dots\text{other terms}$$

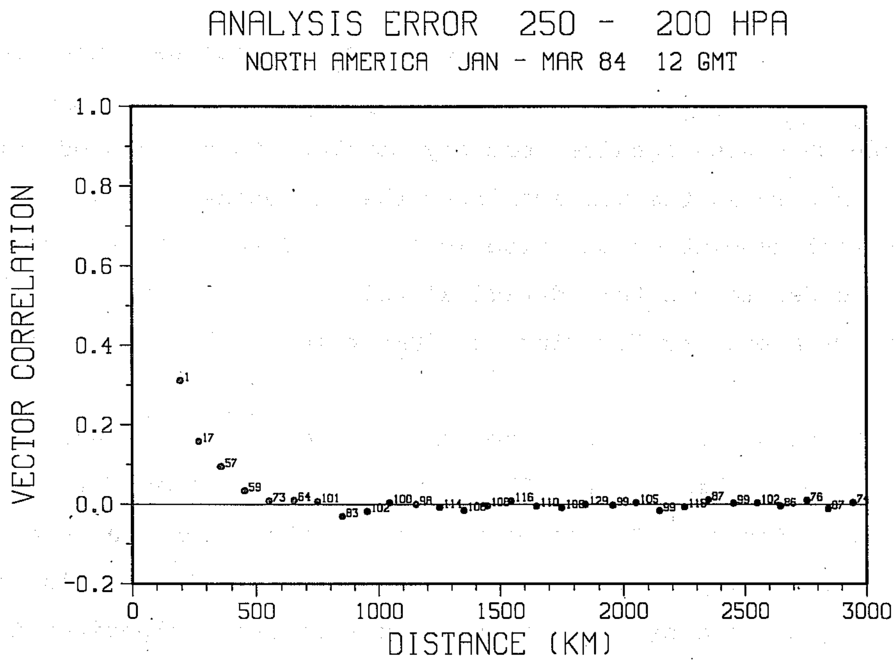

Fig 10 The dependence on station separation of the auto-correlation of the 1983 North American OmA differences for the 250 to 200mb wind shear.

Note the improvement in analysis quality between 1983 and 1986/87.

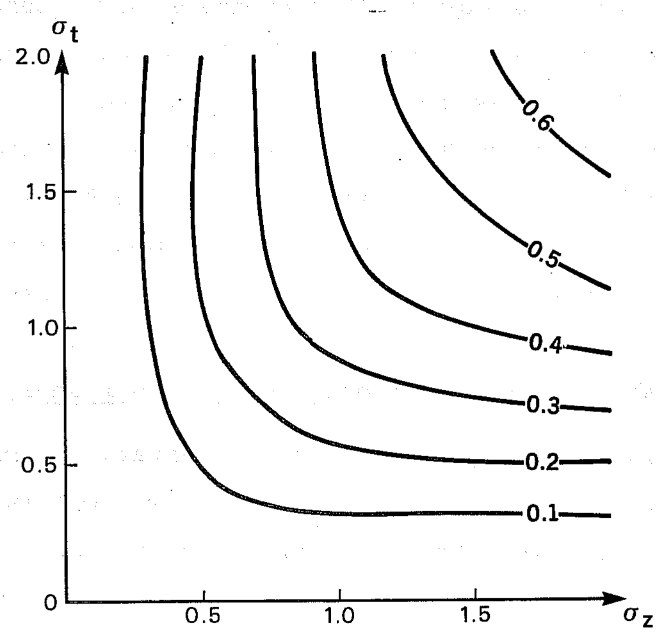

Fig 11 Isolines of the function $G(\sigma_z, \sigma_t)$ defined in the text.

where t_2^o, t_2^a, t_2^p are the observed, analysed, and predicted values of t, at point 2 normalised by the rms first guess error for wind; z_1^o, z_1^a, z_1^p are defined similarly.

Suppose that only these two observations are used in the O/I analysis calculation. Then with

$$\underline{\underline{P}} = \begin{matrix} 1\ , & \mu \\ \mu\ , & 1 \end{matrix} \qquad \underline{\underline{D}} = \begin{matrix} \sigma_z^2 & 0 \\ 0 & \sigma_t^2 \end{matrix}$$

the entry A_{12} in the analysis error covariance matrix is easily calculated from Eq (5), and $-A_{12}$ gives the <t,z> covariance for the OmA differences, provided height and wind observation errors are uncorrelated. The calculation gives

$$<(z_1^o-z_1^a),(t_2^o-t_2^a)> = \frac{-\mu.\sigma_t^2.\sigma_z^2}{[\sigma_t^2+\sigma_z^2+\sigma_t^2.\sigma_z^2+1-\mu^2]}$$

where σ_t, σ_z are the (normalised) observation errors for wind component and height, and μ is the <t,z> correlation for forecast error for that pair of stations. As shown by HL/LH μ is a positive function of station separation. It has a maximum value of about 0.45 at about 400km in the ECMWF system and tends to zero as the separation tends to zero or to infinity.

As more data is made available to the system, one expects the (positive) analysis error covariance $<z_1^a, t_2^a>$ to decrease to zero, and so one expects its negative, the OmA covariance, to increase towards zero. Thus when more than two observations are used, one expects that

$$<(z_1^o-z_1^a),(t_2^o-t_2^a)> \quad > \quad \frac{-\mu\ \sigma_t^2.\sigma_z^2}{[\sigma_t^2+\sigma_z^2+\sigma_t^2.\sigma_z^2\ +\ 1-\mu^2]} \quad > \quad \frac{-\mu\ \sigma_t^2.\sigma_z^2}{[\sigma_t^2+\sigma_z^2+\sigma_t^2.\sigma_z^2]} \qquad (23)$$

and so one has the bounds

$$0 > \langle (z_1^o - z_1^a), (t_2^o - t_2^a) \rangle > \frac{-\mu\ \sigma_t^2 . \sigma_z^2}{[\sigma_t^2 + \sigma_z^2 + \sigma_t^2 . \sigma_z^2]} \qquad (24)$$

The function $G(\sigma_z, \sigma_t) = \dfrac{\sigma_t^2\ \sigma_z^2}{(\sigma_t^2 + \sigma_z^2 + \sigma_t^2 . \sigma_z^2)}$ is plotted in Fig 11 for the range (0.,2.) in each of the arguments, which covers the typical range of variation of the normalised observation errors. The isolines of G are hyperbolic in character. A convenient upper bound for G within this range is obtained by setting the larger of the two arguments to 2., so that

$$G(\sigma_z, \sigma_t) \leq \frac{\sigma^2}{(1+1.25.\sigma^2)} = g(\sigma)$$

where σ is the lesser of the two arguments. The function $g(\sigma)$ takes the values 0.19 when σ=0.5, 0.44 when σ=1, 0.61 when σ=1.5 and 0.66 when σ=2.

b) Geostrophic wind balance at a single level in the ECMWF system

We now assume that the above bounds are relevant to results from a practical analysis system, for which σ_z, σ_t and μ are known with a reasonable degree of confidence. Calculations of the <t,z> correlation for the OmA differences at the 250, and 200 mb levels are shown in Fig 12a,b. At both levels the <t,z> correlation is negative at separations between 250 and 750km, and is almost zero beyond that distance. The minimum values on the plots are of order 0.1, the results being normalised by the OmA differences for z and t at the origin. From Fig 3 the values of σ_z and σ_t at 250mb are 0.65 and 0.92 respectively, while the corresponding values at 200mb are .83 for σ_z and 1.25 for σ_t. From Fig 11, one should expect the OmA <t,z> cross-correlation at 250mb to be bounded below by -.25 μ, so that the minimum value should be about -.1 at about 400km. This is quite close to the empirical result on Fig 12a. At 200mb one expects the OmA <t,z> correlation to be bounded below by -0.32 μ, with a minimum value of about -.12. The empirical data satisfy this bound. We can therefore conclude that the single-level <t,z> correlations are within the expected bounds at both 250 and 200 mb.

c) Thermal wind balance in the ECMWF system

Fig 13 shows a similar calculation for the OmA cross-correlation $\langle \Delta t, \Delta z \rangle$ of the wind shear (more correctly, the vertical wind difference) and the

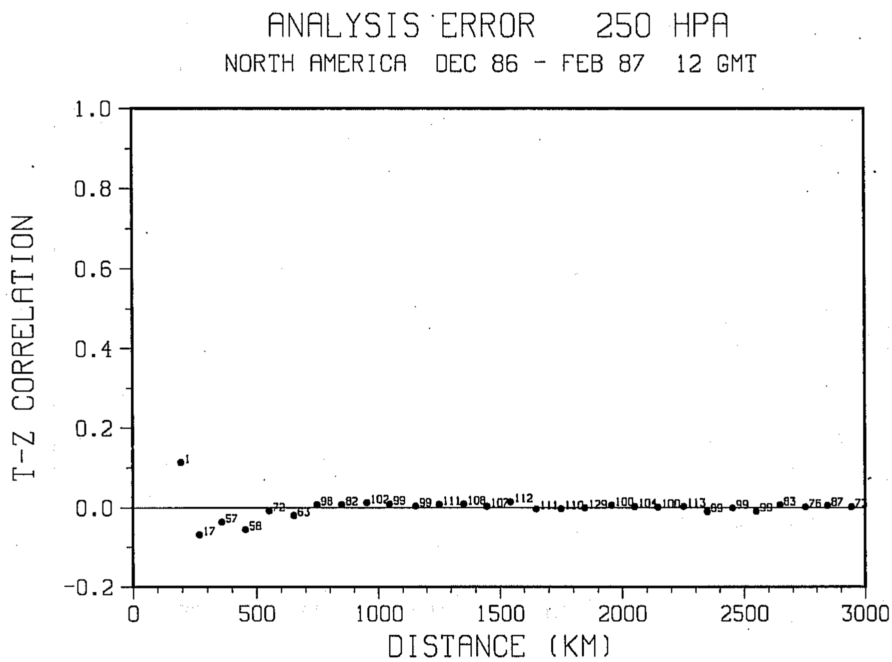

Fig 12a The dependence on station separation of the cross-correlation of the 1986/87 North American OmA differences for the transverse wind and height at 250mb.

If the mass and wind analyses at this level are in balance, the cross-correlation should be zero at large separations and at zero separation; at separations of order 500km it should be negative, but larger than about -0.15.

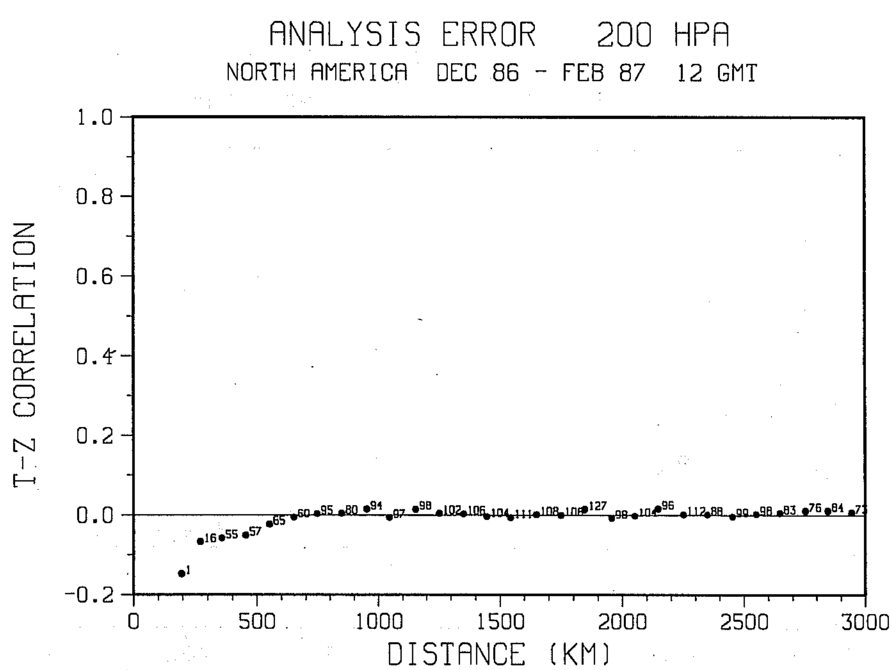

Fig 12b As a) for 200mb.

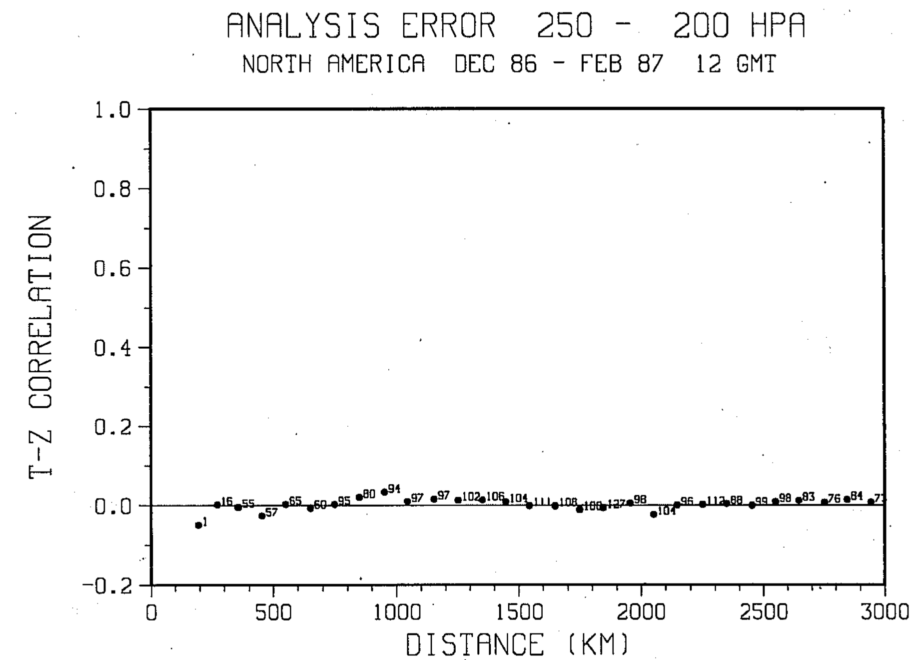

Fig 13 The dependence on station separation of the cross-correlation of the 1986/87 North American OmA differences for the transverse wind differences between 250 and 200mb and the height differences between 200 and 250mb.

If the thickness analysis and wind shear analyses for this layer are in balance, the cross-correlation should be zero at large separations and at zero separation; at separations of order 500km it should be negative, but larger than -0.15.

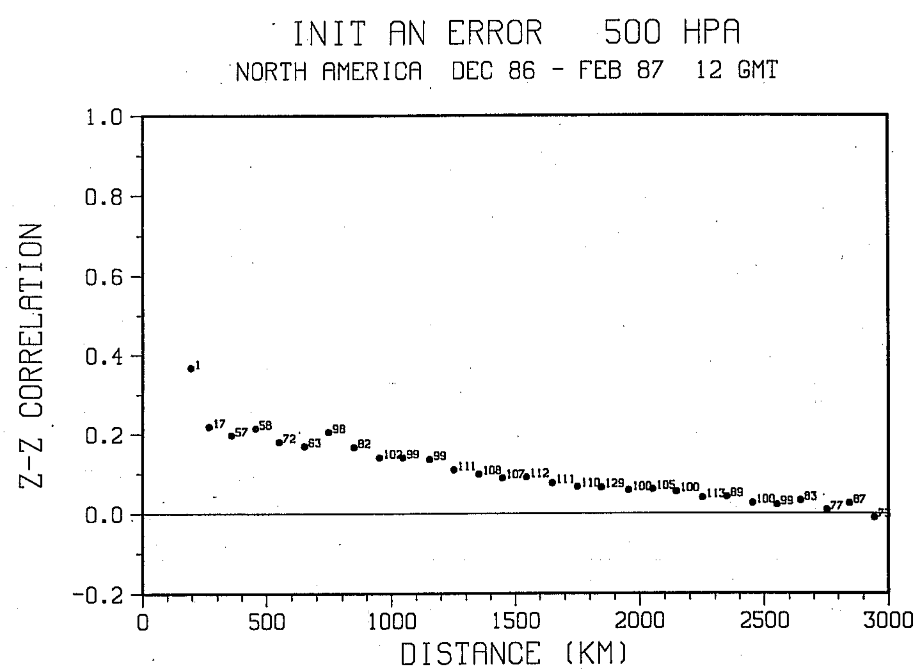

Fig 14 As Fig 6 for the 500mb height OmI differences. The difference between this and Fig 6 indicates data rejection by initialisation. A similar effect is seen at many other levels.

thickness across the 250-200mb layer. The OmA <$\Delta t, \Delta z$> correlation for this layer is zero for large separations, and small separations, but tends to be positive at separations between 750 and 1500km. This last result gives cause for concern as it suggests that there is thermal wind information in the observations which has not been used by the analysis, implying that the thermal wind analysis is inefficient for this layer. The results in Figs 8 and 9 already indicate that both the wind analysis and the thickness analysis are inefficient, and so lend support to the interpretation of Fig 13.

The results of HL/LH and Phillips (1986) show evidence of the need for non-separable structure functions to properly resolve the vertical wind shear and its associated mass field. Non-separable structure functions have yet to be implemented in our operational system. The present results show the need to improve the analysis of baroclinic structures near the tropopause in mid-latitudes.

d) The effect of initialisation on the OmA differences in the ECMWF system

All the empirical results discussed so far have been based on observation minus uninitialised analysis (OmA) data. To study the effect of initialisation we have made the same calculations for the observation minus initialised analysis (OmI) data. Initialisation has very little effect on the wind, thickness, wind-height or wind shear - thickness statistics; the OmA and OmI results for these quantities are almost identical. Initialisation does have a marked effect, however, on the height field statistics. Fig 14 shows the OmI results for the 500mb height field, which may be compared with the corresponding OmA results in Fig 6. The initialisation rejects certain mass information on large scales. A similar effect is seen at many other levels suggesting that the rejected information probably has a broad vertical scale.

At present we can only speculate on the reasons for the occurrence of the unbalanced large scale mass information. At least two possibilities may be mentioned. The effect could arise from inconsistencies between the analyses over land and ocean, each with its characteristic type of data coverage. Secondly the large horizontal and vertical scales involved might also suggest a tidal origin for the rejection. Modifications were made to the system in early 1986 (Wergen, 1986) to improve the retention of tidal structures. The modifications were a distinct improvement on the previous treatment, but may still have limitations.

6. DIAGNOSIS OF THE ECMWF ANALYSIS SYSTEM IN THE TROPICS

Operational data assimilation, and forecasting, are more difficult in the tropics than in the mid-latitudes of the Northern Hemisphere. The main reasons are the paucity of data, the weak coupling between mass and wind fields, the difficulty of parameterising the convection process for large scale models, and the difficulty of analysing the moisture and stability fields. Despite these difficulties progress has been made in recent years; operational analyses and 2 to 3 day forecasts are skilful in some areas of the tropics (Reed et al 1988). Given the paucity of observational data in the tropics,it is essential that it be fully used.

We now apply the methods discussed in earlier sections to verify the background fields and analyses of wind and wind shear in the upper tropical troposphere. In the earlier sections we used plots with station separations up to 3000km to discuss the OmA correlations over North America. For the tropical verifications we extend the plots to 10,000km (taking account of spherical geometry in the calculations) in order to find if there is observational evidence for analysis or forecast problems on large scales.

6.1 Analysis on large scales

The issue of analysis problems on large scales in the tropics was discussed by Cats and Wergen (1982) and by Daley, Cats and Wergen (1986). These authors used the 1982 version of the ECMWF analysis system to analyse large scale Rossby and Kelvin modes, when the data input was provided by simulated radiosondes on a regular grid. They found, inter alia, that the analysis system responded poorly to wind information on very large horizontal scales in the tropics, and that there was poor discrimination between very large scale Rossby and Kelvin modes, because of the difficulty of prescribing a satisfactory mass-wind relation in the tropics. The horizontal structure function used for mass and stream-function in their experiments was a gaussian (negative squared exponential) and the wind law in the tropics was a simple interpolation between simple geostrophic laws for the northern and southern hemispheres.

Hollingsworth and Lönnberg (1986) pointed out that a more general formulation of the correlation model used for analysis will permit the representation of forecast errors on very large scales. Such a representation has been incorporated in the correlation models at ECMWF since early 1985. This was expected to have a beneficial effect on the analyses of height and wind on large scales, but was not expected to have any effect on the problem of discriminating between Rossby and Kelvin modes.

6.2 The wind and wind shear fields

a) Short range forecast errors

Since little has been published on observational verification of short range tropical forecasts, and since such verifications provide a useful background for a discussion of the analysis verification, Fig 15a shows the spatial correlation of the 150mb vector wind forecast errors based on regularly received radiosonde data from 20N to 20S during Dec 1986 through Feb 1987. Fig 15b shows the corresponding results for the spatial correlations of vector wind-shear error between the 200mb and 150mb levels. These levels were chosen to be near the level of maximum shear in the outflow of the Hadley circulation.

In the 150mb wind field forecast error correlation, the main feature is a peak at short separations, indicating that most of the forecast error is on synoptic scales. There is little evidence in these results of a marked forecast error on large scales. The fluctuations in the statistics between 2000 and 10,000km are as likely to be noise as anything else. The magnitude of the perceived rms vector forecast error is 8.1 m/s. If the OmF data are extrapolated to the origin, then they might give an intercept as large as 0.7, implying that the rms forecast error is at most 6.8m/s and the rms vector observation error is at least 4.5m/s. The observation error estimated in this way corresponds reasonably well with the estimate for mid-latitudes on Fig 3.

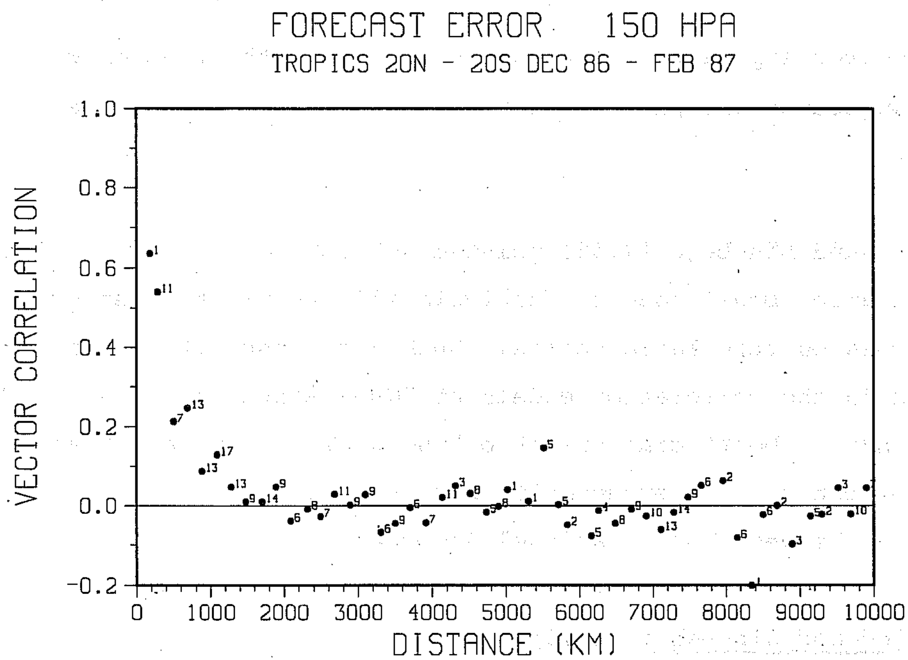

Fig 15 a) The dependence on station separation of the auto-correlation of the 1986/87 Tropical OmF differences for the 150mb vector wind. The number of station pairs contributing to each distance bin is indicated. Most of the wind forecast error is on synoptic scales.

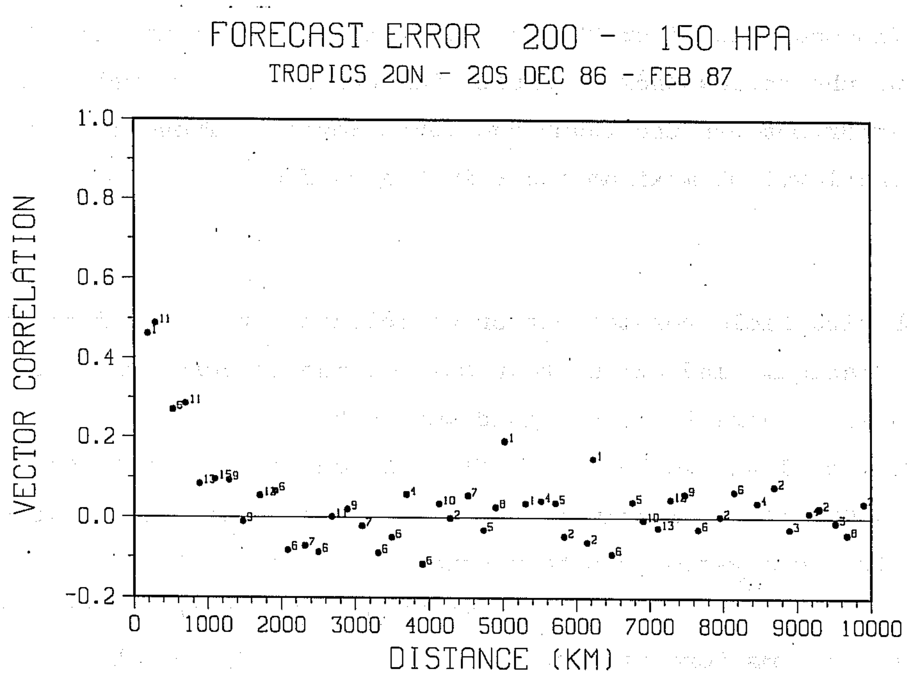

Fig 15 b) As a) for the 200mb-150mb wind difference. Most of the wind shear forecast error is on synoptic scales.

The main feature of the OmF correlation for the 200-150 mb vector wind shear (Fig 15b) is a peak at small separation, rather similar to that in the 150mb wind correlations. Again there is little evidence of a large scale forecast error, and most of the forecast error in the wind shear is on synoptic scales. The perceived forecast error in the vector wind shear is 9.3 m/s. If the OmF data are extrapolated to the origin, then they might give an intercept as large as 0.6, implying that the rms forecast error for the shear is at most 7.2m/s and the rms vector observation error is at least 5.9 m/s. If sonde wind errors are indeed uncorrelated in the vertical then the latter result would imply a vector wind error at a single level of 4.2m/s in good agreement with the estimate from the single level data.

b) Verification of analysed winds and wind shears

The dependence on station separation of the OmA correlations for the tropical 150mb vector wind are shown in Fig 16a. Most of the stations with separations less than 1500km show negative OmA vector wind correlations on average, indicating that the analysis is rather good. Because the results are noisy it is not possible to make a sensible comparison between the weights and the extrapolated value of the OmA correlation at zero separation.

The corresponding OmA results for the wind shear between 200mb and 150mb are shown in Fig 16b. The results again are somewhat noisy. Near the origin one's impression is that the extrapolated intercept must be very close to zero. This indicates that the analyses for the shear are border-line in efficiency. Nevertheless, compared with the mid-latitude results, more account is taken of the observed wind shear data in the tropics than over North America. This is presumably due to the much sharper structure functions used in the vertical in the tropics (Shaw et al., 1987)

Part of the tropical/extratropical difference in analysis performance for wind shear may be attributable to variations in model resolution near the tropopause, where the model levels are at about 102, 142, 191, 252 and 324 mb (for a surface pressure of 1000 mb). The analysis is evaluated multivariately on the model levels and then interpolated univariately to standard levels (100, 150, 200, 250 and 300 mb) for the verifications discussed here. Lorenc

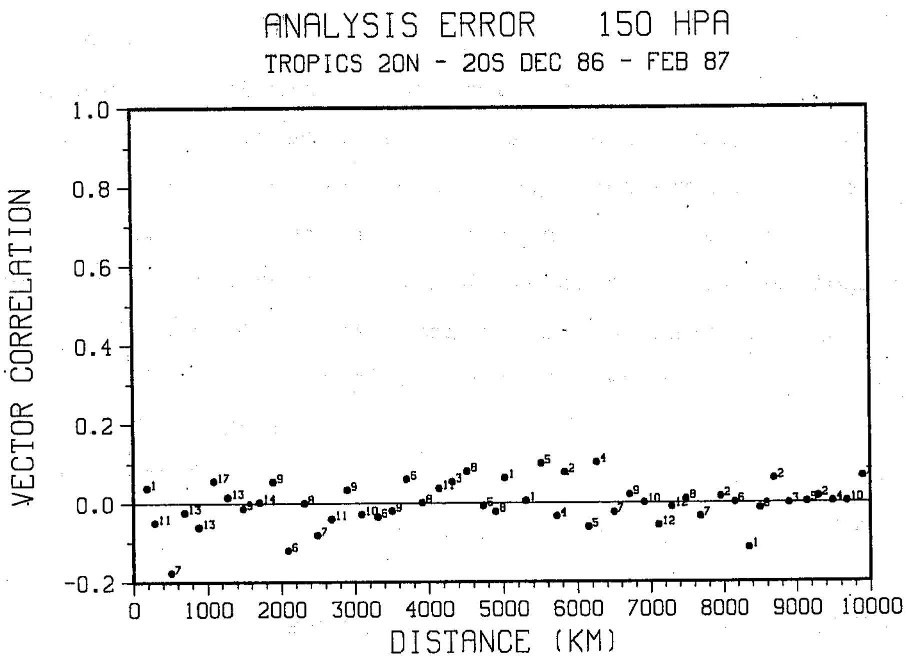

Fig 16 a) The dependence on station separation of the auto-correlation of the 1986/87 Tropical OmA differences for the 150mb vector wind. The number of station pairs contributing to each distance bin is indicated. There is little reliable evidence for a defective wind analysis on large scales.

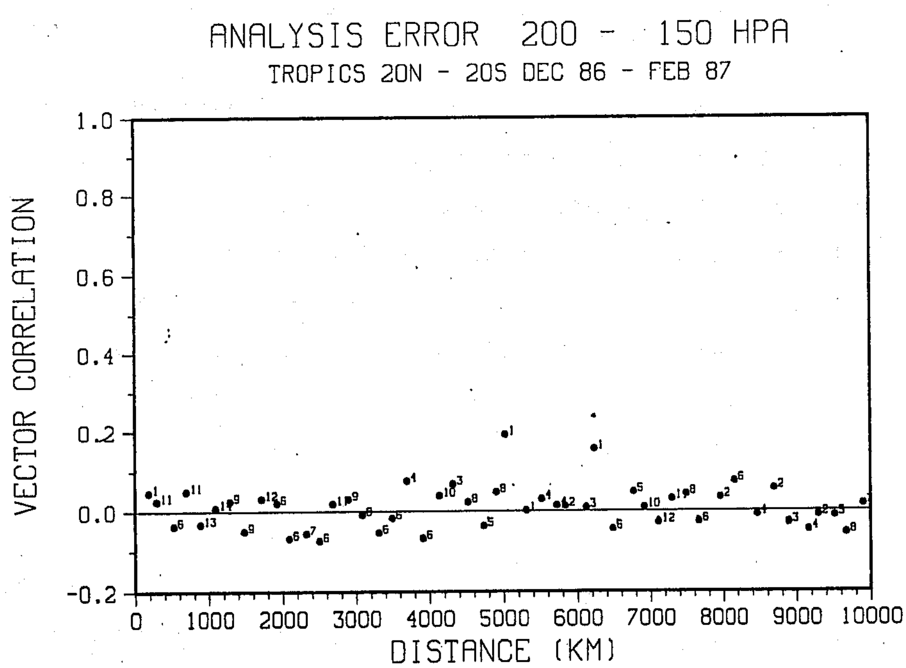

Fig 16 b) As a) for the 200mb-150mb wind difference. The wind shear analyses on short scales seem quite reasonable. There is little reliable evidence for a defective wind analysis on large scales.

(1986) has emphasised the need to take explicit account of such interpolations in formulating an optimal system.

7. SUMMARY AND DISCUSSION

We have presented new diagnostics for evaluating the performance of a practical objective analysis system, based on the spatial correlation structure of the observation-minus-analysis differences. Based on the spatial coherence of the fit to observations, the diagnostics provide answers to such questions as:

* Are the observations used effectively?;

* Are the analyses balanced?

* What is the effective data density?

The horizontal spatial correlation of the OmF differences indicates the presence of information in the observations, and the corresponding lack of that information in the forecast, provided the observation errors are uncorrelated. The analysis algorithm should extract the information from the observations.

The properties of the spatial correlations of the OmA differences are examined for a theoretical optimal-interpolation analysis and for a practical linear analysis system. The results are used to interpret the empirical determinations of the OmA correlations in the ECMWF system, for data with uncorrelated errors. For short separations the correlations of the OmA differences should be negative, and for large separations they should be zero. When extrapolated to zero separation the value of the correlation provides bounds on the analysis error. The extrapolated value of the correlation at zero separation provides a new way of estimating the observation error. The occurrence of a positive intercept is clear evidence that the analysis system does not take enough information from the observations, and so is inefficient.

In a theoretical O/I analysis the negative of the value of the correlation, extrapolated to zero separation, should be equal to the weight given to the observation in the analysis of the observed value at the observation position. In an efficient practical analysis system the area of the weight-intercept plane in which the analysis system will operate is quite restricted. That area is roughly bisected by the line where the negative intercept equals the weight, i.e. the line on which the theoretical O/I analysis operates. This suggests that in a practical system the negative intercept and the weight should be nearly equal. If they are quite different then there may be considerable scope for improvement in the system. In a manner of speaking, this test of an analysis is rather like the method of back-substitution used to check the solution of an equation: If the weight used a-priori agrees with the weight estimated a-posteriori, then the analysis is about as good as it can be. Knowledge of the weight and intercept also provides an estimate of analysis error at the observation position. If a practical analysis is efficient, knowledge of the analysis error provides an estimate of the effective data density, and so an indication of the potential for increased analysis resolution.

The methods outlined above are applicable to wind and thickness errors from radiosondes, for which there is little spatial correlation in the horizontal or vertical; the methods cannot be applied to height analyses, because of the large vertical correlations in radiosonde height errors . The methods are tested in practice by demonstrating that operational mid-tropospheric wind and thickness analyses over North America are quite good, but that the wind-shear and thickness analyses near the tropopause over North America are definitely inefficient.

The studies of the OmA correlations were extended to cross-correlations of mass and wind differences, which provide an observational check on the balance of the analyses. The mass wind balance over North America is good for single level calculations, but there is evidence of inefficient analysis of thermal wind balance for thin layers near the tropopause. If anything the analysis of the wind shear near the tropical tropopause is somewhat better than over North America, probably because of the use of sharper vertical structure functions.

The methods developed in this paper provide a new approach to the problem of analysis verification. The methods are simple, and represent an extension of the standard methods for determining the statistics of forecast error and observation error. The new methods provide an inexpensive and effective set of tools for identifying weaknesses in an analysis system. They are much more incisive than simple statistics of OmA differences which are most useful for identifying blatant problems. The new methods are also much more incisive and much less expensive than tests using forecasts, although the forecast tests are the acid tests for practitioners of NWP.

In addition to addressing the primary question of analysis quality, the techniques developed in this paper provide objective methods for comparing the performance of operational assimilation systems. This has been an area of considerable difficulty and controversy in the past. The tools used to diagnose analysis performance in earlier studies have been measures of the fit of the observations to the data, and studies of the forecast quality when forecasts are run from different analyses. The first of these methods is crude and the second is very expensive. The methods presented here can be used to test if all the data presented to a system is used effectively. If this is not the case, then the results of a data impact study (which withholds or adds a particular observing system) may not be reliable.

The methods for analysis validation discussed here, and the methods for validation of short range forecasts presented in Hollingsworth and Lönnberg (1986), Lönnberg and Hollingsworth (1986) together provide a useful set of indicators on the quality of an assimilation system, and a systematic means of identifying shortcomings in the system. If one could demonstrate that a three-dimensional analysis system had extracted all useful information from a given observing system in a given area, then no further gains in analysis accuracy can come from refinements of three-dimensional analysis technique alone.

Acknowledgements

We are grateful to G. Cats, A.C. Lorenc, J. Pailleux, N.A. Phillips and P. Undén for valuable comments which improved the manuscript. M. Simpson and J. Williams did excellent work on the text and figures, as usual.

APPENDIX: PROOF THAT $\underline{\underline{D}} - \underline{\underline{A}}$ IS POSITIVE DEFINITE

With the same definitions of the prediction error correlation matrix $\underline{\underline{P}}$, and the scaled observation error covariance matrix $\underline{\underline{D}}$ as in section 2, Hollingsworth (1987) showed that there exist non-singular matrices $\underline{\underline{Q}}$ and $\underline{\underline{R}}$ such that

$$\underline{\underline{Q}}^t.\underline{\underline{P}}.\underline{\underline{Q}} = [1]$$

$$\underline{\underline{Q}}^t.\underline{\underline{D}}.\underline{\underline{Q}} = \underline{\underline{R}}[\nu]\underline{\underline{R}}^t$$

where the diagonal matrix $[\nu]$, with entries ν_n, is positive definite, and $\underline{\underline{R}}$ is orthonormal. It then follows, since $\underline{\underline{A}} = \underline{\underline{P}} - \underline{\underline{P}}.(\underline{\underline{P}}+\underline{\underline{D}})^{-1}.\underline{\underline{P}}$, that

$$\underline{\underline{R}}^t.\underline{\underline{Q}}^t.\underline{\underline{P}}.\underline{\underline{Q}}.\underline{\underline{R}} = [1]$$

$$\underline{\underline{R}}^t.\underline{\underline{Q}}^t.\underline{\underline{D}}.\underline{\underline{Q}}.\underline{\underline{R}} = [\nu] \qquad \text{(A.1)}$$

$$\underline{\underline{R}}^t.\underline{\underline{Q}}^t.\underline{\underline{A}}.\underline{\underline{Q}}.\underline{\underline{R}} = [\frac{\nu}{1+\nu}] \ .$$

For any vector $\underline{X}$ of the same order as $\underline{\underline{P}}$, let $\underline{Y} = \underline{\underline{Q}}.\underline{\underline{R}}.\underline{X}$. Then

$$\underline{X}^t.\underline{\underline{P}}.\underline{X} = \underline{Y}^t.[1].\underline{Y} = \sum_n y_n^2 \qquad \text{(A.2a)}$$

where y_n is the n^{th} component of $\underline{Y}$. It follows that

$$\underline{X}^t.\underline{\underline{D}}.\underline{X} = \underline{Y}^t.[\nu].\underline{Y} = \sum_n \nu_n y_n^2 \qquad \text{(A.2b)}$$

$$\underline{X}^t.\underline{\underline{A}}.\underline{X} = \underline{Y}^t.[\frac{\nu}{1+\nu}].\underline{Y} = \sum_n \frac{\nu_n}{1+\nu_n} y_n^2 \qquad \text{(A.2c)}$$

Since $\nu_n > 0$ for all n it follows that $\underline{\underline{A}}$ is positive definite, and that both $\underline{\underline{P}}-\underline{\underline{A}}$ and $\underline{\underline{D}}-\underline{\underline{A}}$ are also positive definite. This completes the main proof. Some useful corollaries follow.

Since the diagonal elements of a positive definite matrix are positive, it follows from the positive-definiteness of $\underline{\underline{P}}-\underline{\underline{A}}$ and $\underline{\underline{D}}-\underline{\underline{A}}$ that if a_k and d_k represent the normalised analysis and observation errors at an observation point k

$$1-\langle a_k^2\rangle \quad > \quad 0 \tag{A.3}$$

$$\langle d_k^2\rangle-\langle a_k^2\rangle \quad > \quad 0 \tag{A.4}$$

so that the expected square error of the analysed value at the observation point is less than the expected square error of either the first guess or the observation.

Moreover from Eq (6) it follows that at every observation point k

$$\langle (d_k-a_k)^2\rangle = \langle d_k^2\rangle-\langle a_k^2\rangle \quad < \quad \langle d_k^2 \rangle \tag{A.5}$$

so that the mean square OmA differences are smaller than the mean square observation errors.

A further deduction from Eq (A.2) is based on the min-max characterisation of eigenvalues (Wilkinson, 1965). If $\underline{\underline{C}}$ and $\underline{\underline{D}}$ are real symmetric positive definite matrices, with eigenvalues g_i, d_i arranged in non-decreasing order, then

$$g_i < d_i, \text{ for all } i$$

provided

$$\underline{X}^t.\underline{\underline{C}}.\underline{X} \quad < \quad \underline{X}^t.\underline{\underline{D}}.\underline{X}$$

for any vector X. Let $\lambda_i, \delta_i, \tau_i$ be the eigenvalues of $\underline{\underline{P}}$, $\underline{\underline{D}}$, and $\underline{\underline{A}}$ respectively, arranged in non-decreasing order. It follows from (A.2) that

$$\tau_i < \lambda_i, \text{ for all } i$$

$\tau_i < \delta_i$, for all i

The implications of this result are striking, in that if the principal components of $\underline{\underline{P}}$, $\underline{\underline{D}}$, $\underline{\underline{A}}$ are ordered by their relative variance, the variance of the ith component of $\underline{\underline{A}}$ is always less than the variance of the ith component of $\underline{\underline{P}}$ or $\underline{\underline{D}}$, even though the principal components themselves are different.

REFERENCES

Cats G.J. and W. Wergen, 1982: Analysis of large scale normal modes by the ECMWF analysis scheme. in Proceedings of ECMWF workshop on 'Current problems in Data Assimilation', pp343-372;available from ECMWF

Daley R., G.J. Cats, W. Wergen, 1986: The objective analysis of planetary scale flow. Mon. Weath. Rev., 114, 1892-1908.

Drozdov O.A., A.A. Shepelevskii, 1946: The theory of interpolation in a stochastic field of meteorological elements and its application to meteorological map and network rationalisation problems. Trudy NIU GUGMS Series 1, No. 13.

Eliassen A. 1954: Spatial covariance and autocorrelation of the pressure field. Rap. No. 5 Videnskaps-Akademiets Institutt for Vaer og Klimaforskning, Oslo, Norway.

Gandin, L.S. 1963 Objective analysis of meteorological fields. Translated from Russian by the Israeli Program for Scientific Translations (1965).

Gilchrist A (ed.), 1982: JSC Study Conference on Observing Systems Experiments, Exeter, April 19-22 1982. Report 4 of the Numerical Experimentation Programme pub GARP and WCRP, WMO, Geneva.

Hollett, S.R. 1975 3 dimensional spatial correlations of PE forecast errors. M.Sc. thesis. Department of meteorology, McGill University, Montreal, Canada. 73 pp.

Hollingsworth, A., 1987: Objective Analysis for Numerical Weather Prediction in: Short and Medium Range Numerical Weather Prediction, Collected Papers presented at WMO/IUGG NWP Symposium, Tokyo, 4-8 August, 1986, ed. by T. Matsuno (Special Volume of the Journal of the Meteorological Society of Japan, p. 11-59).

Hollingsworth A, A C Lorenc, M S Tracton, K Arpe, G Cats, S Uppala, P Kållberg 1985 The response of Numerical Weather prediction Systems to FGGE II-b Data Part I : Analyses. Quart J Roy Meteor Soc 111, 1-66.

Hollingsworth A, D B Shaw, P Lönnberg, L Illari, K Arpe, A J Simmons 1986: Monitoring of observation quality by a data assimilation system. Mon. Wea. Rev., 114, 861-879.

Hollingsworth A and P. Lönnberg 1986: The statistical structure of short range forecast errors as determined from radiosonde data. Part I The wind errors. Tellus, 38A, 111-136.

Julian P R and H J Thiebaux, 1975: On some properties of correlation functions used in optimum interpolation schemes. Mon.Wea.Rev., 103, 605-616.

Lau N-C, 1984 A comparison of circulation statistics based on FGGE level III-b analyses produced by GFDL and ECMWF for the special observing periods NOAA data report ERL GFDL-6; available from NOAA, Executive Blvd, Rockville Md.

Lönnberg P, A Hollingsworth 1986: The statistical structure of short range forecast errors as determined from radiosonde data. Part II Covariance of height and wind errors. Tellus, 38A, 137-161.

Lorenc, A.C., 1986: Analysis methods for numerical weather prediction. Quart.J.Roy.Meteor.Soc., 112, 1177-1194.

Phillips, N.A. 1982 On the completeness of multi-variate optimum interpolation for large-scale meteorological analysis. Mon.Wea.Rev., 110, 1329-1334.

Phillips, N.A., 1986: The spatial statistics of random geostrophic modes and first-guess errors. Tellus, 38A, 314-332.

Reed, R.J., A. Hollingsworth, W.A. Heckley and F Delsol, 1988: An evaluation of the performance of the ECMWF operational forecasting system in analysing and forecasting tropical Easterly Wave Disturbances. To appear in Mon.Wea.Rev., March 1988.

Rutherford, I.D. 1972 Data assimilation by statistical interpolation of forecast error fields. J.Atm.Sci., 29, 809-815.

Shaw, D. B., P. Lönnberg, A. Hollingsworth, P Unden 1987: Data Assimilation: The 1984/85 revisions of the ECMWF assimilation system. Quart.J.Roy.Meteor. Soc., 113, 533-566.

Thiebaux, H. J., D. W. Shantz, H. L. Mitchell, 1986: An algorithm for updating forecast initialisation procedures with periodic adjustment of the objective analysis scheme. Mon. Wea.Rev., 114, 1048-1066.

Wergen, W., 1987: Diabatic non-linear normal mode initialisation for a spectral model with a hybrid vertical coordinate. ECMWF Tech Rept.59; available from ECMWF,Reading U.K.

Wilkinson, J.H., 1965: The Algebraic Eigenvalue Problem, Oxford University Press.