TECHNICAL MEMORANDUM

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Rationale for a new physically based parametrization of subgrid scale orographic effects

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1. INTRODUCTION

The scheme outlined below arose from a desire to adapt results from various laboratory and theoretical studies to NWP, general circulation and mesoscale numerical models, and it constitutes a first attempt in this direction. The recommendations build on the schemes developed and implemented at ECMWF in recent years.

2. CURRENT ECMWF SCHEME (MARCH 1990)

Orography is represented by h(x,y), the height of the earth's surface above mean sea level. Subgridscale orographic effects are represented in the current model in two different ways. Firstly, the mean orographic height at each grid point in the model is raised above the actual mean orography of the earth's surface, $\overline{h}$, by an amount μ_h, the standard deviation of h within the area represented by that grid point, as given by the US Navy (10' x 10') data set. The resulting surface is termed the "envelope orography" h_E (Wallace et al., 1983), where

$$h_E = \overline{h} + \mu_h \tag{1}$$

This type of parametrization therefore represents small sub-grid-scale effects by enhancing the resolved topography of the model, which will in turn enhance the generation of long waves in the atmosphere.

The second type of parametrization, which is used in conjunction with the first, is described in the internal **Memo** by Miller and Palmer (ECMWF Res. Dept. memo R2327/2421, 1989). It involves, firstly, a representation of gravity wave drag based on linear theory, and secondly, a representation of high-drag states when the flow becomes hydraulic at low levels due to the breaking of lee waves. The total (subgridscale) drag on the atmosphere is represented as

$$\tau(p_s) = \tau_w(p_s) + \tau_{fr}(p_s) \tag{2}$$

where

$$\tau_w(p_s) = k_w \, \rho \, U_\ell \, N_\ell \, . \, \mathrm{VAR} \tag{3}$$

(Palmer et al., 1986; McFarlane, 1987) where k_w is a chosen constant, U_ℓ and N_ℓ are low-level wind and buoyancy frequency, and VAR is the variance of the subgridscale orography with a set maximum value. Also

$$\tau_{fr}(p_s) = 0 \qquad \nu \le 2$$

$$\tau_{fr}(p_s) = k_\ell \, \rho \, U_\ell^3 / N_\ell \, (\nu - 2)^2 A \qquad \nu > 2 \qquad (4)$$

where $k_\ell = 4k_w$, $\nu = N_\ell \, \mu_h \, / \, U_\ell$, and A is a function of the directional variation of the subgridscale variance. The vertical distribution of the wave drag, $\tau_w(p)$, is given by

$$\tau_w(p) = (1-\beta)\, \tau\, (p_s)(p-p') \, / \, (p_s-p'), \qquad \text{for } p \ge p'. \qquad (5)$$

$$= \beta \tau_w(p_s) \; f(p) \qquad p < p',$$

where f(p) depends on a Richardson number (Ri) criterion for limiting internal wave amplitudes. The drag for p > p' represents the effect of low-level wave ducting, and for p < p' the effect of upward propagation. The values of β ($\simeq$ 0.3) and p' ($\simeq 0.8\ p_s$) are chosen. The hydraulic drag distribution is given by

$$\tau_{fr}(p) \;\; = \tau_{fr}\,(p_s)\,(p-p\,(z_c))\,/\,(p_s - p\,(z_c)), \qquad p(z_c) < p < p_s, \qquad (6)$$

z_c is given by

$$\int_0^{z_c} \frac{N(z)}{U(z)} \, dz = \frac{3\pi}{2} \; . \qquad (7)$$

We may note several aspects of this procedure:

1. Except for the envelope orography component, all the drag on the atmosphere is in the direction of the low-level wind.

2. While much of the scheme has a physical basis there are several arbitrary aspects. In particular, the subgridscale orographic effects are represented twice, by the envelope orography and by the drag parametrization. The net effect is clearly beneficial, but this mixture is largely for historical reasons, and there is no clear physical basis for choosing the proportions. The proposed scheme below has a physical basis for each of its aspects, and is suggested as a replacement for all of the above.

3. PROPOSED NEW SCHEME

It is proposed that the "envelope orography" be dispensed with, and the orographic height in the model be taken as the mean orographic height of the earth's surface for the region represented by that grid point. This will have two particular advantages. Firstly, special compensation effects which were introduced to allow for a model topography which is not at the correct altitude (Tibaldi, 1986) may be dispensed with. Secondly, it seems possible that, in addition to its beneficial effects, the added "envelope" may introduce spurious drag (or "lift") forces which are non-physical and not easily perceived. There are indications of this from the initial tendency analyses. (E. Klinker, pers. comm.).

3.1 Specification of subgridscale orography

The mean topographic height above mean sea level over the grid point region (gpr) is denoted by $\bar{h}$, and the coordinate z denotes elevation above this level. It is suggested that the topography relative to this height $(h(x,y)-\bar{h})$ be represented by three parameters, as follows:

1. The net variance, or standard deviation μ_h of h(x,y) in the gpr. This is the same quantity as defined above, calculated from the US Navy data set as described by Wallace et al. (1983). μ_h gives a measure of the amplitude of the topography, and $2\mu_h$ approximates the physical envelope of the peaks (Wallace et al. 1983).

2. A parameter γ which characterises the anisotropy of the topography within the gpr, defined below, and

3. An angle ψ, which denotes the angle between the low-level wind direction and that of the principal axis of the topography.

γ and ψ may be defined from the topographic gradient correlation tensor

$$H_{ij} = \overline{\frac{\partial h}{\partial x_i} \frac{\partial h}{\partial x_j}} , \tag{8}$$

where $x_1 = x$, $x_2 = y$, and the terms may be calculated (from the USN data set) by using all relevant pairs of adjacent grid points within the gpr. This symmetric tensor may be diagonalised to find the directions of the principal axes and the degree of anisotropy. If

$$K = \left(\overline{\left(\frac{\partial h}{\partial x}\right)^2} + \overline{\left(\frac{\partial h}{\partial y}\right)^2} \right)/2, \quad L = \left(\overline{\left(\frac{\partial h}{\partial x}\right)^2} - \overline{\left(\frac{\partial h}{\partial y}\right)^2} \right)/2, \quad M = \overline{\frac{\partial h}{\partial x} \frac{\partial h}{\partial y}} , \tag{9}$$

the principal axis of H_{ij} is oriented at an angle θ to the x-axis, where θ is given by

$$\theta = \tfrac{1}{2} \arctan (M/L). \tag{10}$$

This gives the direction where the topographic variations, as measured by the mean square gradient, are largest. The corresponding direction for minimum variation is perpendicular. If we change coordinates to x', y' which are oriented along the principal axes ($x' = x \cos\theta + y \sin\theta$, $y' = y \cos\theta - x \sin\theta$), the new values of K, L and M relative to these axes, denoted K', L' and M', are given by

$$K' = K, \; L' = (L^2 + M^2)^{1/2}, \; M' = 0 , \tag{11}$$

where K, L and M are given by equation 9. We may now specify the anisotropy of the orography by the "aspect ratio" γ, defined by

$$\gamma^2 = \overline{\left(\frac{\partial h}{\partial y'}\right)^2} \Big/ \overline{\left(\frac{\partial h}{\partial x'}\right)^2}, \tag{12}$$

$$= \frac{K' - L'}{K' + L'} = \frac{K - (L^2 + M^2)^{1/2}}{K + (L^2 + M^2)^{1/2}} , \tag{13}$$

If the low-level wind vector is directed at an angle φ to the x-axis, then the angle ψ is given by

$$\psi = \theta - \varphi. \tag{14}$$

The physical interpretation given to γ is clear from the following. If we consider topography of the form

$$h\,(x',y') = h_0\,(1 + \cos kx'.\cos my'), \tag{15}$$

then

$$\bar{h} = h_0,\; \mu_h = h_0/2 \text{ and } \gamma = m/k \tag{16}$$

(Note that with this anisotropic but simple topography, taking directional variances of the form

$$\overline{(\,h(x',y') - \overline{\overline{h(y')}}\,)^2}$$

where $\overline{\overline{h(y')}}$ denotes the mean value of h(x', y') over x' at y', does not give information about the values of k and m). Hence γ is equal to the aspect ratio of these periodic bumps, and equations 12, 13 would appear to give the simplest estimate of the degree of anisotropy of an arbitrary region of topography. Further, we may represent a region of anisotropic topography of given γ by a number of isolated bumps, each with the same aspect ratio.

An example of the effect of the isotropy coefficient on T106 subgridscale orographic standard deviation is shown in Fig. 1. Fig. 1a shows μ_h over Europe, Fig. 1b shows $\mu_h(1-\gamma)$. It can be seen, for example, how the French Alps, largely isotropic, are strongly damped in Fig. 1b, which emphasises the more anisotropic nature of the central and eastern Alps. The magnitude of $\mu_h(1-\gamma)$ for the Pyrenees, is comparable with that of the Alps.

3.2 Parametrization

We represent the total drag in the same form as equation 2, splitting it into a gravity wave part $\underset{\sim}{\tau}_w$ and an hydraulic part $\underset{\sim}{\tau}_{fr}$, except that a vector form is taken - we no longer assume that the total drag is aligned with the low-level wind;

$$\underset{\sim}{\tau}(p) = \underset{\sim}{\tau}_w(p) + \underset{\sim}{\tau}_{fr}(p) \tag{17}$$

We take the topographic height, relative to h to be $2\mu_h$. If the low-level (i.e. at or just above mountain top height $2\mu_h$) wind and buoyancy frequency are denoted U_ℓ and N_ℓ respectively, we classify the flow regimes according to the value of the parameter $\nu = 2\mu_h N_\ell / U_\ell$. This follows results from a number of laboratory studies (see, for example, Baines and Manins (1989) for a recent survey).

<u>$0 < \nu < 1$</u>

In this range we may assume that the dynamics of the flow over the subgridscale orography is predominately linear, and may be described by linear wave theory. Linear wave drag on the atmosphere over anisotropic topography has been calculated by Phillips (1984), and has the form

$$\underset{\sim}{\tau}_w(p_s) = (\tau_1, \tau_2) \tag{18}$$

where τ_1 is the stress in the direction opposed to the low-level wind (direction $\pi+\varphi$), and τ_2 is perpendicular to it (direction $\varphi - \frac{\pi}{2}\,\mathrm{sgn}\,\psi$), and

$$\tau_1 = k_1\, U_\ell\, N_\ell\, \mu_h^2\, (B(\gamma) \cos^2\psi + C(\gamma) \sin^2\psi), \tag{19}$$

$$\tau_2 = k_1\, U_\ell\, N_\ell\, \mu_h^2\, (B(\gamma) - C(\gamma)) \sin\psi.\cos\psi \tag{20}$$

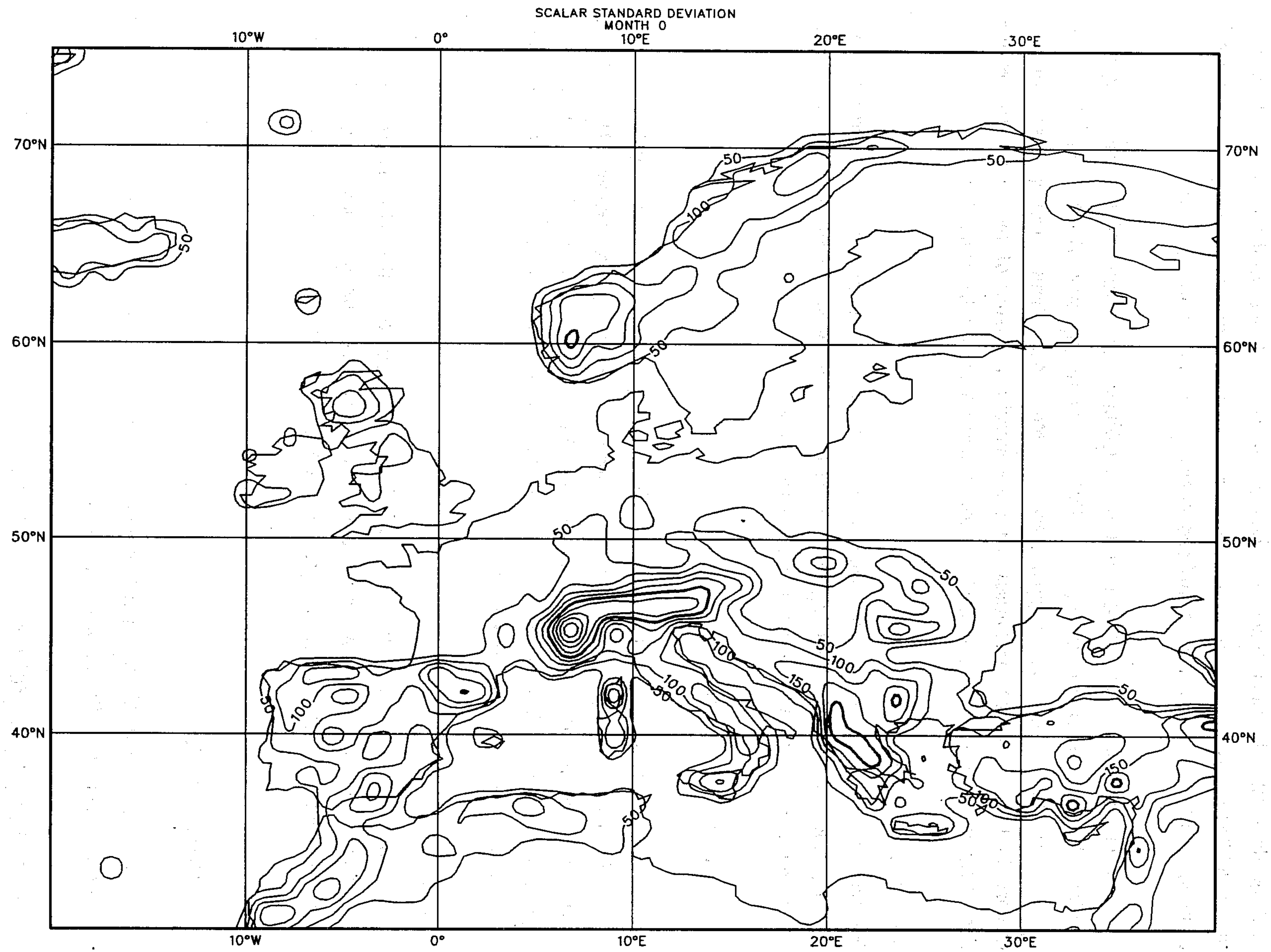

Fig. 1a μ for T106 model (m).

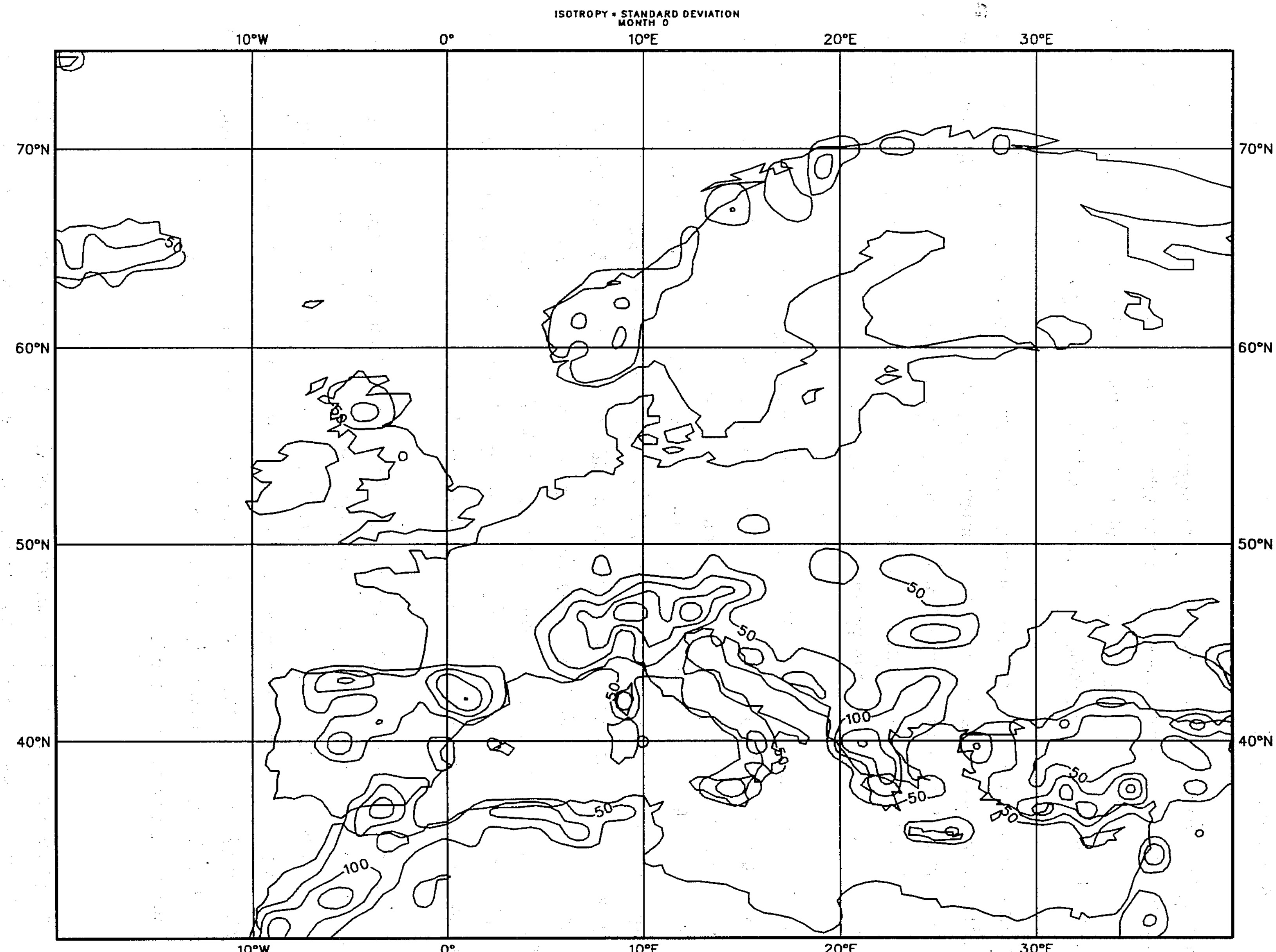

Fig. 1b μ(1-γ) for T106 model (m).

The functions $B(\gamma)$ and $C(\gamma)$ are given approximately by

$$B(\gamma) = 1 - 0.18\,\gamma - 0.04\,\gamma^2, \qquad (21)$$
$$C(\gamma) = 0.48\,\gamma + 0.3\,\gamma^2, \qquad (22)$$

and k_1 is a constant which must be chosen empirically. Note that the transverse wave drag may be quite substantial, unless the topography is nearly isotropic so that γ is close to unity. The net wave drag is in general in a direction which is not aligned with the low-level wind U_{ℓ}, or the topography. $\underline{\tau}_{fr}(p)$ is taken to be zero in this parameter range.

In general the wind U and the buoyancy frequency N vary with height z (horizontal variations may be neglected for present purposes), and the effects of this on the wave drag on the atmosphere must be considered. The steady-state lee wave field produced by a region of complex terrain will be composed of a spectrum of horizontal wave numbers centred on a wavenumber directed opposite to the surface stress.

When U(z) and N(z) vary with height, as the waves propagate vertically the horizontal wave numbers remain unchanged (from linear theory and WKB). We let $U_*(z)$ be the wind resolved in the direction along the surface stress vector, and write $\ell^2 = N^2(z)/U_*^2(z)$ if l decreases with height so that l^2 becomes negative at some level, the wave energy will be reflected downward and will be trapped below this level. The associated drag should be distributed uniformly through this depth range. If, on the other hand, $U_*(z)$ decreases with height and becomes zero at some level (because of rotation of the wind vector, for example), this will constitute a critical level for the waves, and they will be dissipated in a region immediately below this level. The associated drag should therefore be implemented over a relatively narrow range of heights below this level.

If $l \gg 1$ throughout the atmosphere, the Richardson number criterion (Miller et al., 1989) should be applied throughout the whole depth of the atmosphere, up to a critical level, if present. If l has values approaching unity, a transmission coefficient which is a function of l is recommended for this region. Hence we envisage a function $f(l(p), Ri(p))$ for the dependence of the wave-induced stress in the vertical. The form of f(p) will be similar to that of equation 5, with the exception that p' will be dependent on the vertical profile of l.

<u>$\nu > 1$</u>

In this parameter range the flow pattern is generally substantially different from that for $\nu < 1$, for two reasons. Firstly, the internal waves will steepen to the point of breaking, which leads to a hydraulic flow regime at low levels. Secondly, flow at low levels (specifically, for $0 < z < 2\mu_h - U_\ell/N_\ell$) will flow around the topography rather than over it. This will result in extensive "wake" regions downstream of topographic features where the net momentum in the flow will be small. Obstacles within these wake regions will therefore feel very little flow at these levels, and the net effect will be to tend to cause the low-level fluid to flow around the whole region. In order to parametrize this effect, therefore, it is appropriate to introduce a large drag coefficient (or equivalent) which reduces the fluid velocity in the range $0 < z < 2\,\mu_h - U_\ell/N_\ell$ to a small value. The effects of this are expected to be similar to that of the "envelope orography" in many respects, with none of the adverse side effects; flow through the region (for example, in response to a pressure gradient) is still possible.

Hence, for model levels in the range $0 < z < 2\,\mu_h - U_\ell/N_\ell$ the additional term $C_D\,|\,\underset{\sim}{U}\,|\,\underset{\sim}{U}$ should be introduced into the horizontal momentum equation, for suitably chosen C_D.

For greater heights, $z > 2\,\mu_h - U_\ell/N_\ell$, the transverse flow over ridges (with incident flow $U\cos\psi$) will be hydraulic, but the variable topographic configuration (with variable heights and orientations of individual topographic features) will imply that substantial lee wave energy may be generated as well. Evidence from experimental studies indicates that the flow in this non-linear regime may take a very long time to reach a steady-state, so that a definitive scheme (even with specified simple topography) is probably not possible. The optimum scheme for this parameter range may well depend on trial and error.

The parameters γ and ψ will still apply, and it is proposed that the scheme of Miller and Palmer, mentioned in the introduction, be implemented here, with two modifications:

Firstly, the wave drag component $\underset{\sim}{\tau}_w$ has directional components as given above for linear theory, and the mountain amplitude is taken to be $U_\ell\,/\,N_\ell$ (rather than $2\mu_h$). This implies that

$$\underset{\sim}{\tau}_w = (\tau_1, \tau_2)\, f(p), \tag{23}$$

where

$$\tau_1 = k_2\, \rho\, U_\ell^3/N_\ell\, (B(\gamma) \cos^2\psi + C(\gamma) \sin^2\psi), \tag{24}$$

$$\tau_2 = k_2\, \rho\, U_\ell^3\, N_\ell\, (B(\gamma) - C(\gamma)) \sin\psi.\cos\psi, \tag{25}$$

k_2 is another constant, and $f(p)$ is the same function of l and R_i as described above.

Secondly, the Froude number component of the drag is given the vector dependence and is effected in the height range where the hydraulic flow takes place, but is otherwise unchanged in its vertical structure etc. This implies that

$$\underline{\tau}_{fr}(p) = (k_3/k_2)\,(\nu-1)\,g(p)\,(\tau_1, \tau_2), \tag{26}$$

where

$$g(p) = (p-p(z_c))/(p^* - p(z_c)), \tag{27}$$

where p^* is the pressure at the height $z = 2\mu_h - U_\ell / N_\ell$.

In summary, this proposed scheme is based on physical considerations and makes considerable use of existing schemes. It should not be difficult to implement, but some fine-tuning will obviously be necessary.

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