LOCAL SKILL PREDICTION WITH A SIMPLE MODEL

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Abstract: Using a 3-level quasi-geostophic model two skill forecast experiments are performed for Western-Europe. The skill predictor we propose in this paper is the maximal forecast error which can occur for Western-Europe. This maximal error is determined with adjoint techniques. One experiment is within the context of the 3-level model. By modifying the method, we also investigate the skill in predicting the quality of the ECMWF forecast for Western-Europe. Both experiments evidently show the skill of this predictor to select accurate forecasts, upto a forecast period of 72 hours.

1 Introduction.

Over the last two decades research in atmospheric physics has contributed substantially to the improvement of the performance of numerical weather prediction (NWP) models. In recent years it has become more and more difficult to make progress in this. For the state-of-the-art NWP models the deterministic intrinsic growth of small initial errors, is almost as large as the growth of forecast errors (Lorenz, 1982). This is an indication that further improvement of the average skill of NWP models no longer automatically results from model improvements. Yet, the observed variability in skill is quite large. This variability may partly be explained as a purely statistical effect. A particular forecast must be considered as an arbitrary member of a probabilistic distribution. It may also reflect a temporal and regional variability in atmospheric predictability. If the variability in the predictability of the atmosphere is substantial it is useful for the forecaster to have an a priori measure of the quality of the forecasts.

Various methods for predicting the forecast skill have been developed. Epstein (1969) proposed a stochastic-dynamic model for determining the probabilistic distribution of an atmospheric variable. The drawback of this approach is that it involves a large amount of computation. In Monte Carlo forecasting, introduced by Leith (1974), a small ensemble of randomly chosen initial states is integrated with the forecast model. The occurrence of each initial state is considered to be equally likely. The spread between the ensemble members is a measure of the atmospheric predictability. However, the complexity of the current forecast models greatly obstructs the operational use of the Monte Carlo technique. Further it is not obvious that the randomly chosen initial states form a representative sample.

A more feasible way of ensemble forecasting is lagged average forecasting (Hoffman and Kalnay, 1983). Here the forecast ensemble consists of the latest operational forecast together with forecasts based on previous analyses but with the same verification time. By weighting the members statistically the lagged average forecast is obtained. This procedure yields in general a more accurate forecast and because all these forecasts are already available, there are almost no additional computational costs. Dalcher *et al.* (1985) found that the skill of using the spread as a predictor of quality of the operational forecast was only minimal. This could be due to the large verification area, so that regional differences in skill are not taken into account. In Kalnay and Dalcher (1986) ensemble forecasting was performed for smaller areas. They were quite well able to predict the anomaly correlation between the forecast and the corresponding ECMWF analysis. It should be noted that the period which they considered only covered one month. When the verification area was enlarged to the Northern Hemisphere a deterioration of the forecast skill was observed.

Instead of using a predictor based on ensemble forecasting, Chen (1988) proposes to use the persistence of the model atmosphere during the integration as a skill predictor. It appears that for the medium range, the correlation between this simple predictor and the forecast quality is significant for regional as well as for hemispheric verifications.

Motivated by the above results, Leslie and Holland (1990) developed a technique using three different predictors. Two of these, the stastistical regression scheme given by Bennet and Leslie (1981) and modified by Glowacki (1988) and further the persistence predictor introduced by Chen (1989), utilize model output of the Australian region limited area model. The third predictor is based on the spread between forecasts of different weather centres. In predicting the skill of the 36h forecasts, they were able to obtain a high correlation between the predictor and the forecast error.

In this paper we employ adjoint equations to define a possible skill predictor. The concept of adjoint equations was probably first introduced in the meteorological literature by Marchuk (1967), but received serious attention only after the article by Talagrand and Courtier (1987). In Molteni *et al.* (1991), modes are determined with adjoint techniques that have maximal growth of energy within a certain forecast period. Since the tangent equations are involved in deriving the adjoint equations, the applicability of this concept is essentially limited to a forecast period of three days. These modes, computed with a 3-level quasi-geostrophic model as described in Marshall and Molteni (1991), are used to form an ensemble of initial states. This ensemble is integrated with the ECMWF model and the spread is considered as a measure for the skill of the unperturbed forecast. Some case studies for the medium range, beyond the regime where error growth is linear, justify a closer look at this approach, see Molteni *et al.* (1991).

Along these lines we investigated the possibility to obtain a priori knowledge of the skill of the operational ECMWF forecast model for a small area, using the same quasi-geostrophic model. In Barkmeijer (1991), we studied a method which yields the initial error field that has the largest growth in RMS sense for a pre-chosen area and forecast period. This growth showed a large variability depending on the verification area and flow pattern. For that reason it was suggested that this concept could be employed as a possible skill predictor for the short range. The idea is simply to use this predictor of maximal forecast error to distinguish a substantial subset of the accurate forecasts. When the predictor has a small value, the error must be small. For large values nothing definite can be concluded. In this case it is crucial to what extent the initial error projects onto the rapidly growing error mode. In section 2 we give an outline of the techniques we use. The validation of the method as a possible skill predictor is discussed in section 3. Finally, in section 4 we give some concluding remarks.

2 The method.

Suppose that the dynamics of the atmosphere is described by the following dynamical system:

$$\frac{\mathrm{d}}{\mathrm{dt}}x = F(x), x \in \mathcal{H} \tag{1}$$

Let u be a solution of eq. (1) on the time interval $[0, t]$. In this section we are concerned with the growth of initial errors of $u(0)$ for a pre-chosen area and forecast time t.

The time evolution of errors to the reference orbit u is given by

$$\frac{\mathrm{d}}{\mathrm{dt}}\varepsilon = F(u + \varepsilon) - F(u) \tag{2}$$

If we assume that the errors remain small during the forecast time, eq. (2) reduces to

$$\frac{\mathrm{d}}{\mathrm{dt}}\varepsilon = \mathrm{D}F(u)\varepsilon \tag{3}$$

Usually this equation is referred to as the tangent linear equation. The linear operator $\mathrm{D}F(u)$ is obtained by differentiating F along the reference orbit u. There exists a linear operator $R : \mathcal{H} \longrightarrow \mathcal{H}$ such that for solutions ε of eq. (3) we can write:

$$\varepsilon(t) = R(0, t)\varepsilon(0) \tag{4}$$

The operator R is called the resolvent of eq. (3). In the following we are interested in the initial error $\varepsilon(0)$, of some fixed norm, which yields the largest possible RMS error in the stream function field for a small area at time t. It is our intention to relate this maximal error to the skill of the forecast model. For simplicity we first consider the case when the area is reduced to a single grid point p. In the following we denote by $\varepsilon(t)_{|_p}$ the value of $\varepsilon(t)$ in p.

One can choose an inner product $<,>$ on $\mathcal{H}$ and a linear operator $L_p : \mathcal{H} \longrightarrow \mathcal{H}$ such that, see Barkmeijer (1991):

$$< L_p\varepsilon(t), \varepsilon(t) >= (\varepsilon(t)_{|_p})^2. \tag{5}$$

Using eq. (3) we also have

$$< L_p\varepsilon(t), \varepsilon(t) >$$

$$=< L_pR(0,t)\varepsilon(0), R(0,t)\varepsilon(0) >$$

$$=< R^\star(0,t)L_pR(0,t)\varepsilon(0), \varepsilon(0) > \tag{6}$$

The linear operator $R^\star(0,t)$ is the adjoint of $R = R(0,t)$. Further one can show that $R^\star(0,t) = S(t,0)$, where S is the resolvent of the adjoint equation related to eq. (3):

$$\varepsilon = -\mathrm{D}F(u)^\star\varepsilon. \tag{7}$$

We conclude from eqs. (5) and (6) that of all initial errors $\varepsilon(0)$ with some fixed norm $< \varepsilon(0), \varepsilon(0) >$, the error proportional to the eigenvector of R^*L_pR with the largest eigenvalue, results in the maximal absolute value of $\varepsilon(t)$ in p. The inner product $<,>$ we use is given by:

$$< x, y >= \int x \cdot y\mathrm{d}s, \tag{8}$$

where the integration is over the entire sphere.

The above can easily be generalized to a larger area V. Suppose V consists of K grid points p_i, $i = 1,..,K$. Let $\mathcal{L}$ be the operator given by:

$$\mathcal{L} = \sum_{i=1}^{K} L_{p_i}. \tag{9}$$

We have

$$< R^*\mathcal{L}R\varepsilon(0), \varepsilon(0) >$$

$$= \textstyle\sum_{i=1}^{K} < R^*L_{p_i}R\varepsilon(0), \varepsilon(0) >$$

$$= \textstyle\sum_{i=1}^{K}(\varepsilon(t)_{|p_i})^2.$$

It is easy to show that $R^*\mathcal{L}R$ is a positive self-adjoint operator. We determined the eigenvectors and eigenvalues of $R^*\mathcal{L}R$ with the Lanczos algorithm, see Parlett (1980). This algorithm makes it possible to compute the eigenvalues of a linear operator without knowing its explicit form. By choosing $\varepsilon(0)$ proportional to the eigenvector of $R^*\mathcal{L}R$ with the largest eigenvalue λ^2_{max}, we obtain the largest possible RMS of $\varepsilon(t)$ in V. The value of λ_{max} which is proportional to the RMS can be considered as an upper bound to error growth provided that the error remains in the linear regime. Usually many eigenvectors of $R^*\mathcal{L}R$ contribute to the initial error in the analysis resulting in a smaller final error in V.

In the next section we investigate whether small values of λ_{max} imply an accurate forecast for the area V.

3 Numerical experiments.

The model we use for skill forecasting is a 3-level quasi geostrophic model truncated at wavenumber 21. For a complete description of the model we refer to Marshall and Molteni (1991). The levels of this model are at 200hPa, 500hPa and 800hPa. In the sequel we simply denote this model by T21. We perform two experiments to which we refer as the T21-experiment and the ECMWF-experiment.

In the T21-experiment we perform skill forecasts for T21 using the tangent linear model of T21 and its adjoint. In case that the internal error growth of T21 resembles that of the ECMWF forecast model for short range, T21 may also inform about the skill of the ECMWF model. In the ECMWF experiment we validate this idea using the forecast orbit of the ECMWF model. We first discuss the T21-experiment.

(a) *T21 experiment*

For each day in December and January of the years 1989-90 and 1990-91, we retrieved the analyis at 12 GMT from the ECMWF archives. Starting with these analyses, the forward T21 orbit is determined for a forecast time of 48h. This, together with the adjoint model, enables us to compute λ_{max} for the error in the stream function at 500 hPa in an area located between 11°W and 11°E and between 40°N and 60°N. In the following this area is called Western-Europe. Usually three iterations with the Lanczos algorithm, see Parlett (1980), suffice to determine λ_{max}. In numerical costs this corresponds with running the forecast model 9 times. Each iteration more gives the succesive eigenvalue in magnitude.

In fig. 1, the RMS forecast error of the streamfunction at 500 hPa for Western-Europe is plotted against λ_{max}. The forecast error is obtained by verifying the T21 +48h forecast with the corresponding ECMWF analysis. The figure clearly shows the variability in λ_{max} during this period. Sometimes days differ in their value of λ_{max} by a factor of at least two. So by choosing a particular initial error, some flow patterns enable a much larger error growth over Western-Europe than others do. The set of points seems to form a wedge which agrees with the interpretation of λ_{max} as an upper limit on error growth. Usually the analysis error will also project substantially on modes with a slower growth rate than the mode related to the largest eigenvalue. So when λ_{max} is small we expect a clustering of points at small values of the RMS error, while for increasing λ_{max} an increasing range of forecast errors must occur. These expected features show up prominently in fig. 1. It is not clear yet whether or not the cases of small λ_{max} are related to particular types of flow patterns.

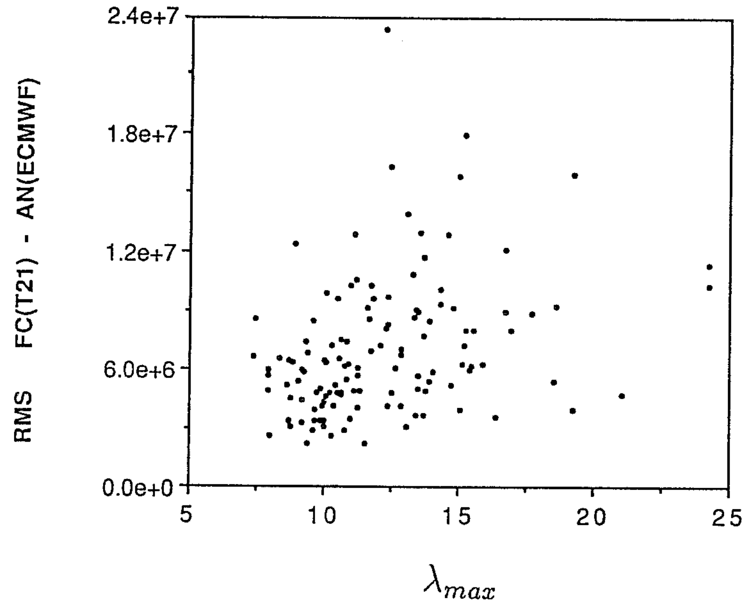

FIGURE 1. For December and January of 1989-90 and 1990-91, the RMS (m^2/s) of the forecast error for Western-Europe is plotted against λ_{max}. The T21 model is used as a forecast model with forecast period 48h.

In order to quantify the relation between λ_{max} and the RMS error over Western-Europe, we choose a RMS value of 9.6 $10^6 m^2/s$ as a distinction between good and bad forecasts. For this RMS value the set of days with an accurate forecast consists of $G = 101$ days. This set is called the good set. Suppose the number of days with λ_{max} less than some pre-chosen value λ is equal to $M(\lambda)$, $G(\lambda)$ of which belong to the good set. In case λ_{max} indeed informs about the good set then the chance of selecting a member of the good set from the subset for which λ_{max} is smaller than λ should exceed $G/124$

In fig. 2 we give for different values of λ the pair $(G(\lambda)/M(\lambda), G(\lambda)/G)$. The first coordinate gives the chance that a selected day indeed belongs to the good set, while the second coordinate denotes the fraction of the good set that is selected if we make the restriction to cases for which λ_{max} is smaller than λ. It is obvious that $G(\lambda)/M(\lambda) \longrightarrow G/124$ for large values of λ. The predictability to select a good forecast considerably exceeds the climatological chance, which is 81% for a RMS equal to 9.6 $10^6 m^2/s$, for all four values of λ. It appears that λ_{max} indeed contains information with respect to the growth of errors.

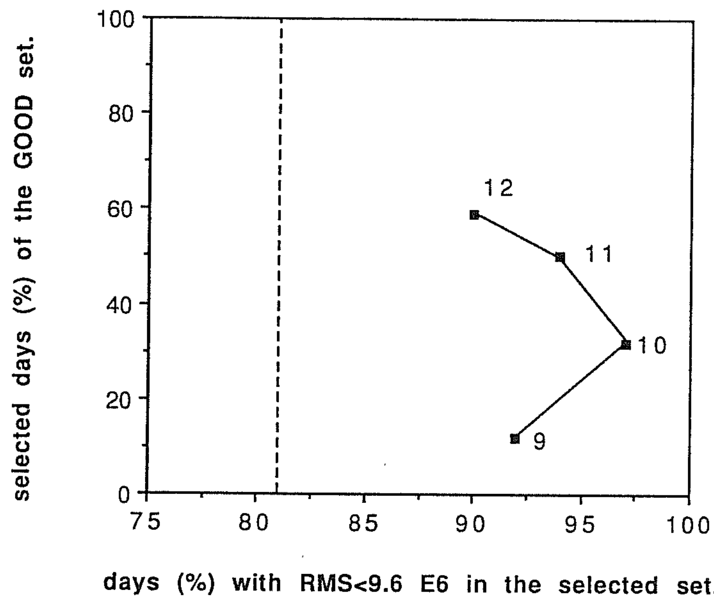

FIGURE 2. Horizontally the percentage of days with $\lambda_{max} < \lambda$ is given that belongs to the good set, i.e. the set of days with RMS $< 9.6\ 10^6 m^2/s$. The percentage of the good set already selected is shown vertically. The dashed line denotes the fraction of the good set in the complete set. The numbers refer to various values of λ.

(b) *ECMWF-experiment*

In the ECMWF-experiment we are interested to what extent the method remains useful in performing skill forecasts for the operational ECMWF model. Because the T21 forecast orbit deviates too much from the ECMWF forecast orbit after 24h, we have to include some dynamics of the ECMWF model. To that end we use ECMWF forecasts every twelve hour and interpolate these with T21, see fig. 3.

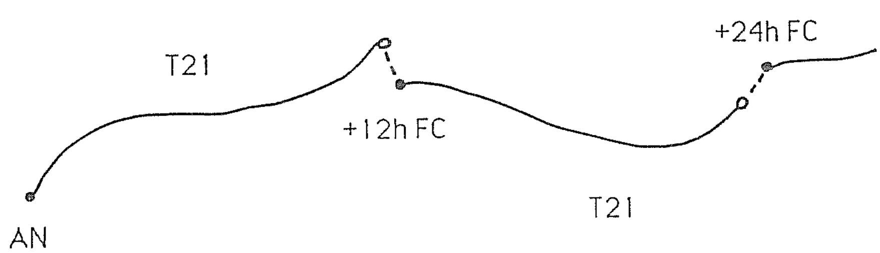

FIGURE 3. Forecast orbit for which λ_{max} is determined in the ECMWF experiment. With AN and FC we denote ECMWF data.

More precisely, starting with an analysis from the same set as is used in the T21 experiment, T21 is integrated for 12h. Then in the final time step the obtained field is replaced by the +12h ECMWF forecast. With this field as new initial field, we again integrate T21 for 12h replacing in the final time step the field by the +24h ECMWF forecast. This procedure is repeated until we reach the forecast time. With respect to this orbit we again determine λ_{max} for Western-Europe as in the T21 experiment. The result is shown in fig. 4. The clustering for low λ_{max} is less apparent than in fig. 1. A shorter interpolation period, thus keeping a better track of the ECMWF forecast orbit, may improve upon this point.

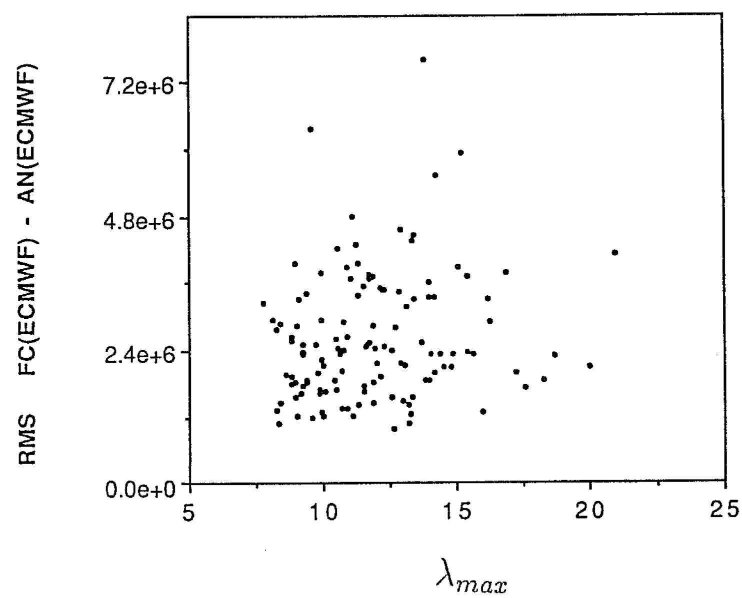

FIGURE 4. Same as fig. 1 but now for the ECMWF forecast orbit interpolated with the T21 model.

In fig. 5, we present for various forecast periods the skill in selecting the good set for Western-Europe as we did in fig. 2 for 48h. For all forecast periods of 48h, 72h and 96h, we choose a fixed RMS value of 3.6 $10^6 m^2/s$ to distinguish between good and bad forecasts. Again there is an improvement over climatology for all five values of λ except at a forecast period of 96h. The gain over climatology is approximately 10 to 15 %, which is not much different from the result of the T21 experiment. When the forecast period is 96h, the chance to select from the good set rapidly drops to the climatological predictability. One of the main reasons probably is that the assumption of linear error growth is not valid anymore. Also the width of the error distributions becomes broader at larger forecast times, leading to a more substantial sampling error in the results at 96h.

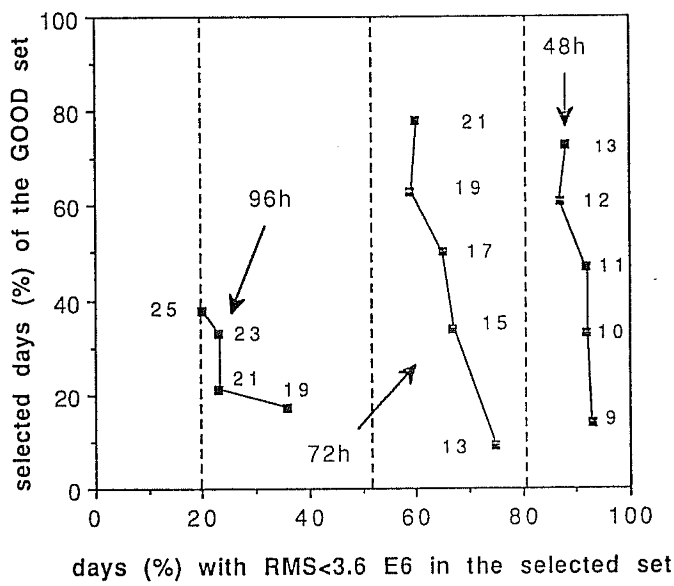

FIGURE 5. Same as fig. 2 for the ECMWF experiment. The forecast period is 48h, 72h or 96h.

4 Final remarks.

We have performed two numerical experiments with a 3-level quasi-geostrophic model to study the feasability of a possible skill predictor. This predictor λ_{max} is the maximal error in RMS sense, as given by the largest eigenvalue of a linear operator, which can occur in the forecast for the stream function at 500 hPa over Western-Europe.

First we tested this idea wholly within the context of the 3-level model. Results show that for low values of λ_{max} we have a substantial inprovement over climatology in predicting that a particular forecast will have a small error. Motivated by this, we investigated whether λ_{max}, as computed with the 3-level model using adjoint techniques, can also be employed to distiguish with a better than climatological chance a subset of the ECMWF forecasts with a small error. The results indicate that this is indeed the case and so the approach may be promising if one is interested in skill forecasts with relatively small investments.

We point out that in this study all information contained in the initial error covariance matrix is neglected. So we assumed that the analysis error does not have a prevailed direction in phase space. As a consequence of this, nothing can be concluded from a forecast with a large value of λ_{max}. It is quite possible that for days with large λ_{max}, the analysis error does not project onto the rapidly growing error modes. Furthermore we did not include in this paper the occurence of model errors either.

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