TECHNICAL MEMORANDUM

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Experiments on 4D-Var assimilation of rainfall data using an incremental formulation

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Abstract

Variational data assimilations of isolated surface rainfall observations are performed using two methodologies compatible with the incremental formulation of the ECMWF operational 4D-Var system. In a first method (defined as “1D-Var+4D-Var”) 1D-Var retrievals of total column water vapour in rainy areas are assimilated in 4D-Var, while a second approach assimilates directly rainfall rates in 4D-Var. In addition, the behaviour of two contrasted convection schemes (mass-flux and adjustment) is examined. The minimization performs generally better for the adjustment scheme and for the “1D-Var+4D-Var” method (i.e. larger decrease of the cost-function). The usefulness of the direct 4D-Var is limited by various inconsistencies present in the current incremental formulation and by the extreme sensitivity of the convection schemes to small changes in humidity. However, it is demonstrated that significant modifications to the dynamical fields can take place when rainfall rates are directly assimilated in 4D-Var. The “1D-Var+4D-Var” method provides analysis fields consistent with the direct 4D-Var but tends to filter the rainfall rate information leading to smaller increments for all model variables.

1 Introduction

Operational Numerical Weather Prediction (NWP) centres are having an increased interest for assimilating rainfall rate observations (Treadon 1997, Mcpherson 2001, Marécal and Mahfouf 2002). The reasons are threefold. Firstly, it is certain that rain measurements contain valuable information on humidity, temperature and wind analyses because precipitating systems contribute substantially to the energy budget of the atmosphere, via latent heat release. Secondly, rainfall rate observations are now available at global scale from spaceborne platforms. In particular, the Tropical Rainfall Measuring Mission (TRMM) was launched in November 1997 to provide accurate rainfall rates in the tropics and subtropics (Simpson et al. 1996, Kummerow et al. 1998). Thirdly, the variational assimilation techniques run operationally in many meteorological centres (NCEP, UKMO, MSC, Météo-France, ECMWF) to produce global analyses and can, in principle, handle precipitation observations. The three-dimensional variational (3D-Var) or four-dimensional variational (4D-Var) methods rely on operators that calculate a model equivalent for each observation. An important requirement is the availability of the tangent linear (i.e. Jacobian vectors) of these operators and their adjoints (transpose of the tangent linear) for an efficient minimization of the objective function of the variational problem. For rain assimilation, the physical operators needed to produce model rainfall rates are the moist convection and the large-scale condensation parameterizations.

The tangent-linear approximation needs to be valid with a reasonable degree of accuracy in order to achieve useful results from a variational data assimilation system. In a variational assimilation of precipitation various non-linearities can be the source of potential problems. Firstly, 3D/4D-Var systems have only been implemented in global NWP models using an incremental formulation (Courtier et al. 1994) to make their computational cost compatible with operational requirements. The incremental formulation solves the minimization problem in terms of increments (difference between the model state and a background state) using simplified tangent-linear and adjoint models. In practice, the use of a low horizontal resolution model with a reduced set of physical parametrizations is the major simplification of the linearised models with respect to the non-linear model computing the innovation vectors. Therefore, the convergence of this formulation can be hampered when the simplified linearised models provide an evolution of the perturbed flow that is inconsistent with the full non-linear model. Secondly, the moist convection schemes describing the effect of sub-grid scale cumulus clouds on the resolved model variables are strongly non-linear and are characterized by on/off processes. Convective parametrization schemes based on the mass-flux approach (Arakawa and Schubert 1974, Tiedtke 1989) require a cloud model increasing the complexity and the non-linearities due to the necessary description of the microphysics. By contrast, relaxation schemes (Manabe et al. 1965, Betts 1986, Betts and Miller, 1993) describe the effect of convection without a cloud stage, leading to simpler schemes that are, a-priori, more linear. Their complexity is compatible with large-scale precipitation schemes based on moist local adjustment. However, because rain formation and evaporation result in a series of threshold processes, all moist parameterization schemes include thresholds, that can limit the validity of the tangent-linear approximation.

Operational rainfall rate assimilation is a target in the forecoming decade for many weather services, in particular for the European Centre for Medium-range Weather Forecasts (ECMWF). At ECMWF, a 4D-Var analysis system has been used operationally since November 1997 (Rabier et al. 2000, Mahfouf and Rabier 2000, Klinker et al. 2000). Various types of data are assimilated but no cloud or rain information yet. Nevertheless, preliminary studies have already been performed on the assimilation of rainfall rates. These studies were based on a two step approach ("1D-Var + 4D-Var"). In a first step, temperature and specific humidity profiles are retrieved from surface rainfall rate observations via a 1D-Var approach (Marécal and Mahfouf 2000). In a second step, the total column water vapour (TCWV) derived from the 1D-Var humidity profiles is assimilated in 4D-Var. This means that the rainfall rate information is first converted into a humidity information before being used in 4D-Var. The "1D-Var + 4D-Var" method was tested on TRMM estimates of rainfall rates. Results have shown a positive impact on global humidity analyses and on wind analysis within and near rainy areas. An improvement of the temperature and wind forecasts in the tropics was also found (Marécal and Mahfouf 2002). These results have proved that rain-derived information is useful in the ECMWF data assimilation system. This is the reason why technical developments have been recently achieved to allow a direct assimilation of rainfall rates in 4D-Var, without going through the 1D-Var step. With this latter approach, it is expected that the performance of the assimilation will closely depend on the convection scheme used and on its corresponding tangent linear and adjoint.

The objective of this paper is to examine two 4D-Var assimilation methodologies of rainfall within operational constraints (i.e. using an incremental formulation), even though we have somewhat modified the operational ECMWF 4D-Var configuration in the present study. The justification of the changes are given in the following section. We also consider the behaviour of two contrasted convection schemes in order to evaluate their suitability for rainfall assimilation. To have a better understanding of the behaviour of the 4D-Var systems, single observations of rainfall rate are assimilated in tropical areas where deep moist convection plays a dominant role in the generation of surface precipitation.

A description of the model configuration is given in section 2. The general behaviour of both schemes is discussed in section 3. Section 4 is devoted to the 4D-Var results. In this section are discussed assimilation experiments with both the "1D-Var + 4D-Var" approach and with the "direct 4D-Var" approach. The main conclusions of this study are given in section 5.

2 Model description

The model configuration described herafter has been designed to improve the consistency between the non-linear and linearised versions used in the incremental 4D-Var system. More precisely, the observations departures are estimated with a model having the same spatial resolution and almost the same physical processes as the one used for the minimization.

In this study we have used a version of the ECMWF model (CY23R3) with a spectral truncation T_L159 corresponding to about 120 km grid-spacing [1] and 60 vertical levels (up to the 0.1 hPa level). It uses a semi-Lagrangian advection scheme together with a linear Gaussian grid (Hortal 1999). The model includes a comprehensive package of sub-grid scale physical processes described by Gregory et al. (2000). Since the tangent-linear and adjoint versions of the model contain only a simplified set of physical processes (Mahfouf 1999),

[1]The current operational truncation is T_L511 (40 km)

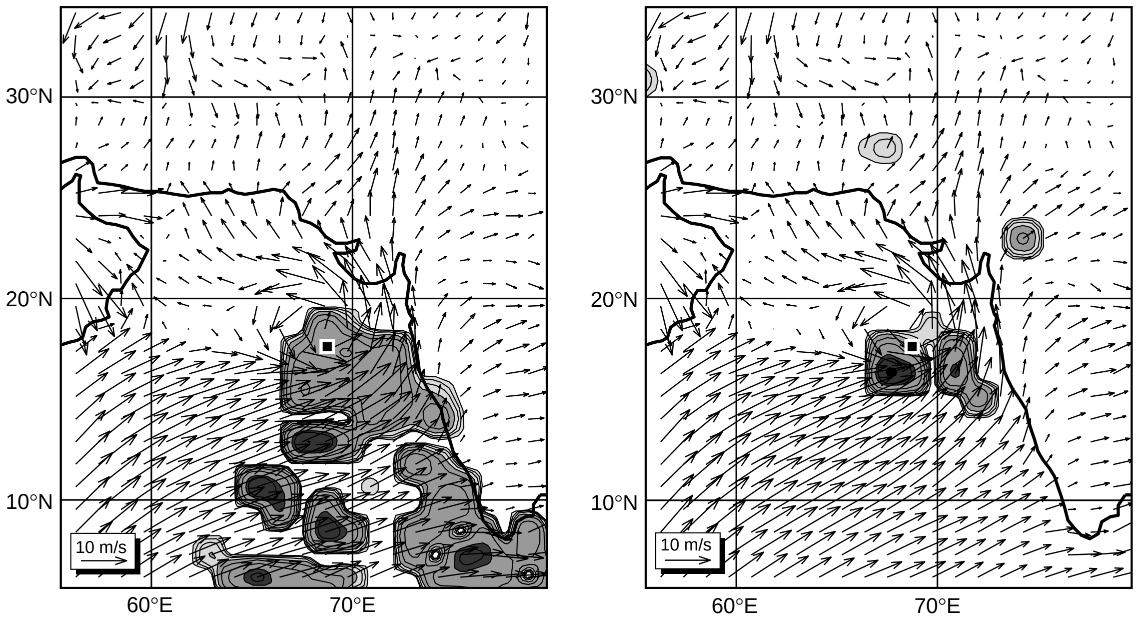

Figure 1: Instantaneous surface convective rainfall rate in mm hr^{-1} from a 6-hour model forecast started at 1200 UTC 26 May 2001 (a) using the MF scheme and (b) using the ADJ scheme (together the wind field at 10 metres). The black square shows the location of the simulated observation. Light grey shading starts above 0.4 mm hr^{-1}, grey shading starts above 4 mm hr^{-1}, dark grey shading starts above 40 mm hr^{-1}

some modifications have been introduced in the non-linear model with respect to the operational package. In particular, a diagnostic large scale condensation scheme is used (Haltiner and Williams 1980) instead of the operational prognostic cloud scheme (Tiedtke 1993). The excess of moisture with respect to saturation at a given level is converted into non-convective precipitation. The adjusted values of temperature and humidity correspond to a saturation state and are obtained by keeping the moist static energy constant. Evaporation of precipitation in subsaturated layers is accounted for.

3 Moist convective processes

The behaviour of two conceptually different convection schemes is examined in this study : the ECMWF operational mass-flux (noted MF hereafter) and the adjustment scheme proposed by Betts (1986) and Betts and Miller (1993) (noted ADJ hereafter). MF was originally developed by Tiedtke (1989). The cloud population is described through a single bulk entraining plume model. For deep convection, Tiedtke's original moisture convergence closure is replaced by a closure based on the reduction of convective available potential energy towards zero over a given timescale depending on the model truncation (Gregory et al. 2000). The Betts-Miller adjustment scheme directly represents the quasi equilibrium between deep convection and the large-scale forcing by relaxing temperature and moisture towards reference profiles deduced from observed soundings in convective situations. A representation of unsaturated downdrafts assumed to originate from a single level (level 50 located around 900 hPa) is also included.

3.1 Surface rainfall

Figure 1 shows a 6-h forecast valid at 1800 UTC 26 May 2001 of instantaneous surface rainfall rate (noted R_b hereafter) over the Gulf of Arabia produced by the two convection schemes. This area has been selected because it encompasses a tropical cyclone in a mature stage as illustrated by the 10-m wind vector. The two fields are significantly different since the ADJ scheme only produces precipitation in the south-eastern part of the cyclone, whereas the MF scheme provides large areas of precipitation in the southern part of the domain. In very active areas, ADJ forecasts rainfall rates are twice as large as with MF. Satellite derived rain rates from the microwave radiometers SSM/I and TMI indicate that the spatial extension of the rainy areas is overestimated by the MF scheme, and that both schemes fail to reproduce a region of intense precipitation along the coast of India near 10^o N (not shown). In the following, we selected the observation location in the area of maximum rainfall for both schemes near the eye of the cyclone (17.383 N, 68.625 E) as indicated in Fig. 1 by the black square. Our choice was also guided by the need to assimilate an observation in a region with a strong dynamical forcing dominated by deep moist convection processes.

3.2 Heating rates and Jacobians

The analysis increments produced in the 4D-Var system will depend both on the background error statistics and on the sensitivity of the observation operator with respect to the control variables (including the model dynamics and the moist physical processes). We examine the heating and moistening profiles produced by the two parameterizations and their associated rainfall rate Jacobians (sensitivity of surface rainfall rate with respect to temperature and humidity profiles). To calculate the Jacobian vectors for T and q, model profiles from 6-hour forecasts started at 1200 UTC 26 May 2001 were taken at the same model grid point as shown in Fig. 1. Note that T and q profiles for MF and ADJ are not identical since they result from a 6-hour forecast run with a different convection parameterisation. The heating and moistening rates produced by the two schemes are presented in Fig. 2a and 2b. The vertical extent of the convective column is similar in the two schemes, the cloud top being found around level 25 (100 hPa). The behaviour below cloud base shows more active downdrafts with MF than with ADJ inducing a significant cooling and drying in one case and negligible modifications of the tendencies in the other case. As already noted on surface rainfall rates, the heating and moistening rates are significantly larger with ADJ than with MF. Whereas the maximum of heating by convection takes place around level 45 (720 hPa) with MF, it is located much higher with ADJ (around 350 hPa). The ADJ scheme produces large cooling and moistening tendencies around the freezing level, whereas a similar behaviour on the large scale variables is noticed with MF around cloud top where significant detrainment of cloud properties in the environment takes place. The relaxation towards prescribed profiles with ADJ produces much smoother profiles than with MF. Figures 2c and 2d show the partial derivatives of the surface rainfall rate with respect to T and q at model levels (Jacobians). In Fig. 2d, the MF scheme exhibits a significant temperature sensitivity between level 50 (around 870 hPa) and level 43 (around 650 hPa) with an extremum of -8 mm day^{-1}/K at level 43, corresponding to the melting layer. Above the melting layer, despite significant tendencies produced by the MF scheme on the mean variables (Fig. 2a), the sensitivity of T and q to the surface rainfall rate is negligible. The largest sensitivity for both T and q is found at the lowest model level, since the thermodynamical properties of that level are used to initiate the cloud ascent and therefore strongly influence the value of the CAPE which in turn determines the overall intensity of the convection through the closure of the scheme. The behaviour of ADJ Jacobian vector (see Fig. 2c) for temperature is very different from MF since it is very fairly constant (around -2 mm day^{-1}/K) between level 49 and level 27 but with a very large peak lower than -10 mm day^{-1}/K at level 50 (900 hPa). This level is associated with the thermodynamical properties at cloud base used to determine the reference profile both for temperature and humidity and therefore affects the whole heating and moistening profiles. A positive sensitivity to changes in specific humidity over the whole profile is noticed with ADJ. In

ADJ, an increase in heating rate is achieved by a cooling of the given layer and an increase in drying is achieved by a moistening of the given layer. Indeed, given the simplicity of this scheme, it is possible to approximate the rainfall Jacobians at a given level k as:

$$\frac{\partial R}{\partial T_k} \approx -\frac{C_p}{\tau} \times \frac{\Delta p_k}{g} \qquad \frac{\partial R}{\partial q_k} \approx \frac{L}{\tau} \times \frac{\Delta p_k}{g} \tag{1}$$

where τ is the relaxation time scale set to 1800 s for the chosen model horizontal resolution.

4 The 4D-Var data assimilation system

4.1 Main features

The operational ECMWF assimilation system is based on an incremental variational method (Rabier et al. 2000, Mahfouf and Rabier 2000, Klinker et al. 2000). 4D-Var seeks an optimal balance between the observations and the dynamics of the atmosphere by finding a model solution that is as close as possible, in a least-square sense, to the background information (model short-term forecast) and to the observations available over a given time period (currently 12 hours). The incremental formulation of 4D-Var (Courtier et al. 1994) consists of computing the background trajectory and the departures (observations minus model) within an *outer-loop* using a comprehensive non-linear model and minimizing the cost function for the increments at initial time within an *inner-loop* using a simplified linearised model.

4.2 Methodologies for rainfall rate assimilation

Figure 3 gives a schematic description of the two methods used to perform the assimilation of surface rainfall rates in the ECMWF model. Both methods are based on the minimization of the following cost-function:

$$J(\mathbf{x}) = \frac{1}{2}(\mathbf{x}-\mathbf{x}_b)^T\mathbf{B}^{-1}(\mathbf{x}-\mathbf{x}_b) + \frac{1}{2}\left(\frac{\mathbf{Y}(\mathbf{x})-\mathbf{Y}_o}{\sigma_o}\right)^2 \tag{2}$$

where $\mathbf{x}$ is the control vector, $\mathbf{B}$ the covariance matrix of background errors, $\mathbf{x}_b$ the background state and $\mathbf{Y}_o$ the observation with an associated error σ_o. The first term of the cost-function is the background term and the second one is the observation term.

A first approach to perform rainfall rate assimilation is a ”direct 4D-Var” assimilation where $\mathbf{x}$ represents the model 3D fields (temperature, specific humidity, vorticity, divergence, surface pressure) and $\mathbf{Y}_o$ the observed surface rainfall rate. $\mathbf{Y}(\mathbf{x})$ is the model rain rate at the time of observation. This quantity is the sum of the surface rain rate provided by the convection and the large scale condensation schemes described in Section 2.

A second method is a ”1D-Var + 4D-Var” approach. In the 1D-Var the control variable $\mathbf{x}$ consists of T and q vertical profiles and $\mathbf{Y}_o$ is an observed surface rainfall rate. The 1D-Var retrieval was tested extensively on TRMM estimates of rainfall rates by Marécal and Mahfouf (2000). We used the MF scheme in our study and found that the 1D-Var adjustments were mainly driven by specific humidity modifications and that temperature is only slightly changed. This is the reason why we retained the total column water vapour (vertical integral of specific humidity) derived from 1D-Var retrieval for assimilation in 4D-Var. In this latter case the observation $\mathbf{Y}_o$ represents the total column water vapour retrieval and $\mathbf{x}$ the model 3D-fields. The ”1D-Var + 4D-Var” approach was successfully tested on two and three week assimilation experiments (Marécal and Mahfouf 2002).

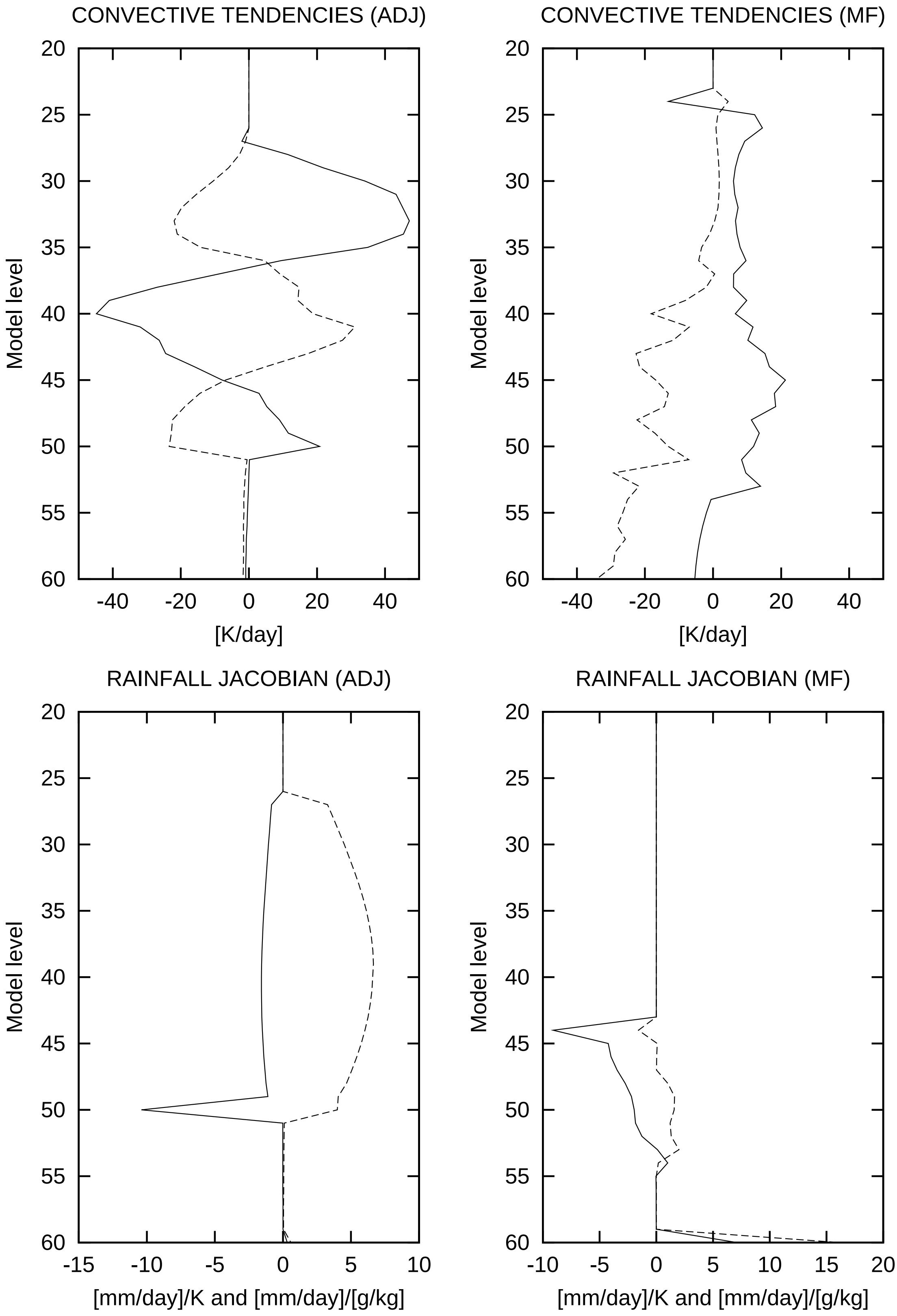

Figure 2: Heating and moistening rates produced by the two convection schemes ADJ (a) and MF (b) expressed in K/day for the selected point (17.383 N, 68.625 E) together with the rainfall Jacobians of the ADJ scheme (c) and of the MF scheme (d). The solid line is for the heating rate and the dashed line for the moistening rate (upper panels). The solid line is for dR/dT and the dashed line is for dR/dq (lower panels). Only the 40 lowest model levels are displayed since convection schemes do not act in the stratosphere.

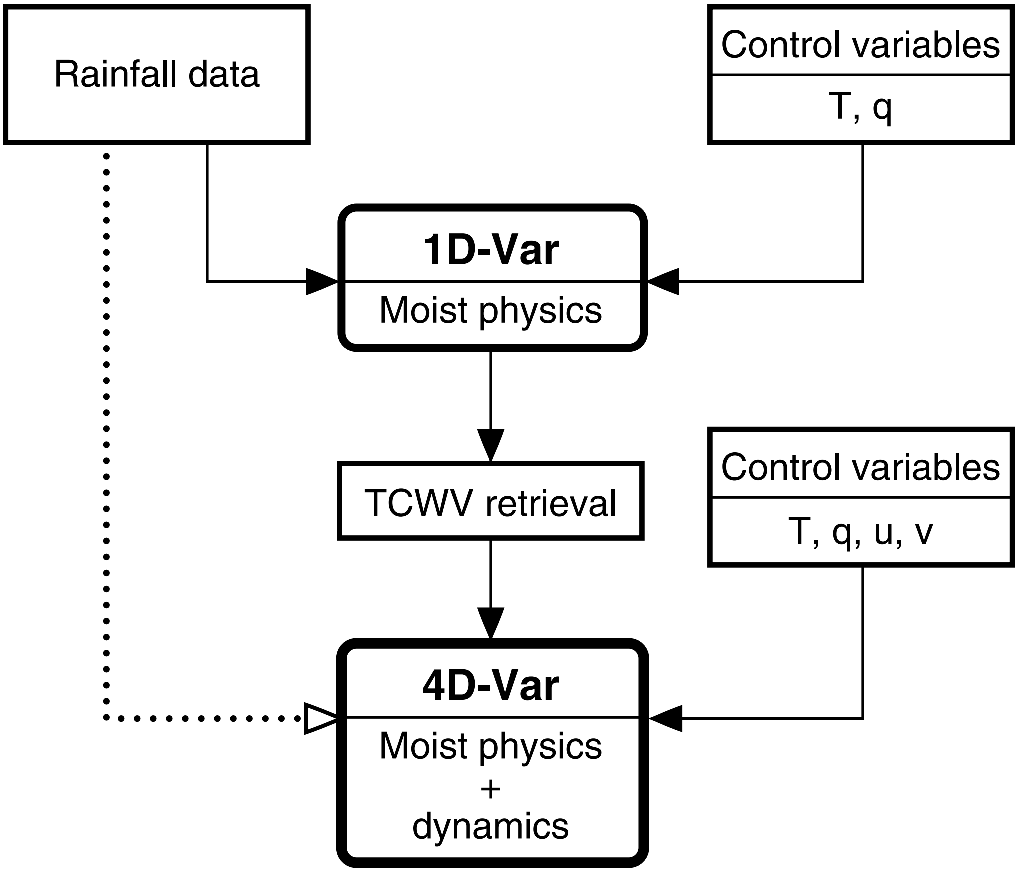

Figure 3: Schematic diagram of the "1D-Var + 4D-Var" (solid lines) and of the "direct 4D-Var" (dotted lines) methods for the assimilation of rainfall rates.

Experiment Name	Assimilation Method	Convection Scheme	Rainfall Observation
ADJ_4P	4D-Var	Betts-Miller	$2.0*R_b$
ADJ_1P	1D-Var+4D-Var	Betts-Miller	$2.0*R_b$
ADJ_4M	4D-Var	Betts-Miller	$0.5*R_b$
ADJ_1M	1D-Var+4D-Var	Betts-Miller	$0.5*R_b$
MF_4P	4D-Var	Mass-flux	$2.0*R_b$
MF_1P	1D-Var+4D-Var	Mass-flux	$2.0*R_b$
MF_4M	4D-Var	Mass-flux	$0.5*R_b$
MF_1M	1D-Var+4D-Var	Mass-flux	$0.5*R_b$

Table 1: Characteristics of the 4D-Var assimilation experiments

4.3 Description of the experiments

To allow a fair interpretation of the assimilation results, the 4D-Var set-up is slightly different from the operational one. Firstly, the same horizontal resolution is used in the incremental 4D-Var. The assimilation experiments were run with a spectral truncation T_L159 for the trajectory in the *outer-loop* and with the same spectral truncation for the minimization in the *inner-loop*. The same moist physical processes are also used in the inner and outer loops. This choice is motivated by the fact that the production of rain by the model parameterizations, and in particular by the convection scheme, is sensitive to the model resolution. By doing so, the differences between the rainfall rate in the *inner loop* and in the *outer loop* are reduced as much as possible. Another modification with respect to the operational configuration is an increase of the number of *outer loops* from 2 to 4 with 10 iterations in each *inner loop*. This was designed to improve the convergence of the minimization process and to highlight problems in finding a proper minimum of the cost-function. For a better understanding of the results and an examination of the implicit structure functions generated by the 4D-Var, only one rainfall rate observation is assimilated. Because rain is generated by deep moist convection and large scale condensation parameterization schemes, the linearised physics used for the propagation in time of the increments and of the gradient of the cost-function is activated during the whole minimization process and not only at the end as in the operational configuration.

A set of eight single observation experiments has been run for the same analysis time at 0000 UTC 27 May 2001. This corresponds to a 12-hour assimilation window starting on 1500 UTC 26 May 2001 and ending on 0300 UTC 27 May 2001. The location of the observation is shown in Fig. 1. In the experiments, either the MF or the ADJ convection schemes is used. Simulated rainfall observations have been set to twice or half the value of the model background rainfall rate. The associated observation error is equal to 25 % of the background rainfall rate as in Marécal and Mahfouf (2000). Since we only consider simulated observations, we did not find it necessary to have a more sophisticated estimation of the error associated to rainfall retrievals (Bauer et al. 2002). The observation is assimilated either through a ”1D-Var + 4D-Var” or a ”direct 4D-Var” approach with the observation at 1800 UTC 26 May 2001 (3-h after the beginning of the assimilation window). The experimental characteristics are summarized in Table 1.

5 Results

Since the ”1D-Var + 4 D-Var” method is a two-step approach, it is necessary to first examine the behaviour of the 1D-Var retrieval before discussing the 4D-Var assimilation results. Fig. 4 displays, for each experiment, the

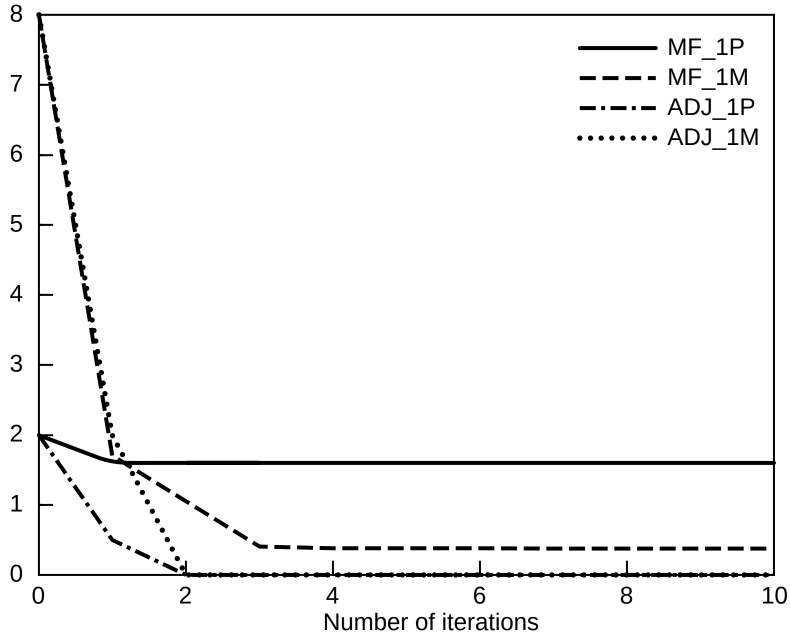

Figure 4: Cost function (observation term) at each iteration of the 1D-Var minimization for experiments ADJ_1P, ADJ_1M, MF_1P and MF_1M.

Experiment Name	Observed Rainfall rate	Observation Error	1D-Var First-guess Rainfall rate	1D-Var Analysed Rainfall rate	TCWV Increments
ADJ_1P	0.454	0.113	0.227	0.454	0.34
ADJ_1M	0.113	0.028	0.227	0.113	-0.17
MF_1P	3.030	0.757	1.515	1.678	0.26
MF_1M	0.757	0.189	1.151	0.784	-1.37

Table 2: Results of the 1D-Var assimilation of rainfall rates for two convection schemes and two simulated observations. Rainfall rates are given in mm h^{-1} and TCWV increments in kg m^{-2}

observation term of the cost-function from the 1D-Var assimilations as a function of the number of iterations. Note that for the selected grid point, MF scheme provides a rainfall rate about 6 times larger than ADJ rainfall rate as shown in Table 2. This is because the cyclone has not exactly the same structure with the ADJ and MF schemes as illustrated in Fig. 1. For all experiments using 1D-Var (ADJ_1P, ADJ_1M, MF_1P and MF_1M), a stable minimum is reached in less than 4 iterations. 1D-Var retrievals with the ADJ scheme (ADJ_1P and ADJ_1M) provide the most efficient minimization with a very large reduction of the cost function (see Fig. 4). This is consistent with Table 2 results where the observed, the 1D-Var first-guess and the 1D-Var analysed rainfall rates are given. The vertical structure of the rainfall Jacobian for MF, particularly the fact that the largest sensitivity corresponds to a minimum value of the background error variance for humidity, leads to difficulties in finding the minimum of the cost function. Among the four experiments, only MF_1P experiment fails to reach a solution close to the observation within the observation error. Nevertheless, in this case, the first-guess rainfall rate is very large ($>$ 3 mm h^{-1}) and doubling this value is difficult to achieve at the chosen model resolution.

TCWV increments resulting from the 1D-Var assimilation are also given in Table 2. These increments are all small (< 1.4 kg m^{-2} in absolute value) compared to the typical value of the first-guess TCWV in the cyclone area (about 70 kg m^{-2}). This means that a small change in the humidity field can modify significantly the resulting rainfall rate. This was also illustrated in the Jacobian study (Section 3) showing a large sensitivity to specific humidity particularly for the MF scheme near the surface. This was also found in previous studies (Fillion and Mahfouf 2000, Marécal and Mahfouf 2000). The very successful minimization for 1D-Var experiments with ADJ leads to fairly small TCWV increments. This is because rainfall rate modifications required to fit observations are not very large. Note also that even if the 1D-Var minimization for MF_1P is not fully satisfactory it provides non-negligible TCWV increments compared to the other three. Thus we proceeded to the second step of the ”1D-Var + 4D-Var” method for all four experiments (ADJ_1P, ADJ_1M, MF_1P and MF_1M).

The minimization results for the eight experiments listed in Table 1 are displayed in Fig. 5 showing the observation term of the cost-function J_o normalised by its initial value as a function of the number of iterations. Let us recall that in ”1D-Var + 4D-Var” experiments (Figs. 5c and 5d), 1D-Var TCWV is the minimized quantity while in ”direct 4D-Var” experiments (Figs. 5a and 5b), it is the rainfall rate. One common feature of the eight experiments is the discontinuity of the cost function that generally appears between the last iteration of each *inner loop* minimization and the following *outer loop* update. This behaviour leads, for experiments MF_4M and ADJ_4M, to a significant increase of J_o between the first and the last *outer loop* values, instead of the expected decrease. These discontinuities can be explained by two main reasons. The first one is that the model configurations for the *inner-loop* and for the *outer-loop* are not fully identical. In particular, grid-point specific humidity is used in the *outer loop* while humidity is calculated in spectral space in the *inner loop* (having the control variable in spectral space leads to an easier formulation of the background error covariance matrix). The differences in the temporal evolution of the specific humidity field between grid-point and spectral spaces lead to small modifications of the TCWV in rainy areas impacting the response of the moist convection schemes. The second reason is related to the incremental formulation of the 4D-Var which assumes that both the observation operator and the model that calculate the trajectory at the observation point at the observation time are linear in the vicinity of the model first-guess trajectory. Using non-linear operators such as moist convective processes or assimilating observations in regions of very active convection, means that this hypothesis is likely to be less valid, than for conventional observations. As a consequence, the minimization procedure cannot always find a minimum of the cost function in the *inner loop* consistent with the behaviour of the model in the *outer loop*. Note that these two identified problems affect both ”1D-Var + 4D-Var” and ”direct 4D-Var” methods. Another result from Fig. 5 is the different behaviour of the minimization depending on which convection scheme is used. Experiments using ADJ scheme tend to produce less *inner loop*/*outer loop* discontinuities and generally lead to a better final adjustment to observations. This result is consistent with the 1D-Var minimization results (Fig. 4) meaning that there is a clear impact of the convection scheme characteristics on the minimization behaviour even when using the ”1D-Var + 4D-Var” approach.

Both the 1D-Var retrieval and the ”direct 4D-Var” method assimilate the rainfall rate. The minimization decrease is much smoother in the 1D-Var assimilation than in the ”direct 4D-Var” as shown by the comparison between Fig. 4 and Fig. 5c-5d. This comes from the incremental formulation of the “direct 4D-Var” producing *innerloop*/*outer loop”* discontinuities and from the larger dimension of the control variables (i.e. three dimensional fields of vorticity, divergence, temperature and specific humidity and surface pressure to be adjusted simultaneoulsy). In a 1D-Var assimilation only individual vertical profiles of temperature and specific humidity are modified. At the end of the minimization process, the 1D-Var generally gives a smaller cost-function than the ”direct 4D-Var”. Nevertheless, this is not always true. For instance, the 1D-Var minimization for MF_1P fails while the ”direct 4D-Var” minimization for MF_4P is successful. In the following, no other results on MF_4M and ADJ_4M experiments will be shown because of their minimization failures (Fig. 5d).

The impact of the TCWV or rainfall rate assimilation on the humidity analysis is illustrated in Figs. 6, 7 and

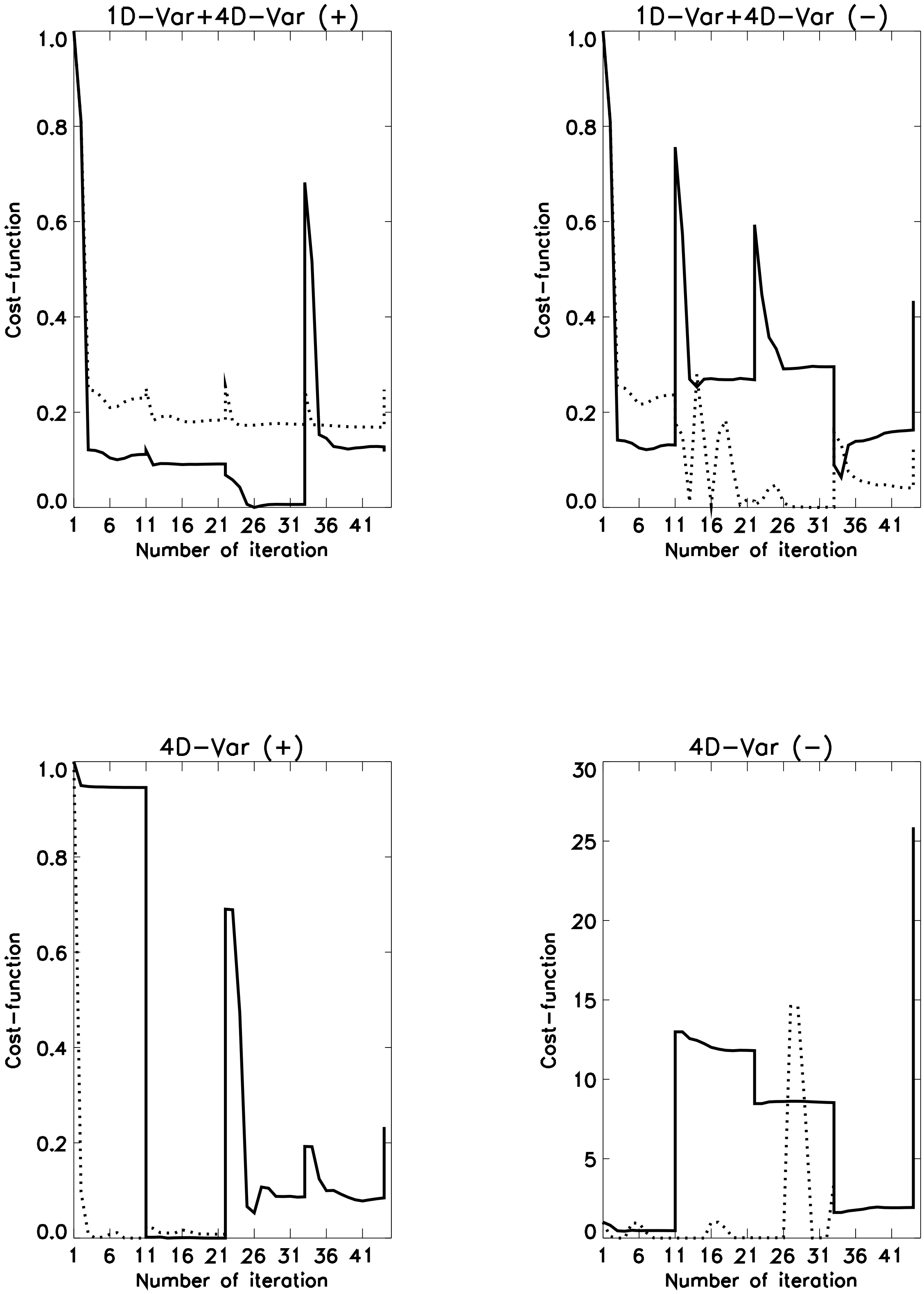

Figure 5: Normalised cost-function (observation term) at each iteration of the 4D-Var minimizations. (a) is for ADJ_1P and MF_1P, (b) for ADJ_4P and MF_4P,(c) for ADJ_1M and MF_1M, and (d) for ADJ_4M and MF_4M. Solid lines are for experiments with MF scheme and dotted lines are for experiments with ADJ scheme.

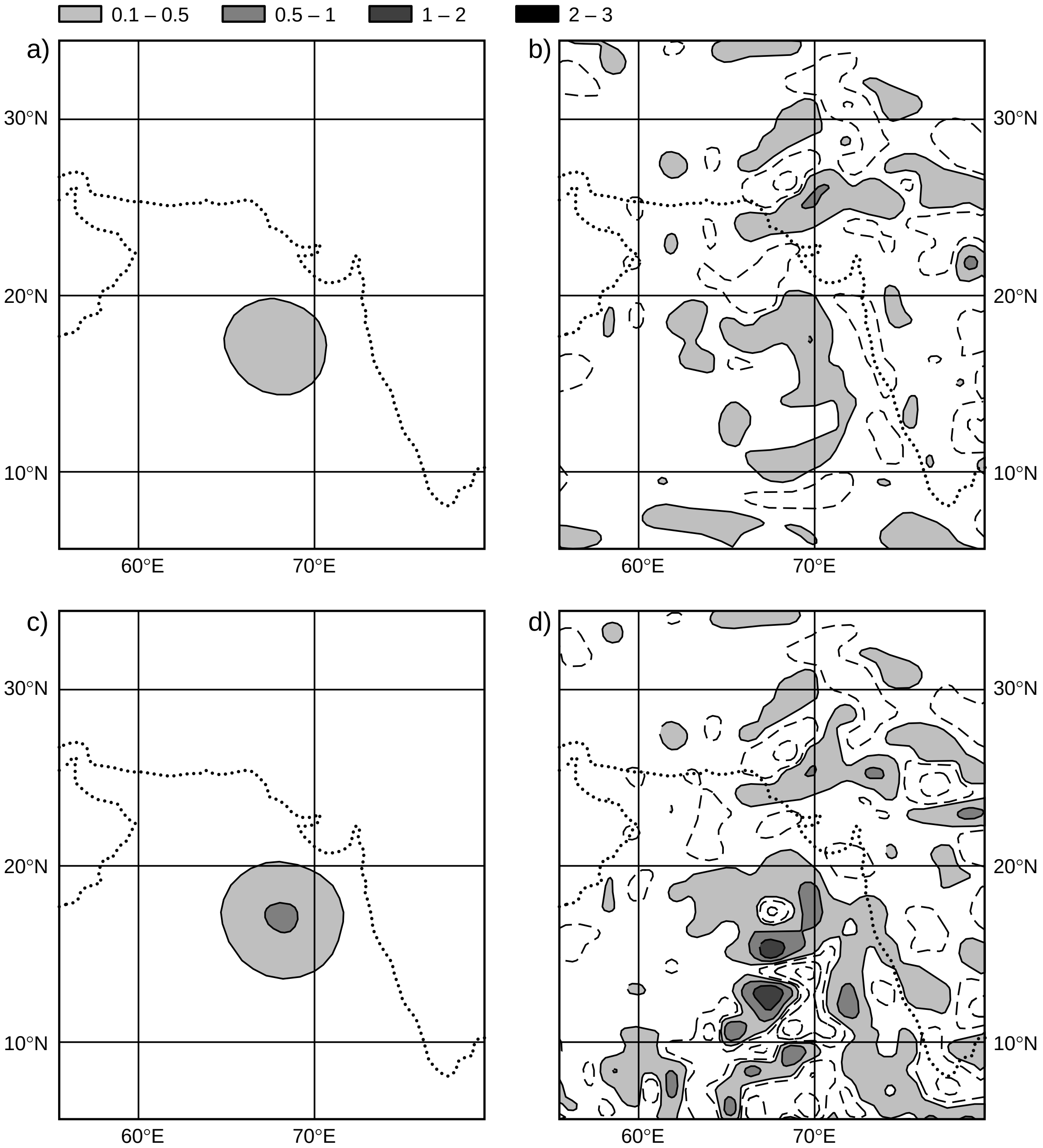

Figure 6: TCWV increments in kg m^{-2} from 4D-Var analysis trajectory (Shaded areas: positive increments; dashed lines: negative increments). The increments are the difference between the analysed TCWV and the first-guess TCWV. (a) ADJ_1P experiment on 26 May 2001 at 1500 UTC (starting time of the analysis window). (b) ADJ_1P experiment on 26 May 2001 at 1800 UTC (time of the observation).(c) MF_1P experiment on 26 May 2001 at 1500 UTC. (d) MF_1P experiment on 26 May 2001 at 1800 UTC.

8 displaying TCWV increment fields at the beginning of the assimilation window (1500 UTC 26 May 2001) and at the observation time (1800 UTC 26 May 2001). As expected at 1500 UTC, when a decrease (resp. increase) of the rainfall rate is needed it is associated with a decrease (resp. increase) of TCWV. The absolute value of the increments is smaller for ADJ experiments compared to MF experiments because of the lower ADJ first-guess rainfall rate. It is even negligible (< 0.1 kg m^{-2}) for ADJ_1M experiment since the rainfall rate decrease required is very small (0.113 mm h^{-1}). The largest increments are obtained for MF_4P experiment with a maximum greater than 2 kg m^{-2} (Fig. 7c). As for the 1D-Var retrieval results, TCWV increments remain small showing the large sensitivity of the rainfall rate to humidity for both convection schemes. The "1D-Var + 4D-Var" experiments provide weaker TCWV increments in absolute values than the "direct 4D-Var" experiments. This means that a two-step approach tends to filter the rainfall rate information. Nevertheless, it remains consistent with the "direct 4D-Var" approach. For the six experiments displayed, the increments at 1500 UTC are characterised by a circular shape with an extremum at the centre and a decrease towards the edges. The extremum corresponds to the rainfall rate observation location transported back in time by the model dynamics. This shape is produced by the structure functions of the background error covariance matrix where the horizontal correlation length scale is prescribed as a function of wavenumber and isotropy on the horizontal is assumed. Therefore the humidity increments are spread over distances between 400 and 800 km. This is a major drawback for rainfall rate assimilation since the associated humidity increments are acting in areas generally larger than the actual rainy areas. This points out the need for appropriate statistics of forecast errors in rainy regions. At 1800 UTC (observation time), the structure of the TCWV increments are very noisy compared to the 1500 UTC fields. When the increment at 1500 UTC (beginning time of the assimilation window) are small (e.g. ADJ_1P and ADJ_1M experiments), it is not possible to identify the corresponding increment on the 1800 UTC TCWV field because it has the same order of magnitude of the noise generated by the 4D-Var analysis. In Fig. 7d, corresponding to MF_4P experiment, the increment the location of the rainfall rate observation appears clearly. Its structure has still a circular shape but distorted by the model "high resolution" trajectory and by the 4D-Var analysis noise. From Figs. 6, 7 and 8, there are no obvious differences between experiments using ADJ or MF schemes even though their Jacobians are different. This is because the incremental 4D-Var generates noise having the same order of magnitude of the differences given by the two convection schemes. Hence, we will only focus hereafter on the comparison between the "1D-Var+4D-Var" and the "direct 4D-Var" methods. For this purpose, we will use MF_4P and MF_1P experiments to illustrate the results since they both provide significant increments in TCWV.

The vertical cross-section of the specific humidity increments for MF_4P and MF_1P in the centre of the cyclone is displayed in Fig. 9. The structure of the two fields is very similar but with different magnitudes. MF_4P provides larger increments than MF_1P by a factor around 3. As for TCWV, this is related to the fact that in the "1D-Var + 4D-Var" approach the rainfall rate information is converted into humidity information before being assimilated in 4D-Var. By doing so, the rainfall rate information is filtered and consequently its impact on the humidity analysis is reduced. The maximum is slightly shifted by about 150 hPa between MF_4P and MF_1P experiments. In MF_1P, the maximum of increments is located where the variance of background errors is the largest, whereas in MP_4P the impact of the dynamics allows the vertical transport of humidity increments at higher levels. It is also interesting to notice that MF_4P produces a more pronounced baroclinic structure of increments in the mid-trosposphere than MF_1P.

Rainfall rate assimilation acts not only on the humidity analysis, and in the 4D-Var framework, thermodynamic and dynamic fields are also modified to be consistent with the humidity analysis. This is illustrated in Fig. 10 displaying the temperature increments. The cross-section shown is the same as in Fig. 9. Only MF_4P field is shown since MF_1P experiment provides a similar structure as MF_4P but with very small values ($<$ 0.005 K). Even for MF_4P experiment, the temperature increments remains weak (< 0.1 K in absolute value). Such a situation is mostly governed by the background error statistics since the rainfall rate Jacobians with respect to temperature shows a non-negligible sensitivity to temperature perturbations (see section 3). The

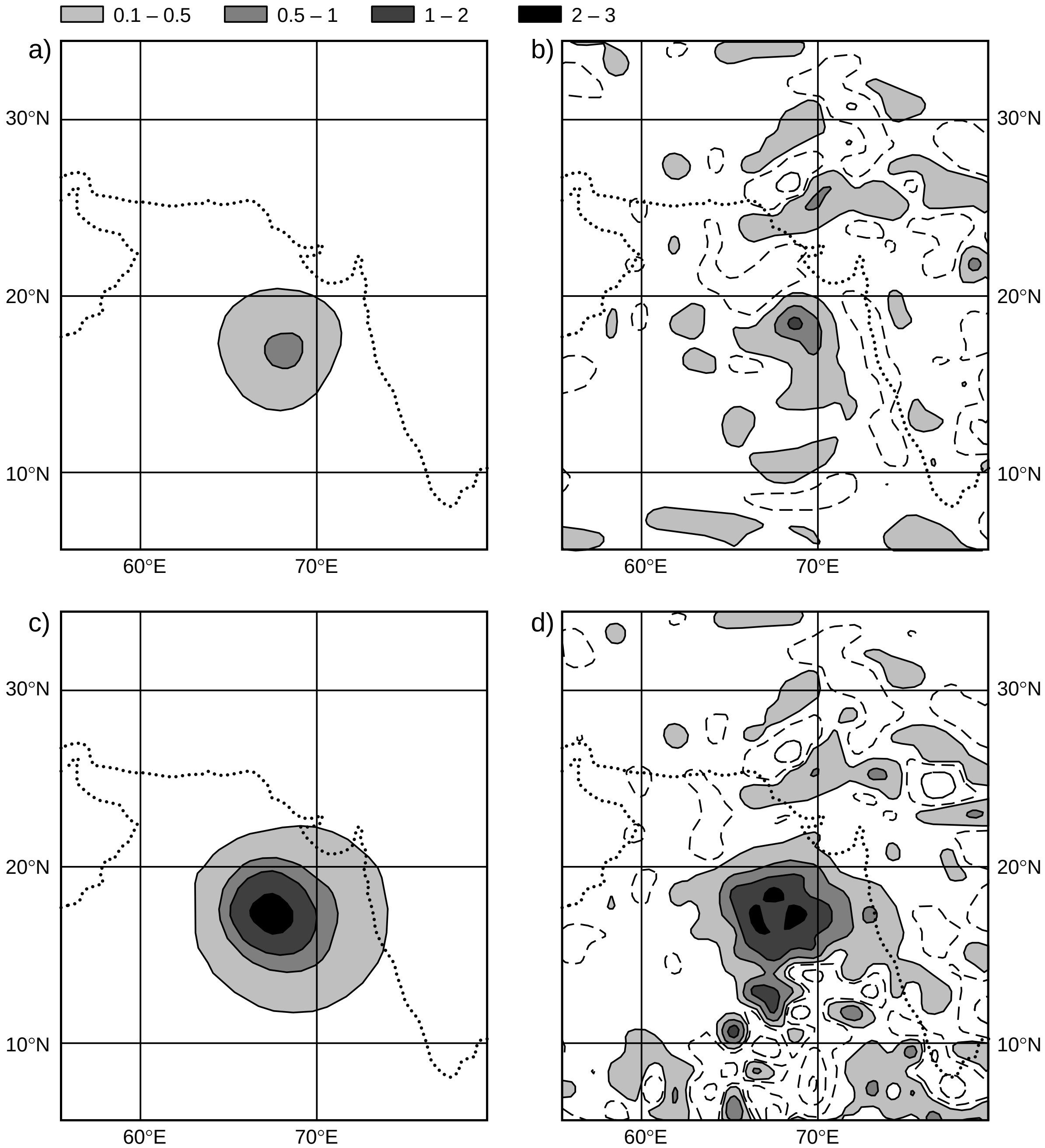

Figure 7: Same as Figure 6 but for ADJ_4P and MF_4P experiments instead of ADJ_1P and MF_1P, respectively.

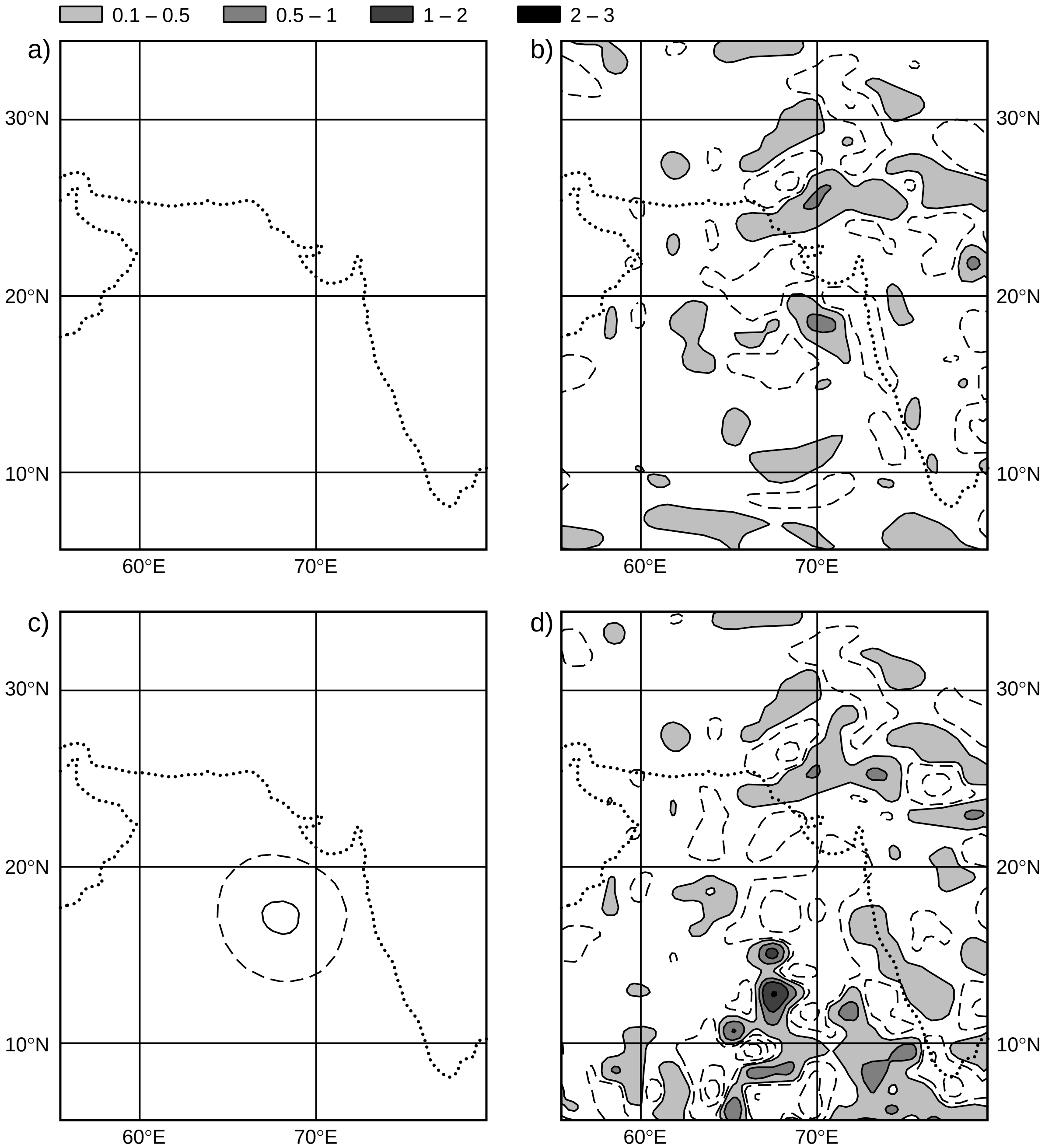

Figure 8: Same as Figure 6 but for ADJ_1M and MF_1M experiments instead of ADJ_1P and MF_1P, respectively.

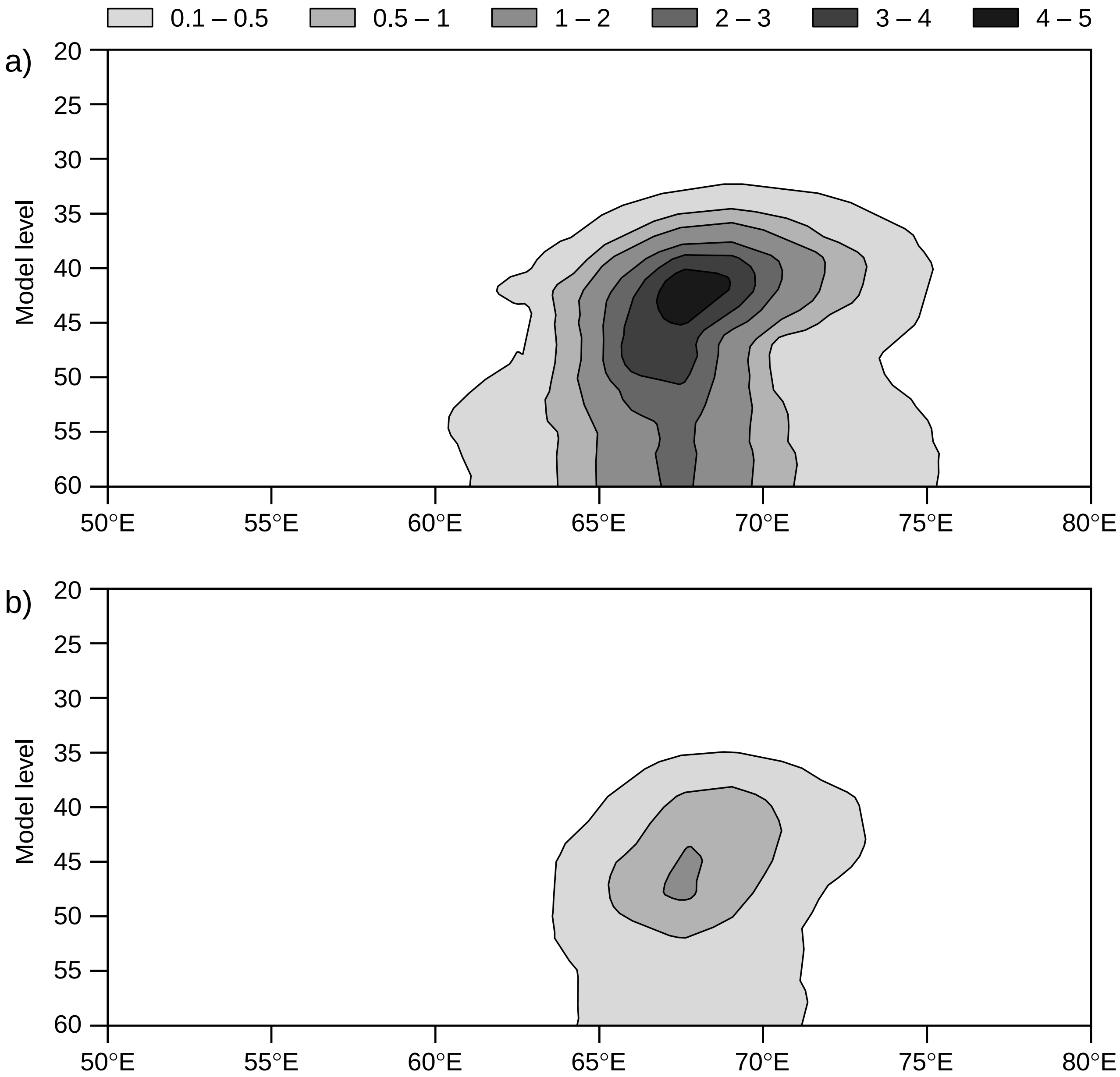

Figure 9: Vertical cross-section of specific humidity increments (10^{-1} gkg^{-1}) in the cyclone at 17.2 ^{o} N on 26 May 2001 at 1500 UTC. (a) MF_4P experiment and (b) MF_1P experiment.

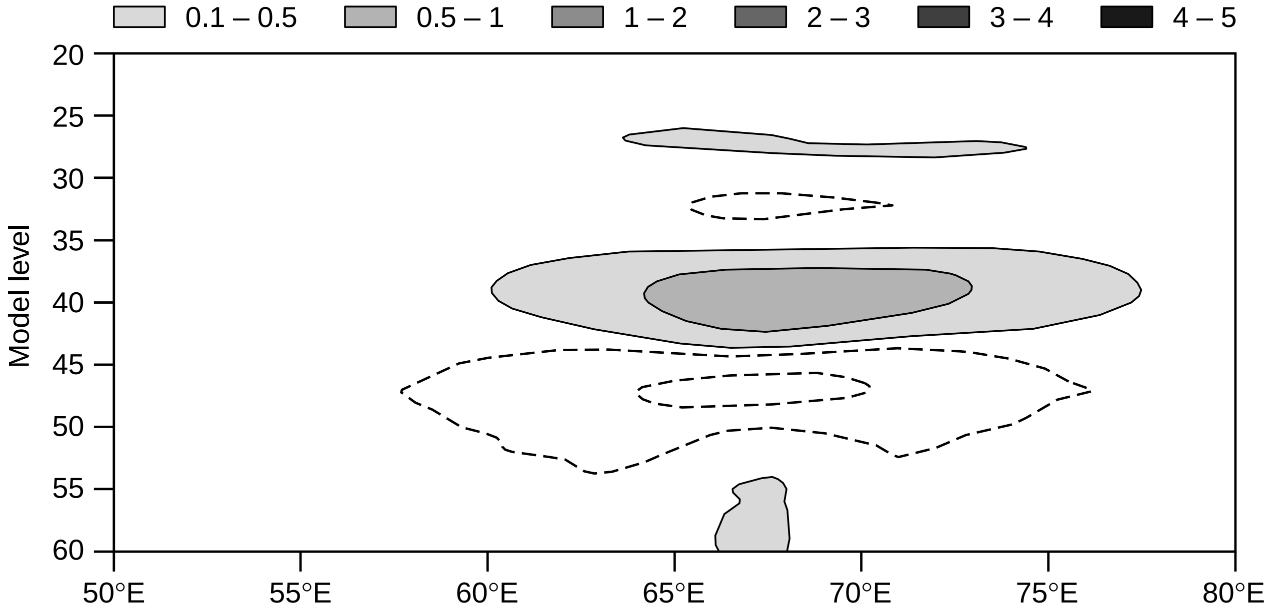

Figure 10: Same as Figure 9 but for the temperature increments for MF_4P experiment. Units are in 0.1 K.

vertical structure of the increments is characterised by an area of negative values around level 50 (850 hPa). This is related to the large sensitivity of the MF scheme around those levels (see Jacobians in section 3). The positive area found around level 40 (500 hPa) and the negative zone above are mainly related to the vertical correlations induced by the background error covariance matrix. This result is consistent with results obtained by Fillion (2002) in a 3D-Var assimilation of surface precipitation. As for the humidity increments, temperature increments are largely spread horizontally due to the 4D-Var structure functions that assumes large horizontal correlations.

Figure 11 displays the results for MF_4P and MF_1P on the increments of the divergence field at levels 30 (around 200 hPa) and 55 (around 950 hPa) at 1500 UTC 26 May 2001. Both experiments exhibit a reinforcement of the convergence at low levels and of the divergence at high levels. This means that there is a strengthening of the cyclone dynamics. The size of the divergence increments is larger for MF_4P experiment than for MF_1P (by a factor 10) as for all other fields previously discussed. The structure of the increments is generally similar with a more isotropic shape for MF_1P experiments. This is correlated with the shape of the humidity increments that are also slightly distorted by the dynamics in MF_4P experiment.

Another interesting field to discuss is the rainfall rate. This is the field that should in principle be improved (i.e. be closer to our simulated observation) in all experiments. Rainfall rate is not a prognostic variable in the analysis but a diagnostic product. Figure 12 shows results from MF_4P and MF_1P experiments. In both cases the rainfall rate at the observation location has been increased noticeably as expected. This reinforcement of the model rainfall rate does not last very long and disappears after a few hours. This suggests that to provide a valuable information to the 4D-Var system (i.e. that lasts), it is necessary to assimilate a series of rainfall rate observations at consecutive times. Another possible option would be to provide accumulated rainfall in order to give a constraint over the whole analysis window. Note also that in the experiments where the humidity increments are of the order of magnitude of the analysis noise, the rainfall rate field is not improved by the use of the rainfall rate observation whatever method is applied (”1D-Var + 4D-Var” or ”direct 4D-Var”).

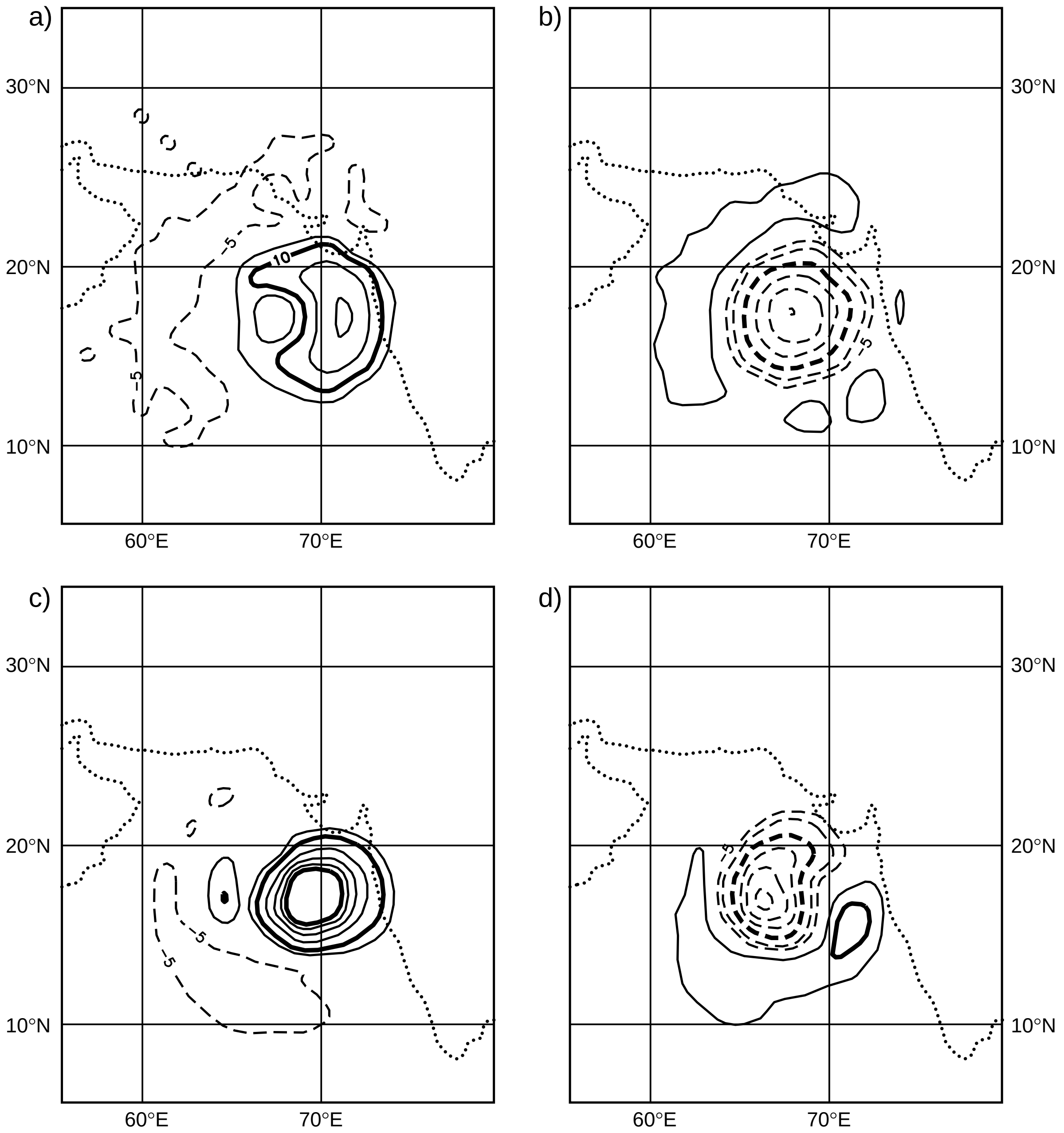

Figure 11: Horizontal cross-sections of increments of divergence in the cyclone area on 26 May 2001 at 1500 UTC. (a) MF_1P experiment at model level 30 (around 200 hPa), (b) MF_1P experiment at level 55 (around 950 hPa), (c) MF_4P experiment at level 30 (around 200 hPa) and (d) MF_4P experiment at level 55 (around 950 hPa). Units are in 10^{-7} s^{-1} for MF_4P experiment and 10^{-8} s^{-1} for MF_1P experiment.

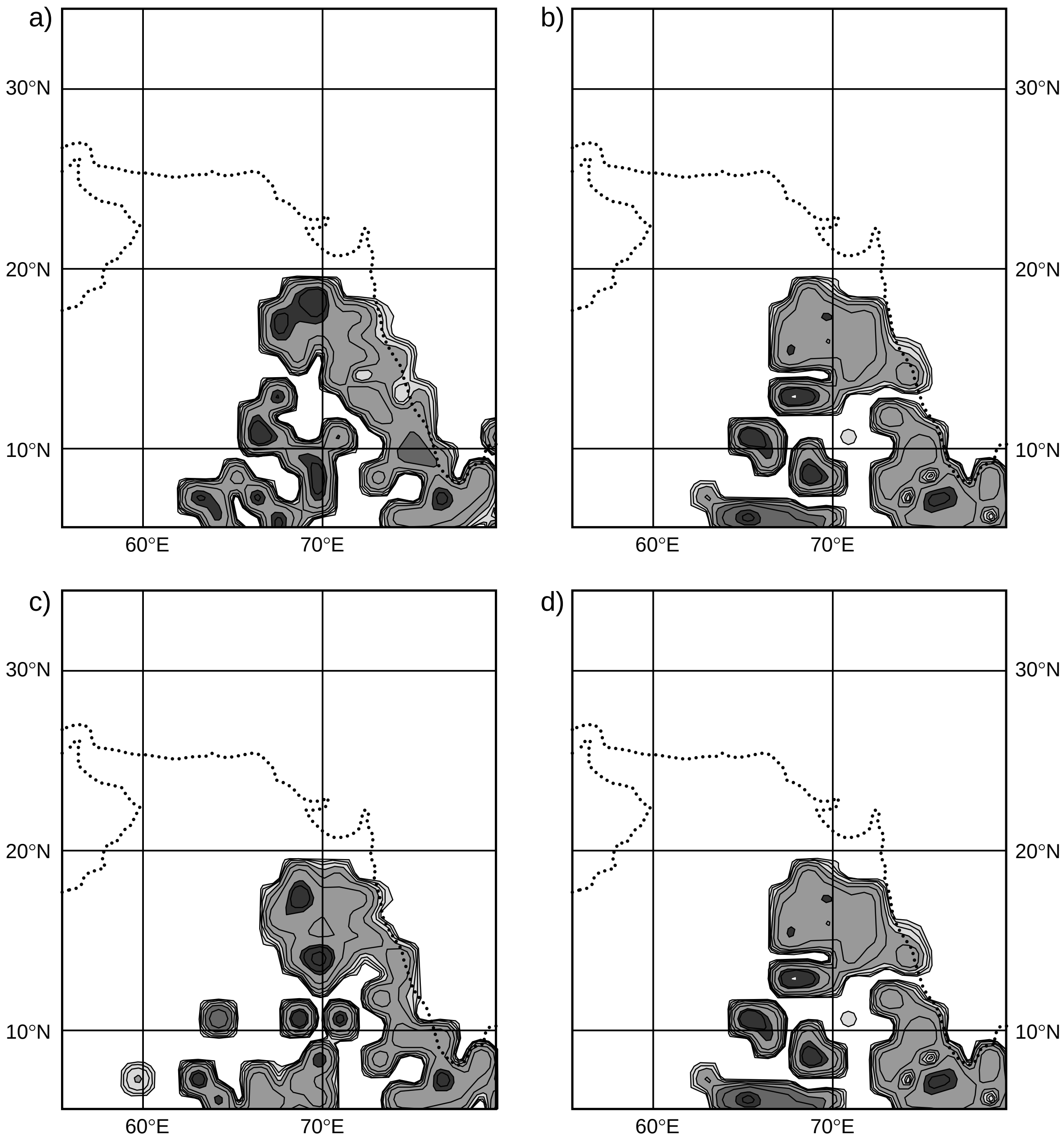

Figure 12: Instantaneous convective surface rainfall rate in mm h^{-1}. (a) from MF_1P analysis trajectory , (b) from MF_1P first-guess, (c) from MF_4P analysis trajectory and (d) from MF_4P first-guess.Light grey shading starts above 0.4 mm hr^{-1}, grey shading starts above 4 mm hr^{-1}, dark grey shading starts above 40 mm hr^{-1}

6 Conclusion

The aim of this study was to investigate the behaviour of two assimilation techniques compatible with the ECMWF 4D-Var incremental approach : a ”1D-Var + 4D-Var” approach and a ”direct 4D-Var” approach. The first method consists of a 1D-Var assimilation of surface rainfall rates which provides adjusted model profiles of specific humidity and temperature. In a second step, 1D-Var TCWV retrievals are assimilated in the ECMWF 4D-Var. The second approach consists of using directly the rainfall rate observation in the 4D-Var system. Moreover these assimilation experiments were perfomed using two conceptually different convection schemes : the adjustment scheme from Betts (1986) and Betts and Miller (1993) and the ECMWF operational mass-flux scheme (Tiedtke 1989, Gregory et al. 2000)

In the first part of the paper, the behaviour of the two convection schemes has been analysed, firstly in terms of intensity and secondly in terms of Jacobians. Even though the surface rainfall in the central part of the cyclone are comparable, the MF scheme tends to produce larger patterns that are not supported by satellite derived rain rates. In the common rainy region, the heating profiles and the sensitivity of surface rainfall to changes in T and q are very different, the vertical structure of the Jacobians being simpler with ADJ than with MF.

Eight assimilation experiments were run using only one simulated observation to allow a detailed analysis of the behaviour of the two convection schemes and of the two assimilation methods. 1D-Var retrieval results have shown that the 1D-Var minimization does not always provide an analysed rainfall rate within the rainfall rate observation error. This is related to the non-linearities of the convection scheme. It mostly happens for MF scheme since its Jacobian is very discontinuous and therefore the solution found is sometimes only a local minimum of the cost function. This result was also found in the 4D-Var experiments. For all eight 4D-Var experiments, the cost function exhibits generally large discontinuities between the last iteration of the *inner loop* minimization and the *outer loop* trajectory. This is due to the model configuration which is slightly different in these two computations. An other reason is the strong non linearities of the model and of observation operator which are somewhat inconsistent with the 4D-Var incremental formulation.

The analysis provides TCWV increments at the initial time of the analysis window that are generally small (< 2 kg m^{-2}) but in agreement with the rainfall rate adjustment required. When these increments are lower than 0.5 kg m^{-2} they are within the noise generated by the analysis after few hours of forecast. No noticable differences are found in TCWV increments between ADJ and MF schemes except for the intensity. This last point can be explained by the smaller rainfall rate modifications required with ADJ scheme since its first-guess rainfall rate from which the observation rainfall rate was simulated was much smaller. At this stage, it was found impossible to investigate any further the differences between the two convection schemes because they are of the same order of magnitude as the noise generated by the ECMWF 4D-Var analysis system. Hence the last part of the paper was devoted to a comparison between the two assimilation methods proposed. The two methods provide consistent results. The ”1D-Var + 4D-Var” approach produces smaller increments for specific humidity, temperature and divergence. This is because the 1D-Var step tends to filter part of the information contained in the observed rainfall rate. Even with the ”direct 4D-Var” the magnitude of the increments are small for all variables when a single rainfall rate observation is assimilated. A possibility to increase the influence of the rain data on the analysis would be to assimilate accumulated rainfall rates instead of instantaneous ones. It was also found that because of the 4D-Var structure functions, humidity increments are spread horizontally isotropically over distances between 400 and 800 km. This is a major problem since humidity increments are thus acting in areas generally larger than the rainy zone of interest. A solution would be to have appropriate statistics of forecast errors in rainy regions.

The aim of this paper was to study in detail the assimilation of rainfall rate in the ECMWF incremental 4D-Var. Several issues have arisen from the results and have motivated on-going studies to improve some features of 4D-Var. Firstly, the possibility to use grid-point humidity in the inner loop is currently under test. This is expected to reduce the noise generated by the minimization on the humidity field. Secondly, a revision of the structure functions for humidity is planned to decrease the horizontal correlation lengths.

This study has clearly shown a more robust behaviour of the minimization for the ”1D-Var + 4D-Var” method. Therefore, this method appears more appropriate for the time being for an operational implementation. Moreover, the ”1D-Var + 4D-Var” was tested extensively by Mahfouf and Marécal (2002) using TRMM data over two periods of 15 and 21 days. We have shown an improvement of the humidity and wind analysis in rainy areas and of the forecast performances in the Tropics of the wind and upper tropospheric temperature.

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