Deutscher Wetterdienst

The Lokal Modell (LM) of DWD / COSMO

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ECMWF-seminar on recent developments in numerical methods for atmosphere and ocean modelling

Reading, 6. Sept. 2004

COSMO (Consortium for Small-scale Modeling)

was formed in October 1998.
Goals: develop, improve and maintain a non-hydrostatic limited-area atmospheric model, to be used both for operational and research applications by the members of COSMO.

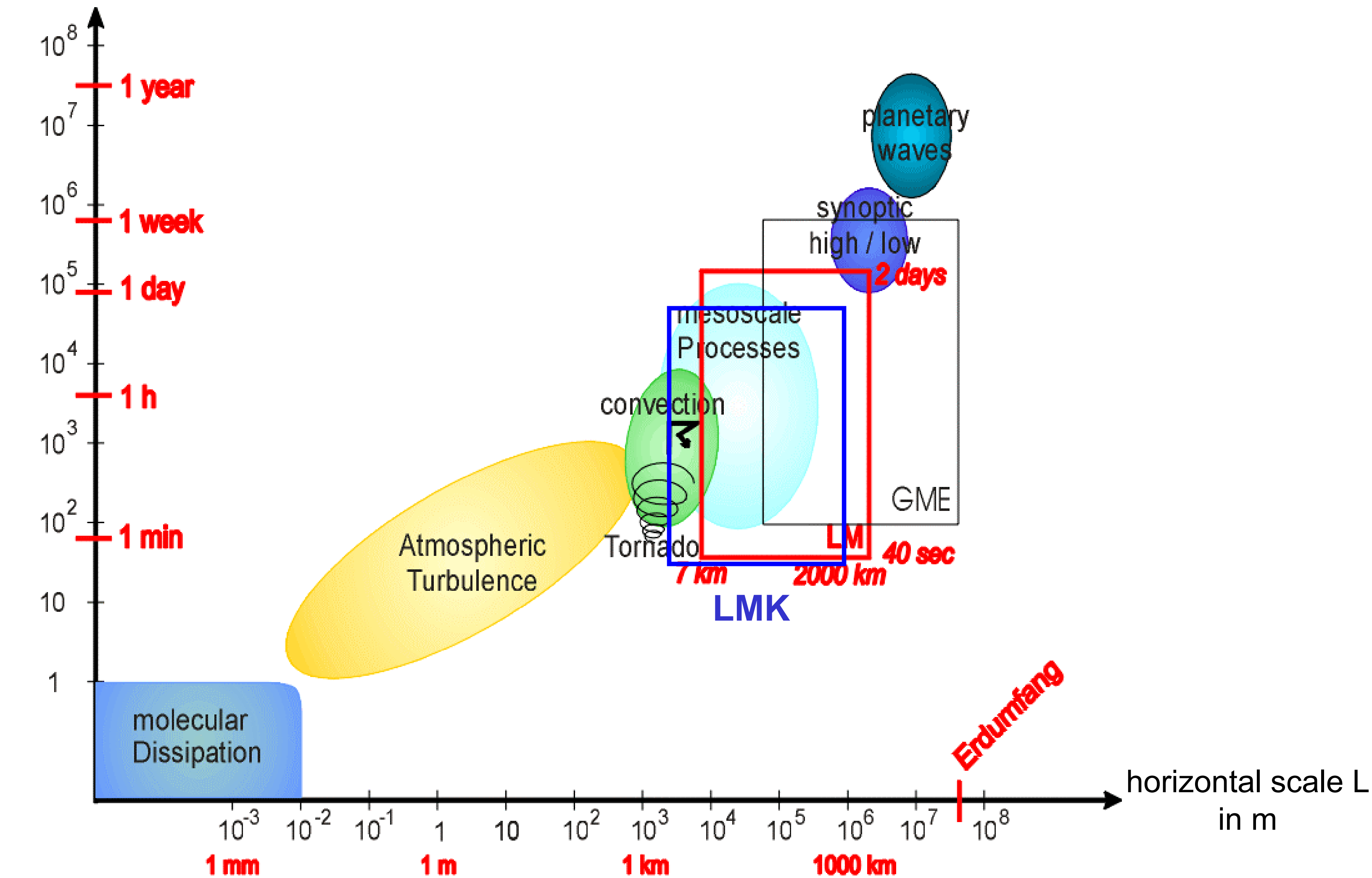
Today, the COSMO consortium has as members the national meteorological services (and additional regional or military services) of:

Germany	Deutscher Wetterdienst (DWD)
	Amt für Wehrgeophysik (AWGeophys)
Switzerland	MeteoSchweiz (MCH)
Italy	Servizio Meteorologico Regione Emilia-Romagna (ARPA-SMR)
	Ufficio Generale per la Meteorologia (UGM)
Greece	Hellenic National Meteorological Service (HNMS)
Poland	Institute of Meteorology and Water Management (IMGW)

Summary

- The Lokal Modell (LM) of DWD / COSMO - overview
- The new prognostic precipitation scheme in LM
- Time splitting schemes
- LMK - a new development

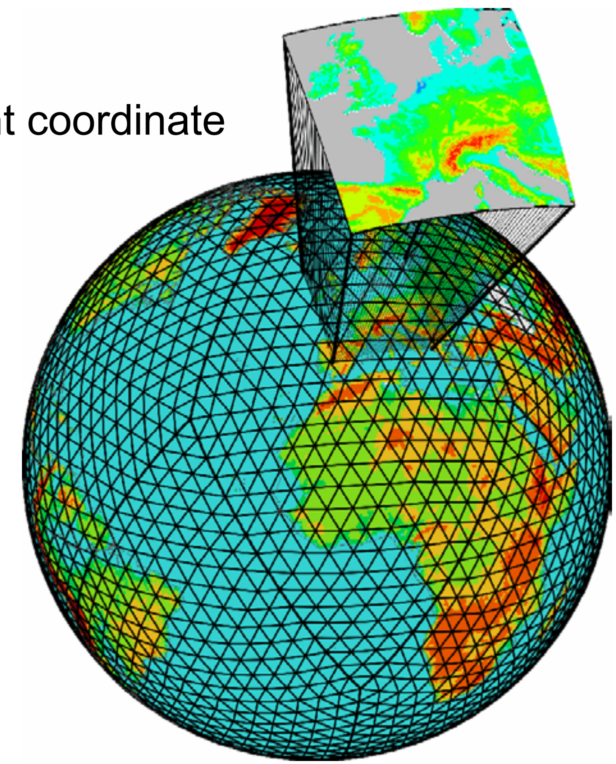
spatial and temporal scales in the atmosphere



LM - Dynamics

Model equations:	non-hydrostatic, full compressible, advection form
Base state:	hydrostatic
Prognostic variables:	cartesian wind components u, v, w pressure perturbation p‘, Temperature T humidity var. qv, qc, qi, qr, qs TKE
Coordinate systems:	generalized terrain-following height coordinate user-defined vertical stretching rotated geographical coordinates

current operational use:
Δx, Δy = 7 km, Δt = 40 sec.
325 * 325 * 35 GPs



LM- Numerics

Grid structure:	horizontal: Arakawa C
	vertical: Lorenz
Spatial discretisation:	second order finite differences
time integrations:	1.) time-splitting: Leapfrog (Klemp, Wilhelmson, 1978)
	2.) time-splitting: 2-time-level Runge-Kutta 2. order (Skamarock, Klemp, 1992)
	3.) 3-timelevel 3-D semi-implicit
Numerical smoothing:	horizontal diffusion 4. order
	3D divergence damping

LM - Initial and Boundary Conditions

initial cond.:	interpolated initial data (from GME, ECMWF, LM)
	or from continuous LM data assimilation stream
lateral bound. cond.:	1-way nesting by Davies-type
	optional: periodic
top boundary:	Rayleigh damping layer

The new prognostic precipitation scheme in LM v3.9

in operational use since 26.4.2004 (COSMO, Work Package 2.1.1)

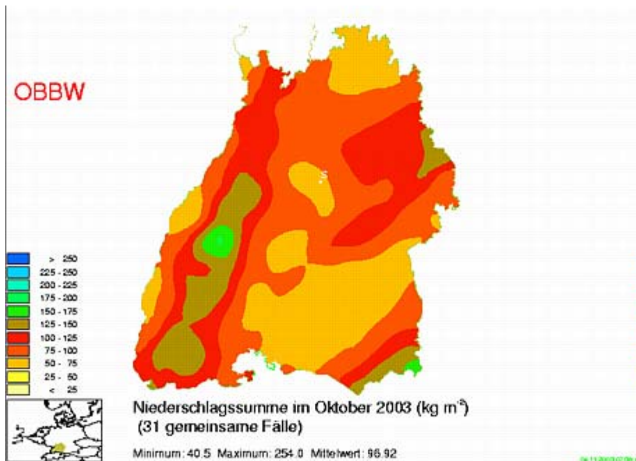
Task:

replacement of the **diagnostic scheme** for rain/snow in the former LM v3.5
by a **prognostic scheme**.

Aim:

improvement of the precipitation distribution in orographically structured areas
due to horizontal **drifting** of rain/snow (solving the ‚windward-lee-problem')

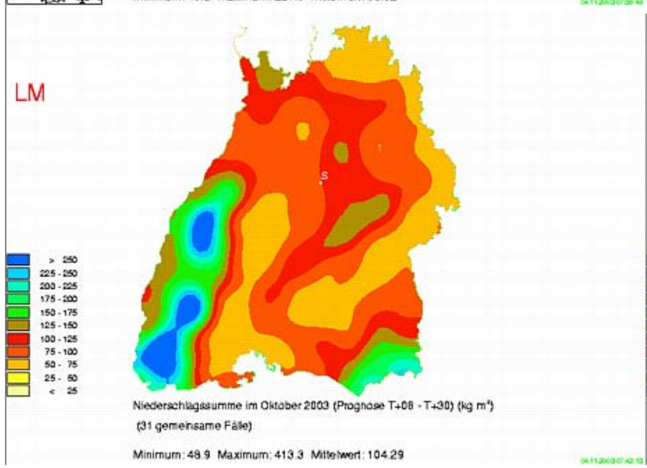
Monthly precipitation sum over Baden-Württemberg (SW Germany) in October 2003

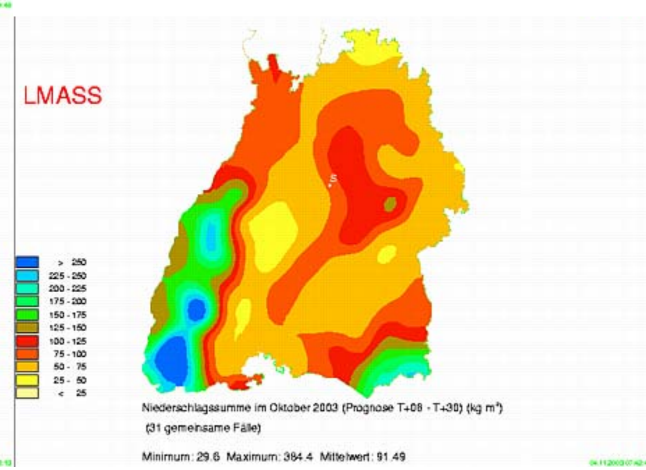


LM without prognostic precipitation

windward-lee-problem:

- precipitation maximum too far in the windward direction
- precip. maximum too strong
- too few precipitation in the lee





Dr. U. Damrath, FE 14

Conservation equation for humidity variables

diagnostic scheme for rain/snow

$$0 = -\nabla \cdot \mathbf{P}^x - \nabla \cdot \mathbf{F}^x + S^x$$

‚column-equilibrium'

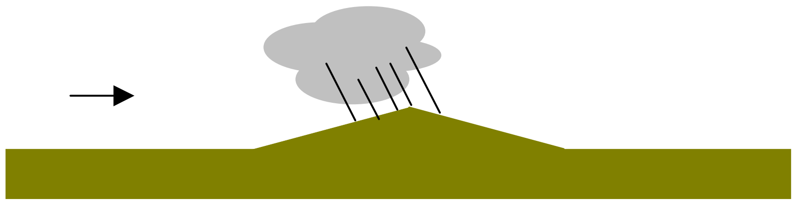
ρ	[kg/m³]	density of air
$q^x = \rho^x/\rho$	[kg/kg]	specific mass
$\mathbf{P}^x$	[kg/m²/s]	sedimentation flux of *x* (only *x*=*r*,*s*)
$\mathbf{F}^x$	[kg/m²/s]	turbulent flux of *x*
S^x	[kg/m³/s]	sources/sinks of *x* (cloud physics)

x=*v*	water vapour
x=*c*	cloud water
x=*i*	cloud ice
x=*r*	rain drops, $v_{sedim} \approx 5$ m/s
x=*s*	snow, $v_{sedim} \approx 1..2$ m/s

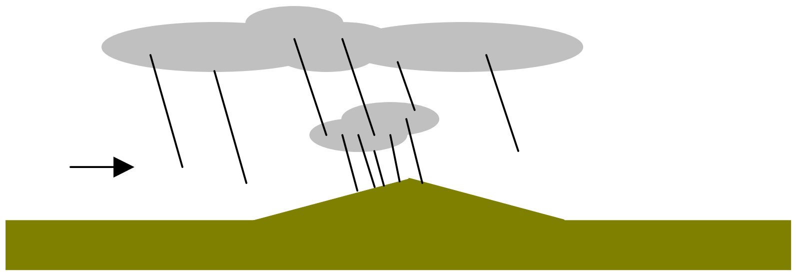
Mechanisms of orographic precipitation generation

(Smith, 1979)

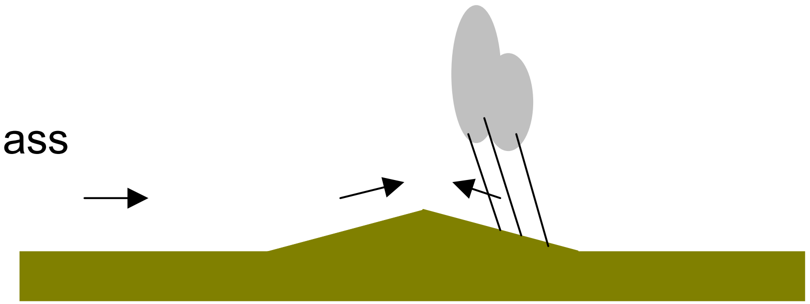
Large-scale upslope precipitation



Seeder-Feeder-mechanism



Cumulonimbus in conditionally unstable airmass



precipitation index: **spillover-factor**

$$SP := \frac{\text{precipitation over the lee side}}{\text{total precipitation}}$$

precipitation of snow only (Jiang, Smith, JAS, 2003)

$$SP = \frac{1}{1 + \frac{\tau_a}{\tau_f} + \frac{\tau_a}{\tau_{Subl}}}$$

- advection time scale $\tau_a := a_M / U$
- sublimation time scale τ_{Subl} $\qquad (\tau_a / \tau_{Subl} \approx h_M / H_{Subl})$
- falling time scale for hydrometeors

$$\tau_f := \frac{D + H_B}{2V_t}$$

V_t = terminal fall velocity
D = box-model-height, H_B = height of cloud base

qualitatively:

$a_M \searrow$ or $\tau_f \nearrow$ or $h_M \searrow$ $\Rightarrow$ $SP \nearrow$

Semi-Lagrange-schemes (I)

Lit.: Staniforth, Côté (1991)
Durran (1999)

Basic idea (1-dim. advection-equation) :

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = 0$$ can be written as: $$\frac{d\phi}{dt} = 0, \qquad \frac{dx}{dt} = u$$

Numerical formulation (e.g. for 3-timelevels): $$\phi(x_j, t^{n+1}) = \phi(\tilde{x}_j^{n-1}, t^{n-1})$$

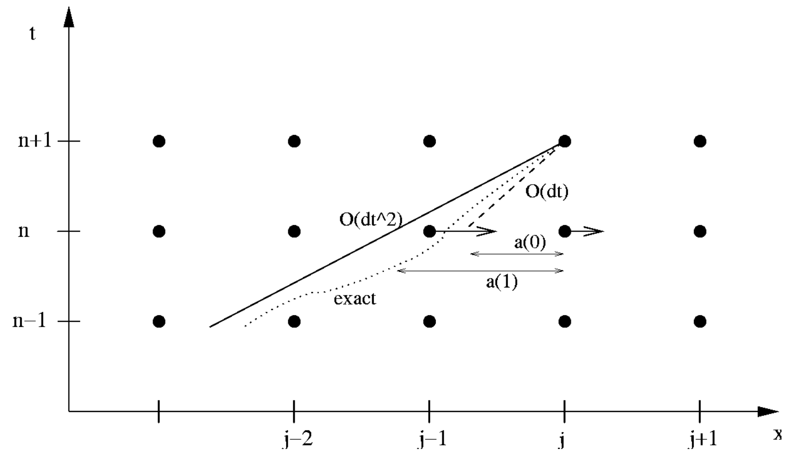
2 steps:

1.) determine the backtrajectory

In principle: every ODE-solver can be used
alternative: iteration-eq. (Robert, 1981)

$$a = \Delta t \; v(x_j^{n+1} - a, t^n)$$

1. iteration step --> 1. order O(Δt)
2. iteration step --> 2. order O(Δt²)



remark: time consuming in staggered 3-dim. grid

Semi-Lagrange-schemes (II)

2.) interpolate ϕ at the starting point

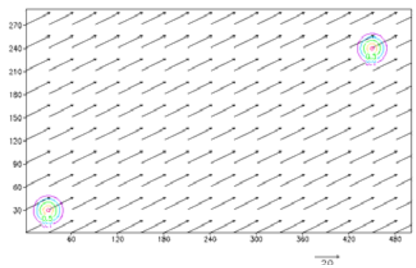
• linear interpolation: (-) strong numerical damping
(+) monotone --> positive definite
• quadratic interpolation: (often used in 60's, 70's)
• cubic interpolation: (+) good compromise between accuracy and numerical costs
• cubic spline interpolation: (+) mass conservation

rule: max. n+1.st order interpolation for n.th order backtrajectory

Properties:
• stability: for u=const. + without source terms: unconditionally stable!
stability constraint by velocity deformation
• simple use in irregular grids
• avoids non-linear instabilities by advection

Advection tests (2-dim.)

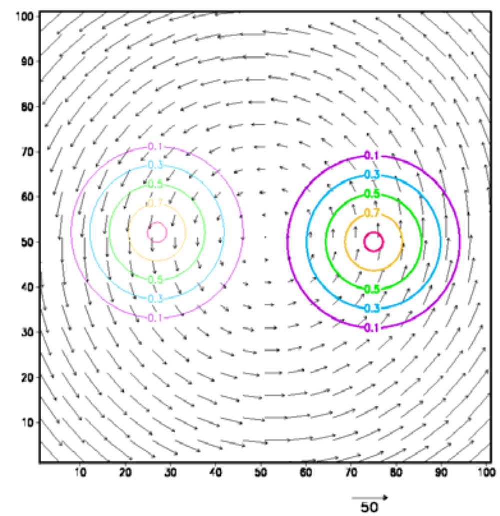
‚constant v‘



Courant numbers

$C_x = 0.4, \; C_y = 0.2$

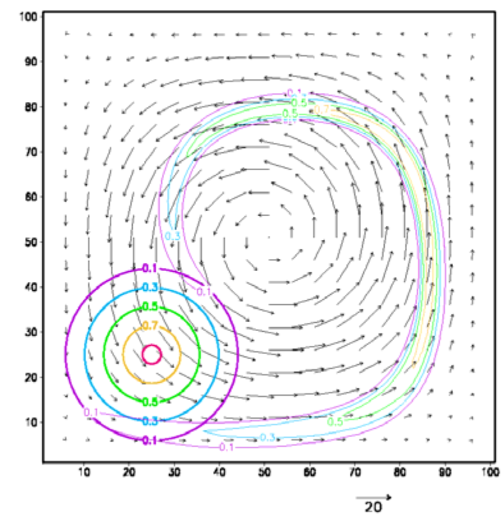
‚solid body rotation‘



Courant numbers

$C_{cone} = 0.4, \; C_{max} = 0.8$

‚LeVeque (1996)‘

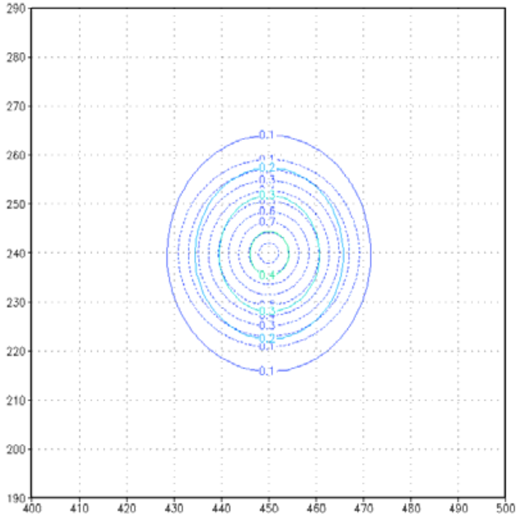


Courant numbers

$C = 0 \ldots 0.4$

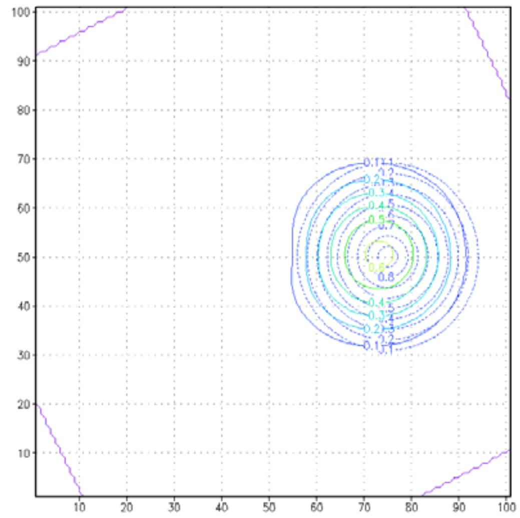
Semi-Lagrange-advection, backtrajectory O(Δt), bilinear interpolation

‚constant v‘



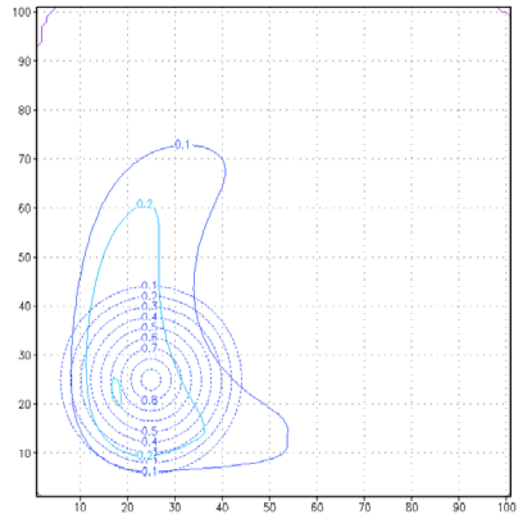
Min. = 0.0
Max. = 0.4204
rel. cons.= -0.000079

‚solid body rotation‘



Min. = 0.0
Max.= 0.62
rel. cons. = -0.18

‚LeVeque‘

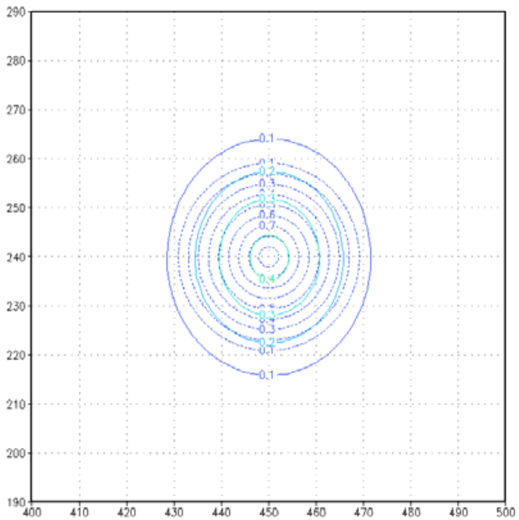


Min. = 0.0
Max. = 0.3018
rel. cons. = -0.0027

computer time relative to upwind 1. order = 2.5

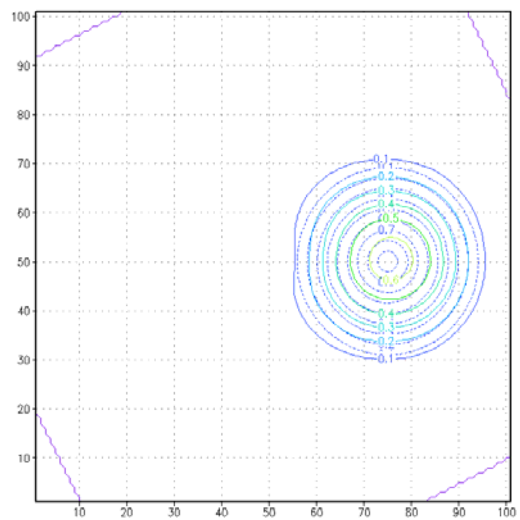
Semi-Lagrange-advection, backtrajectory O(Δt^2), bilinear interpolation

‚constant v‘



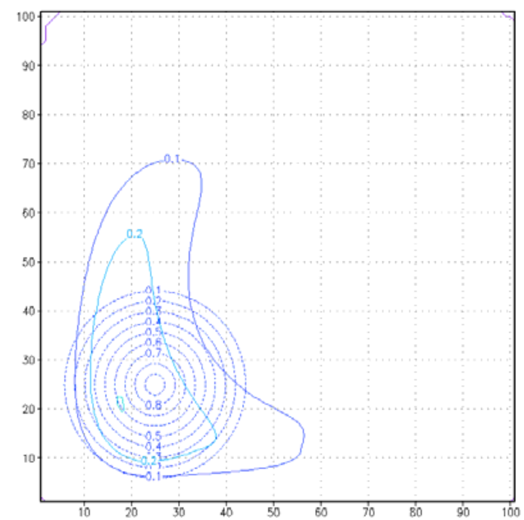
Min. = 0.0
Max. = 0.4204
rel. cons.= -0.000079

‚solid body rotation‘



Min. = 0.0
Max.= 0.6437
rel. cons. = -0.0049

‚LeVeque‘

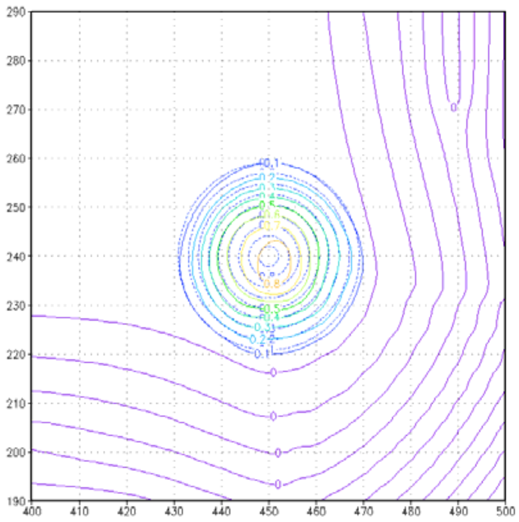


Min. = 0.0
Max. = 0.301
rel. cons. = -0.059

computer time relative to upwind 1. order = 3.75

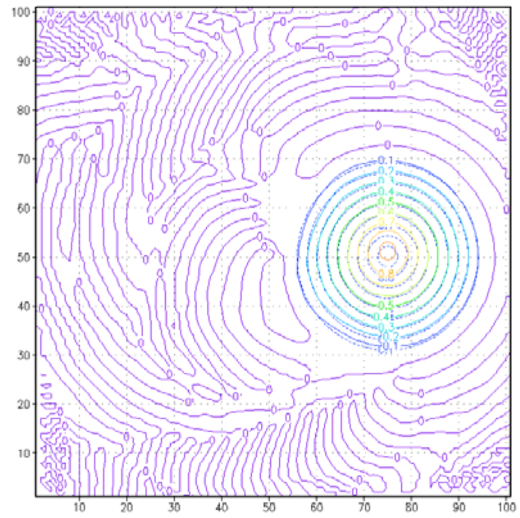
Semi-Lagrange-advection, backtrajectory $O(\Delta t^2)$, biquadratic interpolation

‚constant v'



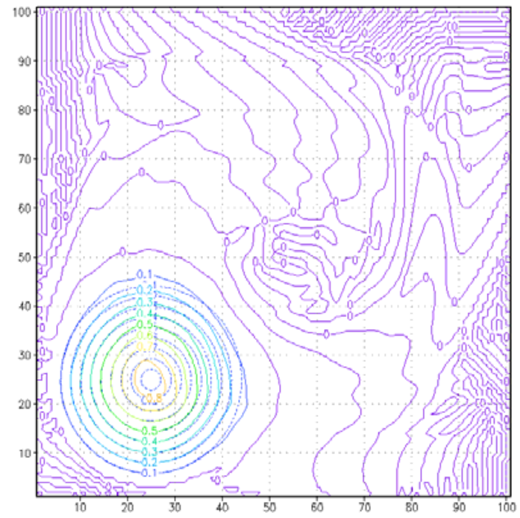
Min. = -0.053
Max. = 0.875
rel. cons.= -0.000061

‚solid body rotation'



Min. = -0.026
Max.= 0.9263
rel. cons. = -0.00020

‚LeVeque'



Min. = -0.0263
Max. = 0.8652
rel. cons. = 0.000019

computer time relative to upwind 1. order = 5.3

In LM used for prognostic precipitation:

Semi-Lagrange Advection

- backtrajectory in 2. order $O(\Delta t^2)$ (about 80% comp. time)
- trilinear interpolation (about 20% comp. time)

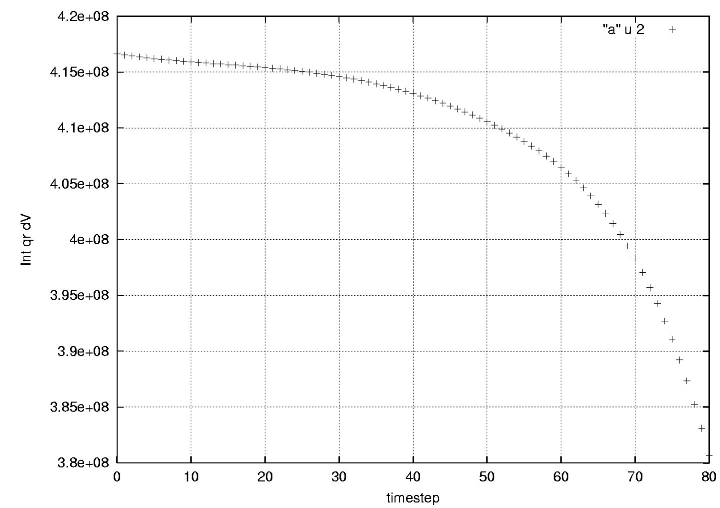
Properties:

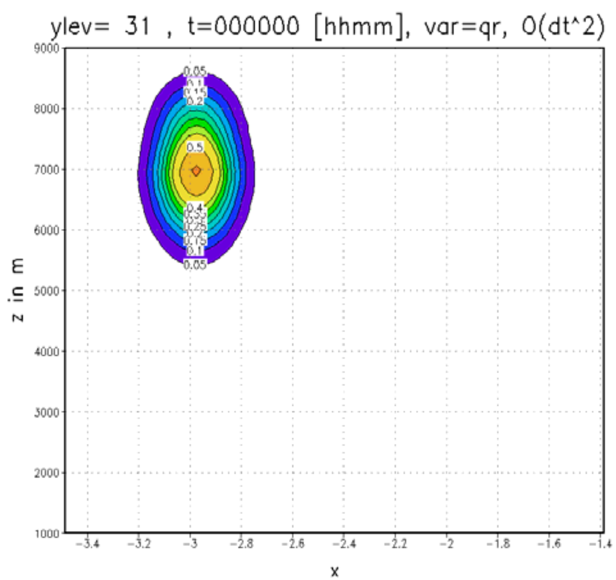
- positive definite
- conservation properties sufficient for rain/snow
- relatively strong numerical diffusion

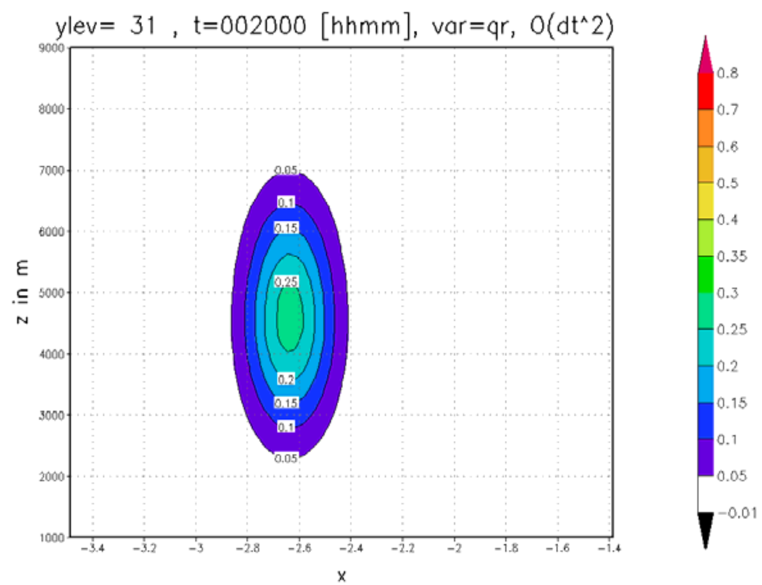
3D Test of Semi-Lagrange-Adv. in LM

backtrajectory in 2. order $O(\Delta t^2)$,
trilinear interpolation

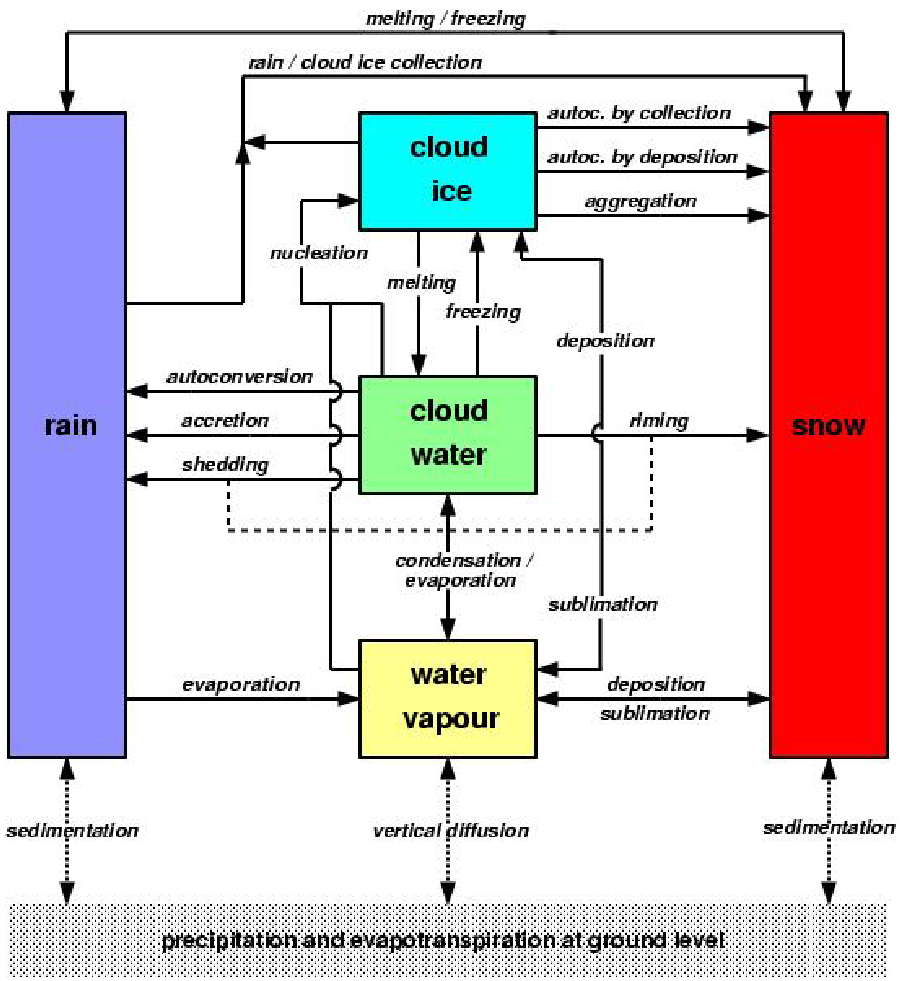
plane, (u,v,w) = (30, 0, -2) m/s = const.







Parameterization of cloud microphysical processes



The current LM (Class-5) scheme is designed for stratiform clouds

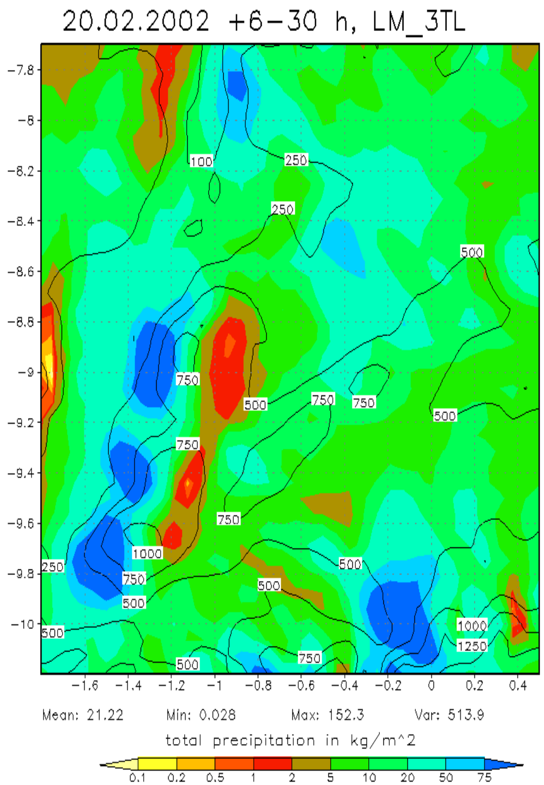
Application to convective clouds requires adjustments and extension to include graupel (hail):
- a Class-6 scheme in the new LMK

Implementation of a double moment ice scheme for research and benchmark (A.Seifert, NCAR)

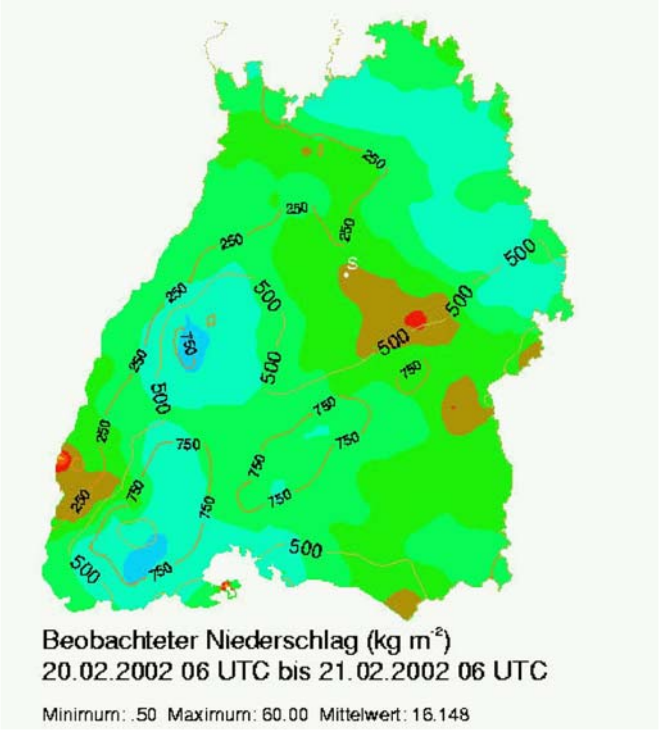
Test case: 20.02.2002 +06-30 h

total precipitation in 24 h over Baden-Württemberg (SW Germany)

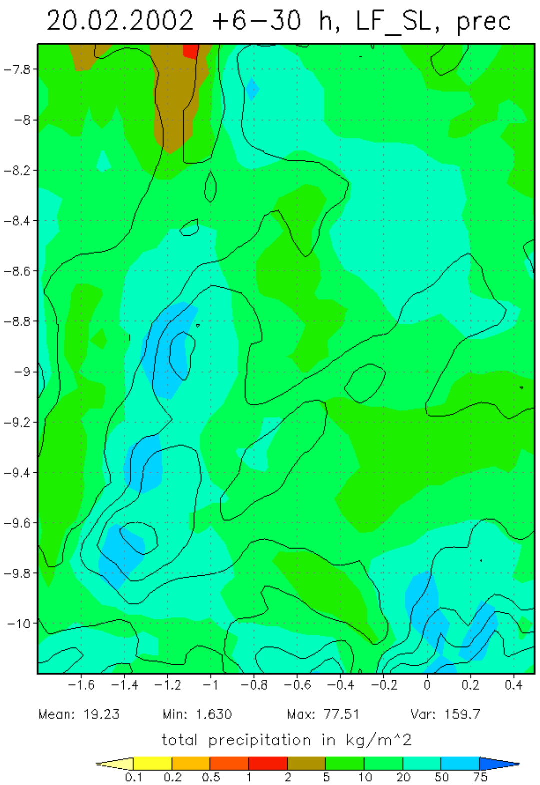
LM with diagnostic precip.



observations

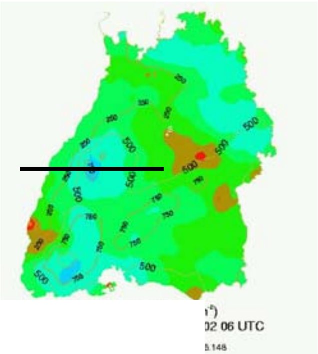


LM with prognostic precip.



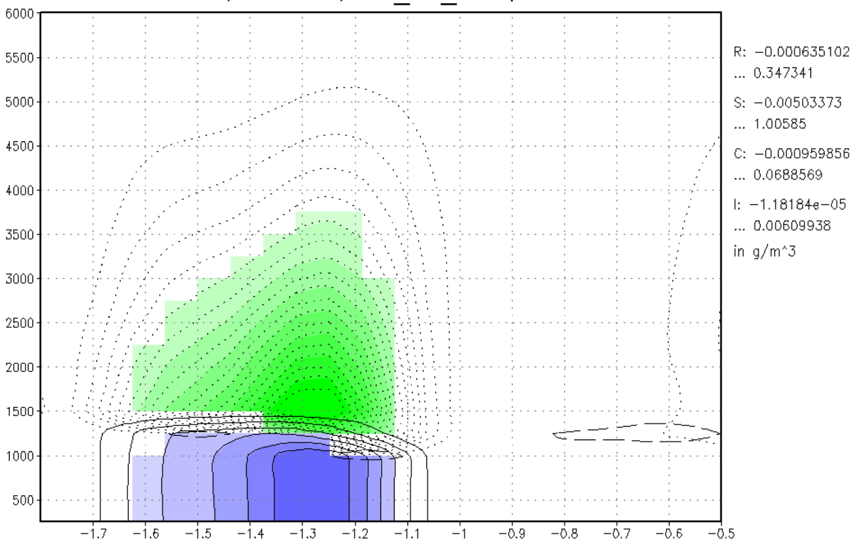
Test case: 20.02.2002

vertical cut (t=16:00)



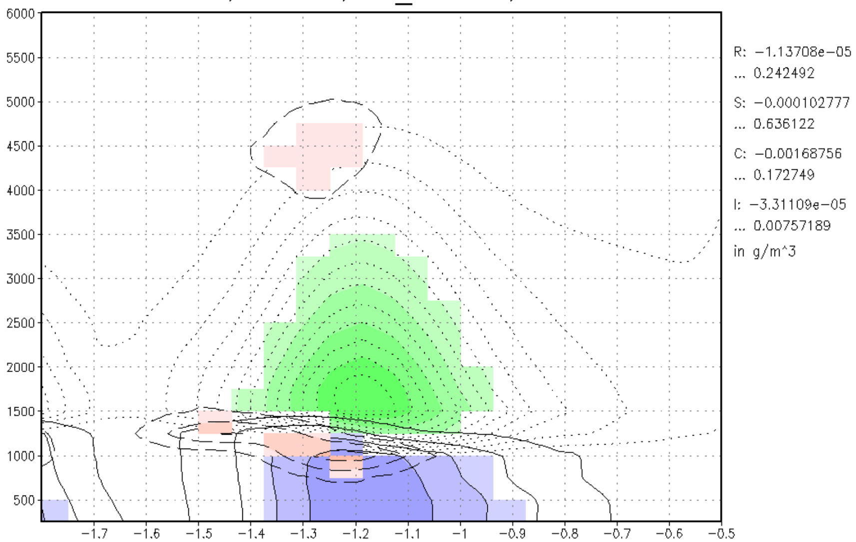
prognostic precip. with v=0

20.02.2002, 016:00, LM_SL_v=0, lat=−8.93'



prognostic precip.

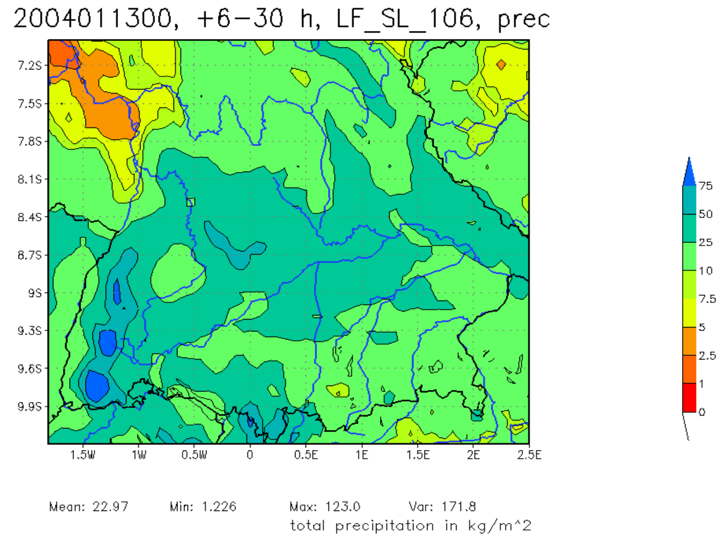
20.02.2002, 016:00, LM_3TL+SL, lat=−8.93'



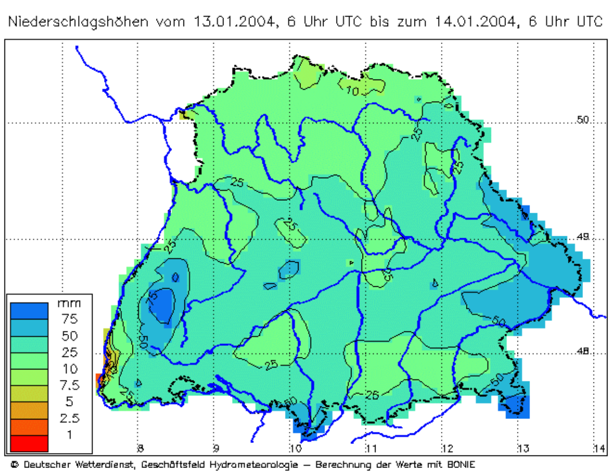
ρ_c ρ_r ρ_s

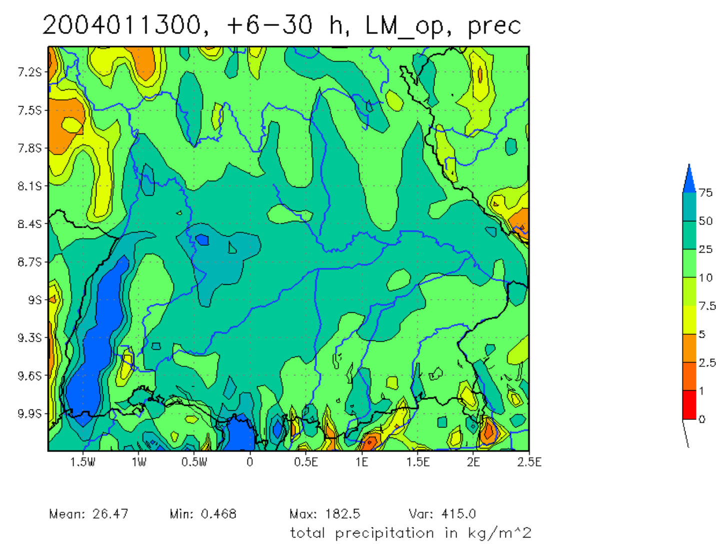
Deutscher Wetterdienst

Numeric experiment:
day 13.01.2004 +06-30 h



BONIE-Analysis

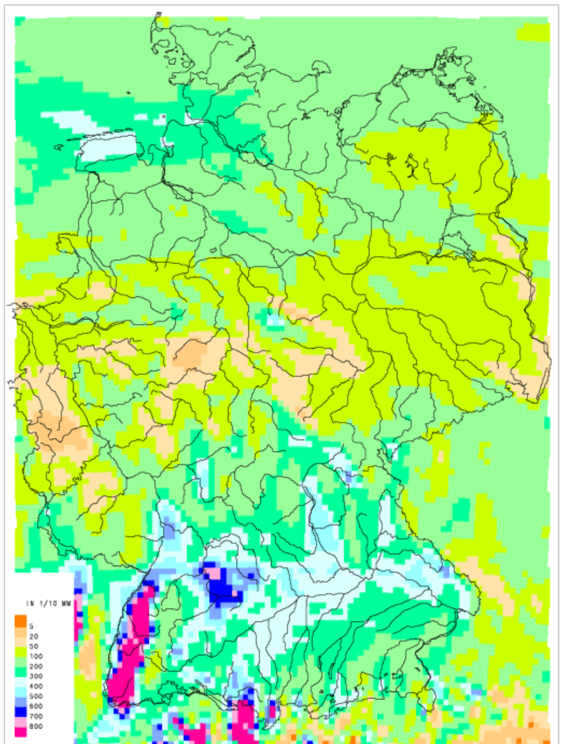




Numeric experiment:

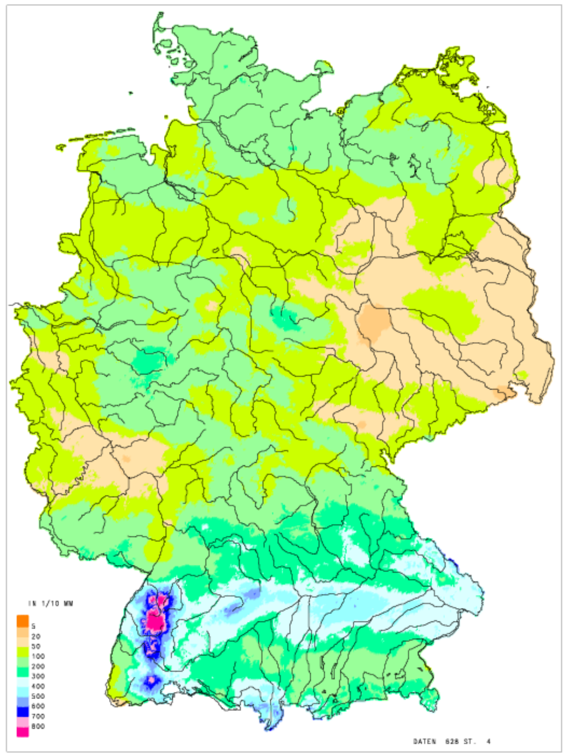
day 13.01.2004 +06-30 h

current operational LM



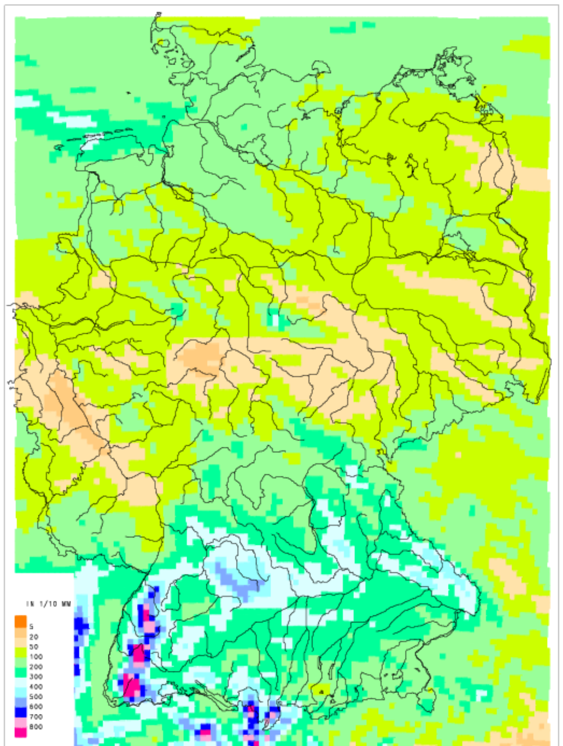
LM 13.01.2004 00UTC 06 UTC bis 06 UTC FT

REGNIE-Analysis



24. STD. ND.HOEHE GEMESSEN AM 14.01.2004 06UTC

with prognostic precipitation

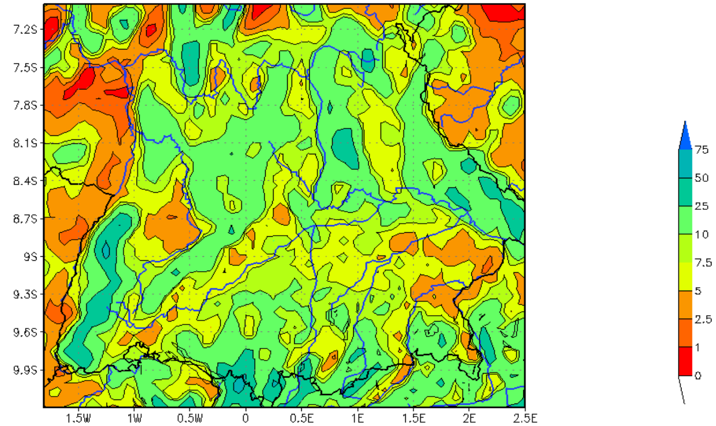


LF_SL 13.01.2004 06 UTC bis 06 UTC FT

Numeric experiment:

day 08.02.2004 +06-30 h

2004020800, +6–30 h, LF_SL_106, prec

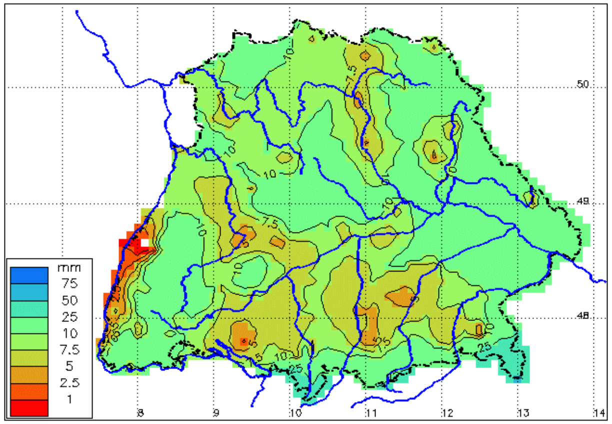


Mean: 10.50 Min: 0.251 Max: 59.81 Var: 65.43

total precipitation in kg/m^2

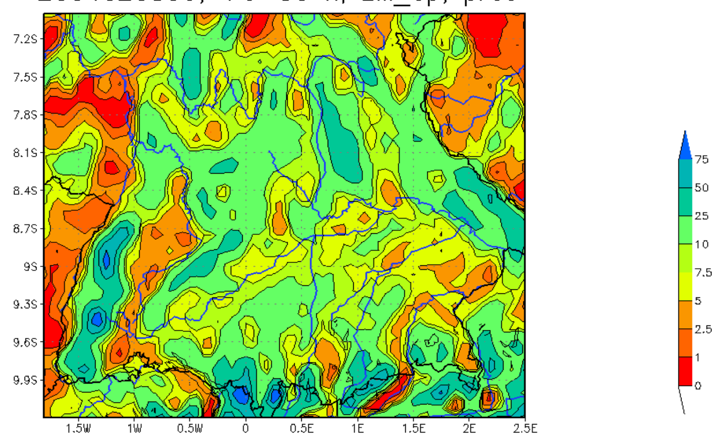
BONIE-Analysis

Niederschlagshöhen vom 8.02.2004, 6 Uhr UTC bis zum 09.02.2004, 6 Uhr UTC



© Deutscher Wetterdienst, Geschäftsfeld Hydrometeorologie – Berechnung der Werte mit BONIE

2004020800, +6–30 h, LM_op, prec



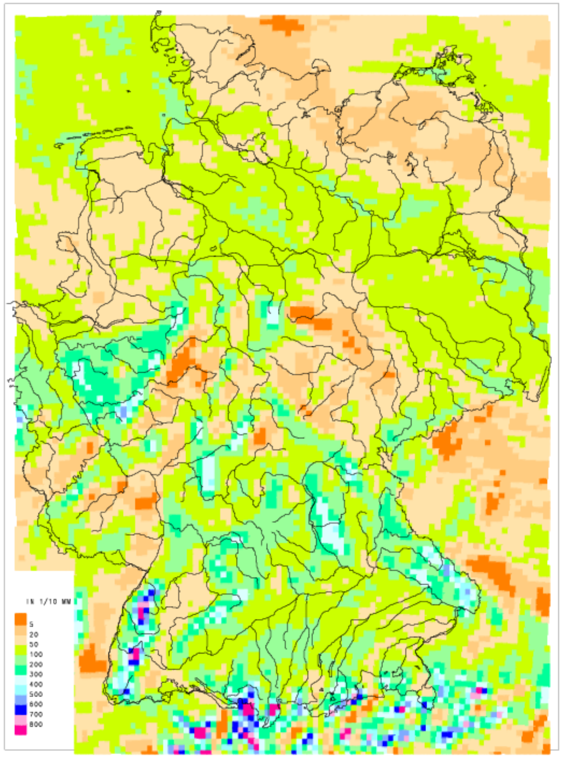
Mean: 12.53 Min: 0 Max: 117.2 Var: 164.6

total precipitation in kg/m^2

Numeric experiment:

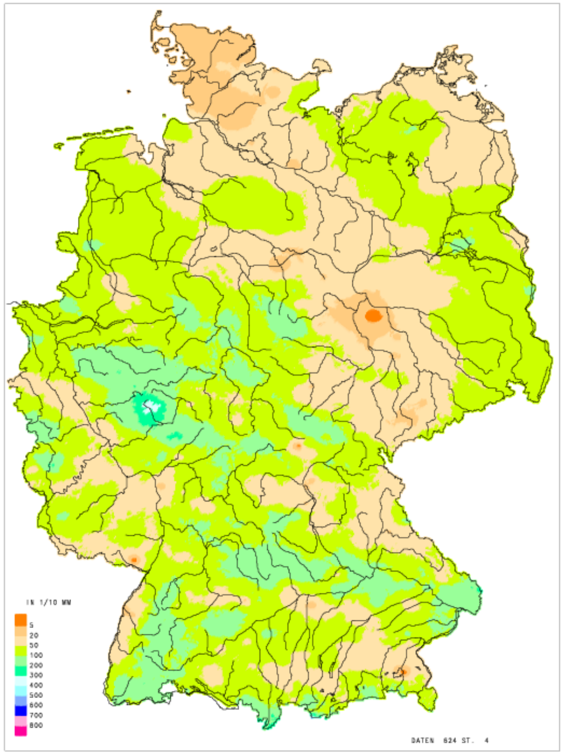
day 08.02.2004 +06-30 h

current operational LM



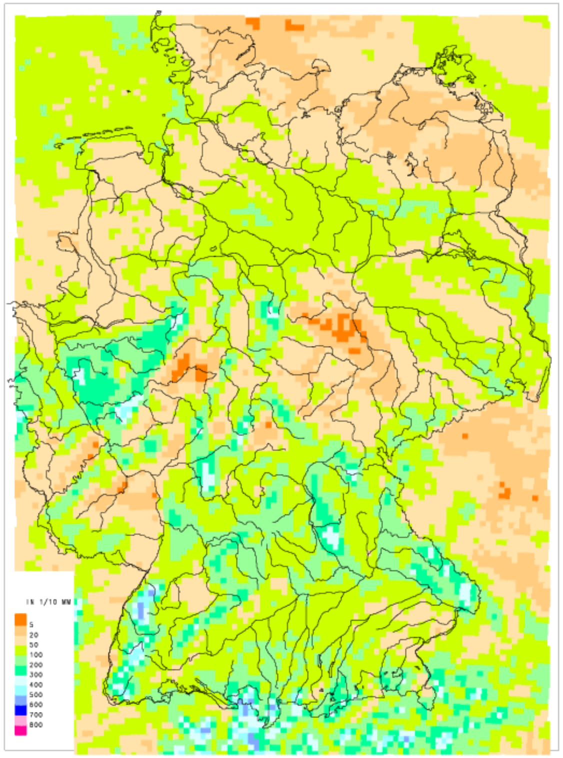
LM 08.02.2004 00UTC 06 UTC bis 06 UTC FT

REGNIE-Analyse



24. STD. ND.HOEHE GEMESSEN AM 09.02.2004 06UTC

with prognostic precipitation



LF_SL 08.02.2004 06 UTC bis 06 UTC FT

Results

from the actual numeric experiment
(analysis over South-Germany in 06.01.-08.02.2004)

compared to the current operational LM:

- Windward-Lee-distribution improved in most cases
- spatial averaged precipitation is reduced by about 15-25%
- precipitation maxima are reduced by about 20-40%
- computation time increased by about 20%

Lit.: Baldauf, Schulz (2004), COSMO-Newsletter

Time splitting schemes in atmospheric models

Some meteorological processes

Process	velocity range	timestep (for $\Delta x \approx 3$ km)
Coriolis (Rossby-waves)		$\tau_f \sim 50000$ sec. ($\sim 1/f$)
Advection	$c_a \sim 10..50$ m/s	$\tau_a \sim 60..300$ sec.
Buoyancy		$\tau_b \sim 600$ sec. ($\sim 2\pi/N$)
Gravity waves	$c_g \sim 30..100$ m/s	$\tau_g \sim 30..100$ sec.
Sound	$c_s \approx 330$ m/s	$\tau_s \sim 5$ sec.
Diffusion		

2 additive processes

$$\frac{\partial}{\partial t} q = \mathcal{P}_s q + \mathcal{P}_f q$$

$\mathcal{P}_s$: Advection, Diffusion, Coriolis, (Buoyancy)

$\mathcal{P}_f$: Sound (Buoyancy)

Time integration methods

- Integration with small time step Δt (and additive splitting)
- Semi-implicit method
- Time-splitting method

  main reason: fast processes are computationally ‚cheap'
  - Additive splitting
  - Klemp-Wilhelmson-splitting
    - Euler-Forward
    - Leapfrog
    - Runge-Kutta 2. order
    - Runge-Kutta 3. order

Time splitting

assume stable discretisations (2-level-schemes):

$$\text{for } \mathcal{P}_s \;:\quad q^{t+\Delta T} = Q_{s,\Delta T}(q^t) \quad = q^t + \Delta T\, P_s(q^t) + \Delta T^2\, P_s^{(2)}(q^t) + \dots$$

$$\text{for } \mathcal{P}_f \;:\quad q^{t+\Delta t} = Q_{f,\Delta t}(q^t) \quad = q^t + \Delta t\;\, P_f(q^t) + \Delta t^2\;\, P_f^{(2)}(q^t) + \dots$$

time splitting ratio:

$$n_s := \frac{\Delta T}{\Delta t}$$

Consistency condition: (2-level-schemes)

$$Q(P_s, P_f) = 1 + \Delta T\, P_s + n_s\, \Delta t\, P_f + O(\Delta T^2, \Delta t^2)$$

Stability: (von Neumann analysis)

$$Q_{lin} q = \lambda q$$

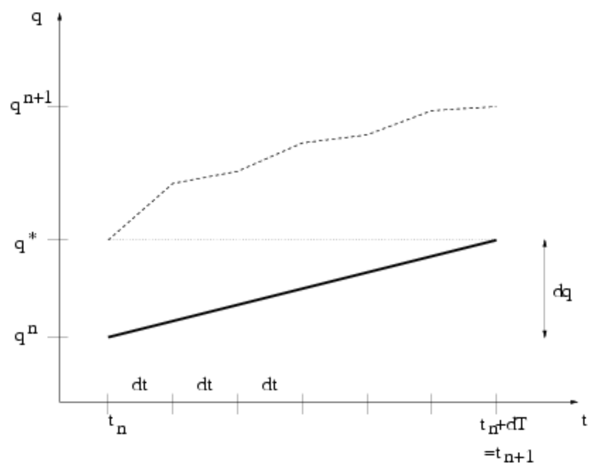
accuracy: ...

Additive Splitting

(complete operator or multiplicative splitting, method of fractional steps)

Peaceman, Rachford (1955) (ADI-Method),
Marchuk (1974)

$$
\begin{aligned}
q^* &= Q_s(q^t) \\
q^{t+\Delta T} &= Q_f(Q_f(\ldots Q_f(q^*)\ldots))
\end{aligned}
$$



- operator formulation

$$Q_{ges} = Q_f^{n_s} \cdot Q_s$$

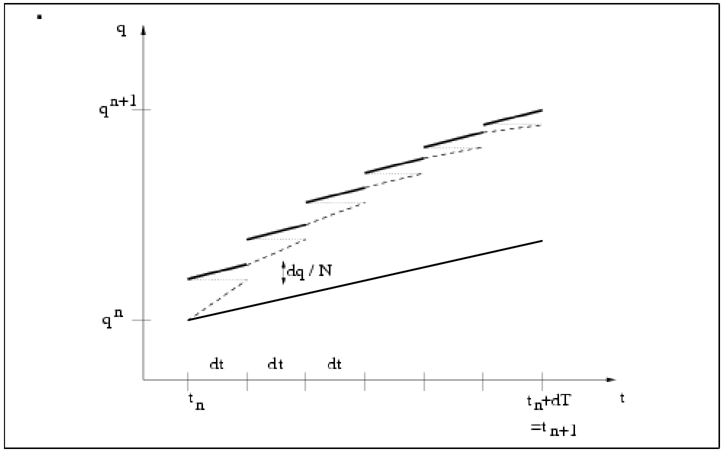
- costs: $1 \times Q_s$, $n_s \times Q_f$
- stability is guarantied only, if P_s and P_f both are stable *and* commutable. (Leveque and Oliger, 1983)
- noisy $\Leftarrow$ only weak coupling between modes
- of 1. order in ΔT (for non-commutable operators)
- method by Strang (for non-commutable operators)

$$q^{t+\Delta T} = (Q_f)^{n_s/2} \cdot Q_s \cdot (Q_f)^{n_s/2}\, q^t$$

$\Rightarrow$ of 2. order in ΔT

Klemp-Wilhelmson-Time-Splitting, Euler-forward-method

(partial operator splitting)



$$q^* = Q_s(q^t), \quad dq_s = \frac{q^* - q^t}{\Delta T}$$

and

$$\begin{aligned} q^{t+\Delta t} &= Q_f(q^t) + \Delta t \, dq_s \\ q^{t+2\Delta t} &= Q_f(q^{t+\Delta t}) + \Delta t \, dq_s \\ &\dots \\ q^{t+\Delta T} &= Q_f(q^{t+(n_s-1)\Delta t}) + \Delta t \, dq_s \end{aligned}$$

- 2-timelevel scheme
- costs: $1 \times P_s$, $n_s \times P_f$
- Skamarock, Klemp (1992), MWR: scheme is not stable (but analysis only in the time domain)
- scheme can be stabilized by divergence filtering (Baldauf (2002), COSMO-Newsletter)

Test case: density current (Straka et al., 1993)

KAMM2 (Karlsruher Atmospheric Mesoscale Model, Version 2)

non-hydrostatic, full compressible
option: KW-Euler-Forward-Time-Splitting

Klemp-Wilhelmson-time-splitting, Leapfrog-method

Klemp, Wilhelmson (1978), JAS

Leapfrog-scheme for ODE $dq/dt = f(q)$:

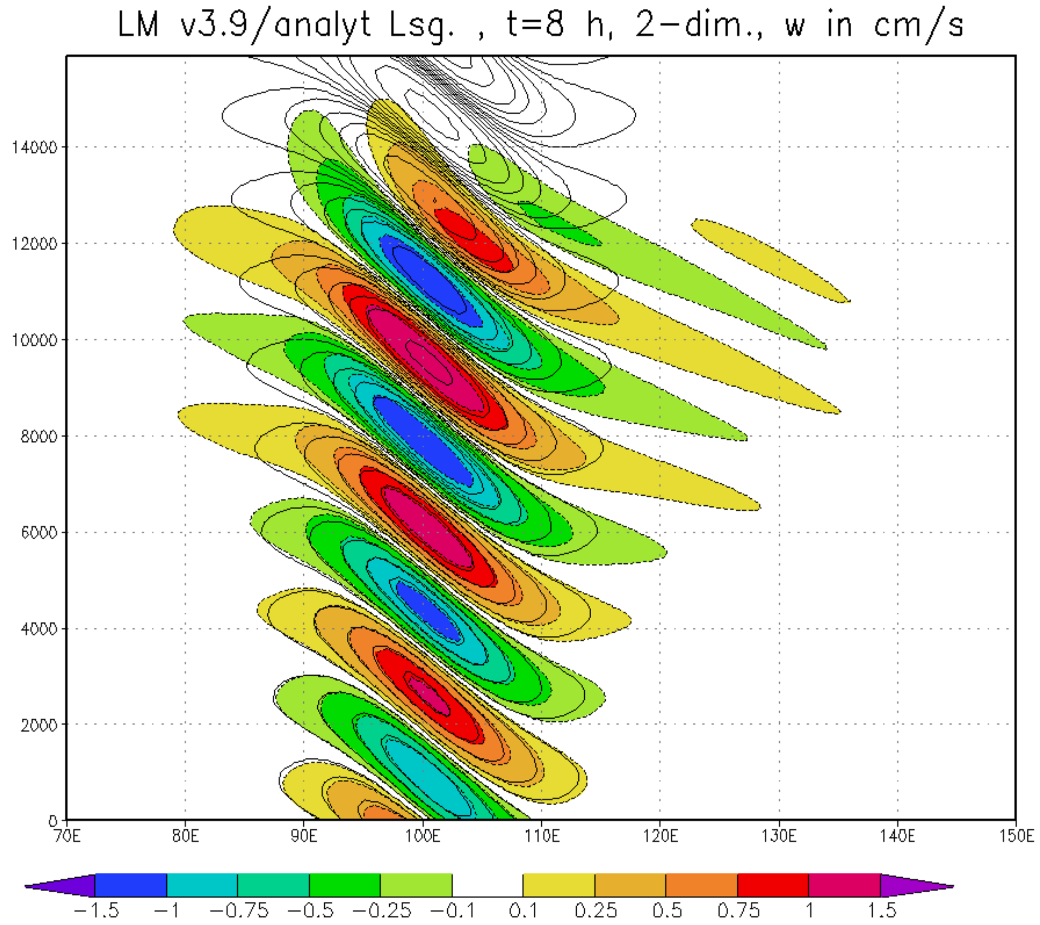
$$q^{n+1} = q^{n-1} + 2\Delta T \; f(q^n)$$

Leapfrog-scheme combined with splitting idea:

1. calculate the tendency of slow processes with q^t
2. perform a KW-Euler-forward-splitting from $t - \Delta T$ to $t + \Delta T$ with this slow tendency

- 3-timelevel scheme
- costs: $1 \times P_s$, $2n_s \times P_f$
- disadv.: only special advection schemes are applicable
- scheme is stable with (weak) Asselin-filter and divergence damping (Skamarock and Klemp, 1992)

Test case: Hydrostatic mountain wave



h=10m, a=10 km
u=10 m/s
isothermal atmosph., f=0
→ Fr_v=55, Fr_h=0.055

LM v3.9, KW-leapfrog-scheme

Δx=2.5 km, Δz=100m
400 * 5 * 160 GPs

Klemp-Wilhelmson-Runge-Kutta-2.order-splitting

Wicker, Skamarock (1998), MWR

RK2-scheme for ODE $dq/dt = f(q)$:

$$\begin{aligned} q^* &= q^n + \frac{\Delta T}{2} f(q^n) \\ q^{n+1} &= q^n + \Delta T\, f(q^*) \end{aligned}$$

RK2-scheme combined with splitting idea:

1. KW-Euler-forward-splitting from t to $t + \Delta T/2 \;\Rightarrow q^*$ as a new state
2. calculate the slow tendency with q^*, perform a KW-Euler-forward-splitting from t to $t + \Delta T$ with this slow tendency

- 2-timelevel-scheme
- costs: $2 \times P_s$, $1.5 n_s \times P_f$.

- only advection (upstream 5. order): stable for C < 0.3 (Wicker, Skamarock, 2002)

'shortened' RK2-scheme: first RK-step only with fast process (Gassmann (2002), COSMO-Newsletter)
costs: $1 \times P_s$, $1.5N \times P_f$

Question: can time splitting schemes deliver a correct stationary solution?

Example: **Scalar Relaxation equation with external force**

$$\frac{d\phi}{dt} = \underbrace{-\beta\phi}_{Relax.} + \underbrace{g}_{Force}, \quad \beta > 0,\ \beta, g \text{ constant}$$

stationary solution:

$$\phi_s = \frac{g}{\beta}$$

Discretisation of single processes (a) and (b) with simple Euler-forward-schemes; resulting stationary solution of the numerical splitting scheme:

Splitting	Relax. fast, Forcing slow	Forcing fast, Relax. slow
Additive	$\frac{g}{\beta}\left(1 + \frac{1}{2}\frac{n_s-1}{n_s}\beta\,\Delta t + \ldots\right)$	$\frac{g}{\beta}(1 - \beta\,\Delta T)$
KW-EV	$\frac{g}{\beta}$	$\frac{g}{\beta}$
KW-Leapfrog	$\frac{g}{\beta}$	$\frac{g}{\beta}$
KW-RK2	$\frac{g}{\beta}$	$\frac{g}{\beta}$
KW-RK2-short	$\frac{g}{\beta}$	$\frac{g}{\beta}(1 - \beta\,\frac{\Delta T}{2})$
KW-'RK3'	$\frac{g}{\beta}$	$\frac{g}{\beta}$
KW-RK3b	$\frac{g}{\beta}$	$\frac{g}{\beta}$

consistent solutions for Δt, $\Delta T \rightarrow 0$, but in practice: $\beta\Delta t \approx 1$ or $\beta\Delta T \approx 1$.

Linearised, 1-dim. sound-advection-equations

$$\frac{\partial u}{\partial t} + c_A \frac{\partial u}{\partial x} = -c_S \frac{\partial p}{\partial x}$$
$$\frac{\partial p}{\partial t} + c_A \frac{\partial p}{\partial x} = -c_S \frac{\partial u}{\partial x}$$

2 single processes

$$\mathcal{P}_s = -c_A \begin{pmatrix} \partial_x & 0 \\ 0 & \partial_x \end{pmatrix}, \; \mathcal{P}_f = -c_S \begin{pmatrix} 0 & \partial_x \\ \partial_x & 0 \end{pmatrix}$$

$\Rightarrow$ operators commute

analytic solution

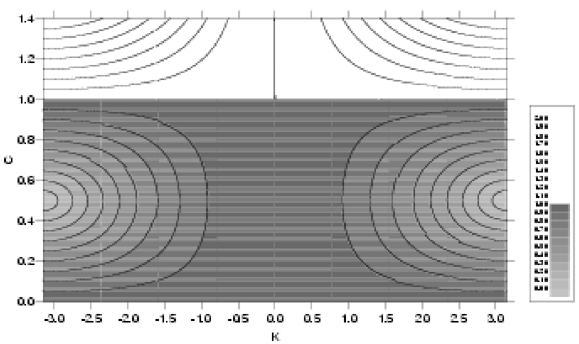
$$p(x,t) = f(x-(c_S+c_A)t) + g(x+(c_S-c_A)t)$$
$$u(x,t) = f(x-(c_S+c_A)t) - g(x+(c_S-c_A)t)$$

- Tatsumi (1983), JMS Japan
- Skamarock, Klemp (1992), MWR
- Wicker, Skamarock (1998), MWR

Advection

$$\frac{\partial \phi}{\partial t} + U_0 \frac{\partial \phi}{\partial x} = 0$$

Courant-number: $C_{adv} := U_0 \frac{\Delta T}{\Delta x}$



stability diagram,
upwind 1. order

upwind-scheme 1. order

- stable for $C_{adv} < 1$
- amplitude error biggest for $C_{adv} = 1/2$
- truncation error: $O(\Delta x)$, $O(\Delta t)$

Lax-Wendroff-scheme (Crowley 2. order)

- stable for $C_{adv} < 1$
- amplitude error biggest for $C_{adv} = 1/\sqrt{2}$
- truncation error: $O(\Delta x^2)$, $O(\Delta t^2)$

Advection-schemes of higher order

$$\frac{q_i^{n+1} - q_i^n}{\Delta t} = -\frac{F_{i+1/2}^n - F_{i-1/2}^n}{\Delta x}$$

Fluxes of 3rd to 6th order

$$
\begin{aligned}
F_{i-1/2}^{(3)} &= F_{i-1/2}^{(4)} - \frac{|u_{i-1/2}|}{12}\left(3(q_i - q_{i-1}) - (q_{i+1} - q_{i-2})\right) \\
F_{i-1/2}^{(4)} &= \frac{u_{i-1/2}}{12}\left(7(q_i + q_{i-1}) - (q_{i+1} + q_{i-2})\right) \\
F_{i-1/2}^{(5)} &= F_{i-1/2}^{(6)} - \frac{|u_{i-1/2}|}{60}\left(10(q_i - q_{i-1}) - 5(q_{i+1} - q_{i-2}) + (q_{i+2} - q_{i-3})\right) \\
F_{i-1/2}^{(6)} &= \frac{u_{i-1/2}}{60}\left(37(q_i + q_{i-1}) - 8(q_{i+1} + q_{i-2}) + (q_{i+2} + q_{i-3})\right)
\end{aligned}
$$

- Hundsdorfer et al. (1995), JCP
- Wicker, Skamarock (2002), MWR

Sound

$$\frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{\rho_0}\nabla p', \qquad \frac{\partial p'}{\partial t} = -\frac{c_p}{c_V} p_0 \nabla \cdot \mathbf{v}$$

Sound-Courant-number (for direction x_i): $C_{S,i} := c_S \frac{\Delta t}{\Delta x_i}$, $c_S := \sqrt{\frac{c_p}{c_V}\frac{p_0}{\rho_0}}$

Discretisation: Forward-Backward (Mesinger, 1977), stable ranges (with first unstable wavelengths)

1-dim., unstaggered:	$C_{S,x} < 2$	$(4\Delta x)$
1-dim., staggered:	$C_{S,x} < 1$	$(2\Delta x)$
2-dim., unstaggered:	$\sqrt{C_{S,x}^2 + C_{S,z}^2} < \sqrt{2}$	$(\pm 4\Delta x, \pm 4\Delta y)$
2-dim., staggered:	$\sqrt{C_{S,x}^2 + C_{S,z}^2} < 1$	$(\pm 2\Delta x, \pm 2\Delta y)$

Discretisation: Forward-Backward, vertical implicit:

2-dim., unstaggered:	$C_{S,x} < 2$, $C_{S,z} < \infty$
2-dim., staggered:	$C_{S,x} < 1$, $C_{S,z} < \infty$

Von-Neumann stability analysis

Linearised PDE-System for $u(x,z,t)$, $w(x,z,t)$, ... with constant coefficients

Discretization u_{jl}^n, w_{jl}^n, ... (grid sizes Δx, Δz)

single Fourier-Mode

$$u_{jl}^n = u^n \, e^{i(k_x j \Delta x + k_z l \Delta z)}$$

2-level-schemes

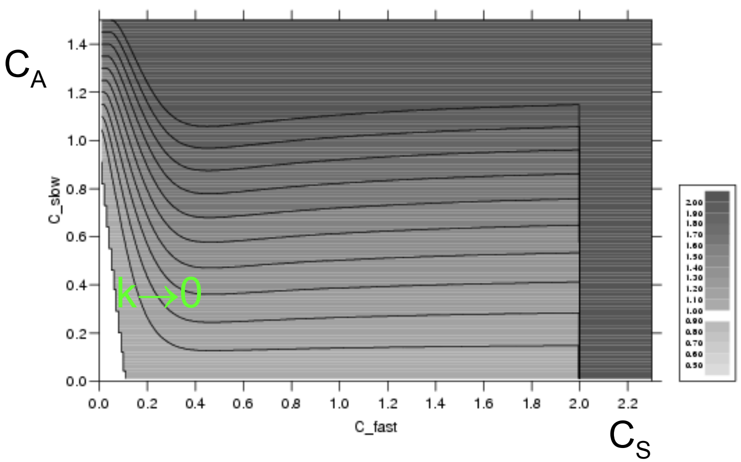
$$\begin{pmatrix} u^{n+1} \\ w^{n+1} \\ p^{n+1} \\ T^{n+1} \end{pmatrix} = Q \begin{pmatrix} u^n \\ w^n \\ p^n \\ T^n \end{pmatrix}$$

Determine eigenvalues λ_i of Q

Stability $\Leftrightarrow |\lambda_i| \leq 1$

Klemp-Wilhelmson-Euler-forward-scheme, non-staggered-grid

$|\lambda|_{max}(C_S, C_A),\ n_s = 8$



- $2\Delta x$-waves stable for

$$0 < C_A < 1.$$

- long waves ($k \to 0$) are stable for

$$C_A < 1 - C_S(1 + n_s)$$

$\Rightarrow$ stability range limited by long waves

$\Rightarrow$ try divergence filter!

Divergence damping

$$\frac{\partial \mathbf{v}}{\partial t} + \ldots = \ldots + \alpha_{div} \nabla (\,\mathrm{div}\, \mathbf{v})$$
$$\frac{\partial p}{\partial t} + \ldots = \ldots$$

$\Rightarrow$ Diffusion of Divergence

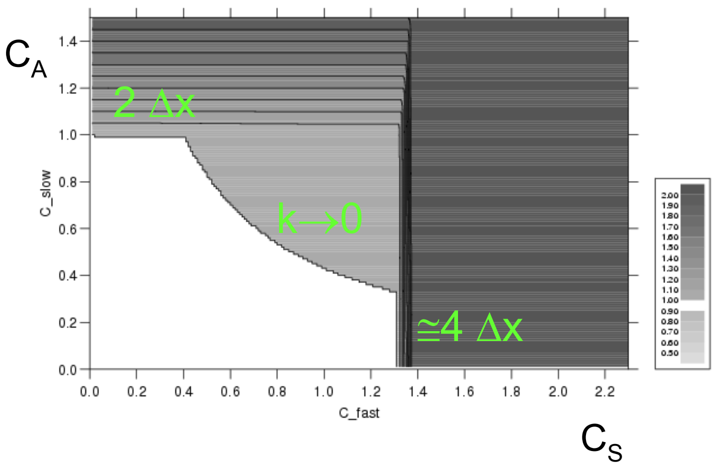
Stability of an explicit Differencing-scheme:

1D, explicit: $C_{div,x} < \frac{1}{2}$

2D, explicit, staggered grid: $C_{div,x} + C_{div,z} < \frac{1}{2}$

Courant-number: $C_{div,x} := \alpha_{div} \frac{\Delta t}{\Delta x^2}$

Klemp-Wilhelmson-Euler-forward-scheme, with divergence damping



$|\lambda|_{max}(C_S, C_A)$,
$C_{div} = 0.45$,
$n_s = 8$

long waves ($k \to 0$) are stable for

$$1 - C_A - C_S > \left(C_S - \frac{C_{div}}{C_A} \right) n_s$$

stability for strong divergence damping:

$$C_{div} > C_S C_A \qquad \text{and} \qquad n_s > \frac{1 - C_A - C_S}{C_S - \frac{C_{div}}{C_A}}$$

Conclusions from stability analysis of the 1-dim., linear Sound-Advection-System

- Klemp-Wilhelmson-Euler-Forward-scheme can be stabilized by a (strong) divergence damping
  --> stability analysis by Skamarock, Klemp (1992) too carefully
- No stability constraint for n_s in the 1D sound-advection-system
- Staggered grid reduces the stable range for sound waves.

  Stable range can be enhanced by a smoothing filter.

Linearised, 2-dim. Sound-Buoyancy-Advection-System

$$
\begin{aligned}
\frac{\partial u}{\partial t} + U_0 \frac{\partial u}{\partial x} &= -\frac{1}{\rho_0}\frac{\partial p'}{\partial x} && && + Q_x \\
\frac{\partial w}{\partial t} + U_0 \frac{\partial w}{\partial x} &= -\frac{1}{\rho_0}\frac{\partial p'}{\partial z} && -\left(\frac{p'}{p_0} - \frac{T'}{T_0}\right) g && + Q_z \\
\frac{\partial p'}{\partial t} + U_0 \frac{\partial p'}{\partial x} &= -\frac{c_p}{c_v} p_0 \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) && + \rho_0 g w && \\
\frac{\partial T'}{\partial t} + U_0 \frac{\partial T'}{\partial x} &= -\frac{R}{c_v} T_0 \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) && - \frac{\partial T_0}{\partial z} w &&
\end{aligned}
$$

Tend. Adv. Sound Buoyancy

- primitive variables

$$u, \quad w, \quad p = p_0 + p', \quad T = T_0 + T'$$

- Q_x, Q_z e.g. a divergence damping

$$Q_x = \alpha_{div} \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right), \quad Q_z = \alpha_{div} \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right)$$

Buoyancy terms:

$$\frac{\partial w}{\partial t} = -g\left(\frac{p'}{p_0} - \frac{T'}{T_0}\right)$$
$$\frac{\partial p'}{\partial t} = \rho_0 g w$$
$$\frac{\partial T'}{\partial t} = -\frac{\partial T_0}{\partial z} w$$

Forward-Backward scheme;
stable for $C_{buoy} := \omega_a \, \Delta t < 2$
no amplitude error in the stable range

acoustic cut-off frequency $\omega_a := \sqrt{N^2 + \frac{g^2}{c_S^2}}$; $\qquad C_\beta = \frac{1}{T_0}\frac{\partial T_0}{\partial z}\frac{c_S^2}{g}$

Commutation with other numerical operators:
$Q_{buoy} \cdot Q_{sound} \neq Q_{sound} \cdot Q_{buoy}$, $\qquad Q_{buoy} \cdot Q_{div} \neq Q_{div} \cdot Q_{buoy}$

Parameter study of the 2-dim., lin. Sound-Buoyancy-Advection-System

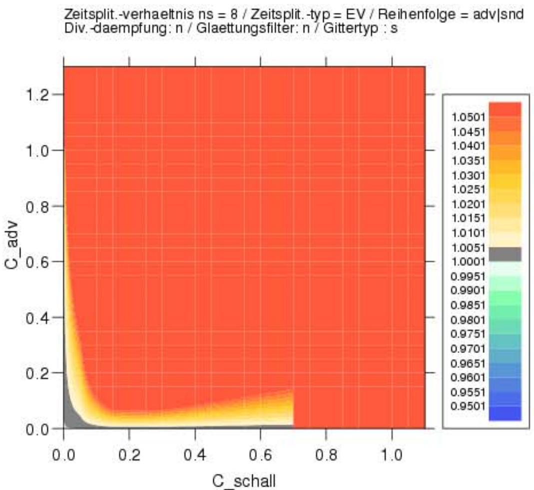
- Courant-numbers: $C_{adv,x}$, $C_{S,x}$, $C_{S,z}$, C_{buoy}, $C_{div,x}$, $C_{div,z}$
- other Parameters: n_S, C_β
- Splitting schemes: Additive, KW-Euler-Forward, KW-Runge-Kutta 2. order, KW-RK2_short
- Smoothing Filter
- Sound vertical implicit/explicit
- Advection: upwind 1. order, Lax-Wendroff
- Buoyancy in slow or fast processes
- Sequence of single processes

Maximum Timesteps for single processes $U_0 = 10$ m/s, Standard-Atmosphere

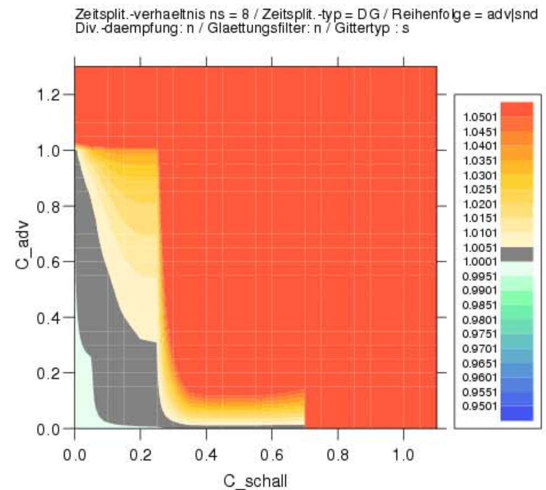
		$\Delta x = 300$ m $\Delta z = 10$ m	$\Delta x = 3$ km $\Delta z = 10$ m
Sound	$\Delta x/c \approx$	1 s	10 s
	$10\Delta z/c \approx$	0.3 s	0.3 s
Advection	$\Delta x/U_0 \approx$	30 s	300 s
Buoyancy	$2/\omega_a \approx$	60 s	60 s

no buoyancy, no divergence damping, $n_s = 8$

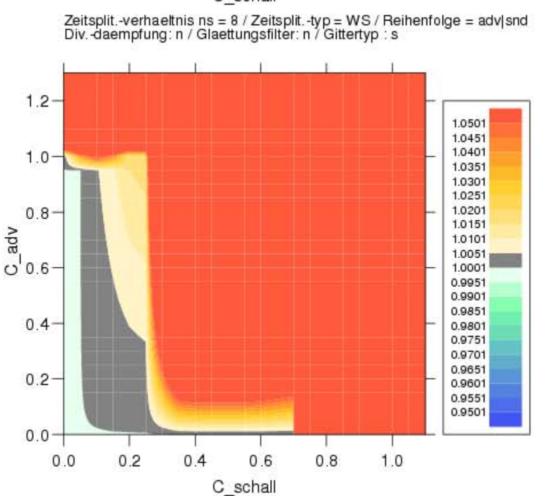
KW-EV



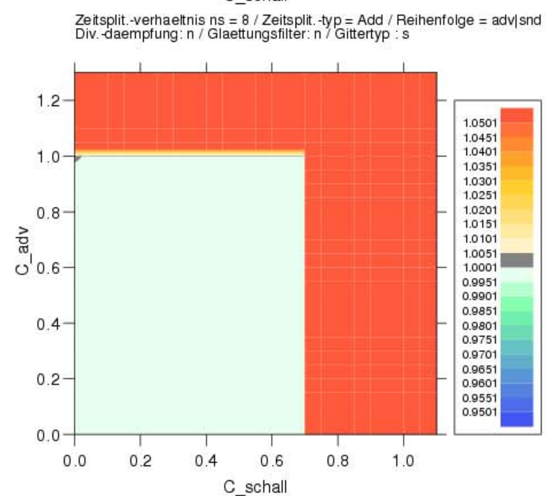
KW-RK2-short



KW-RK2



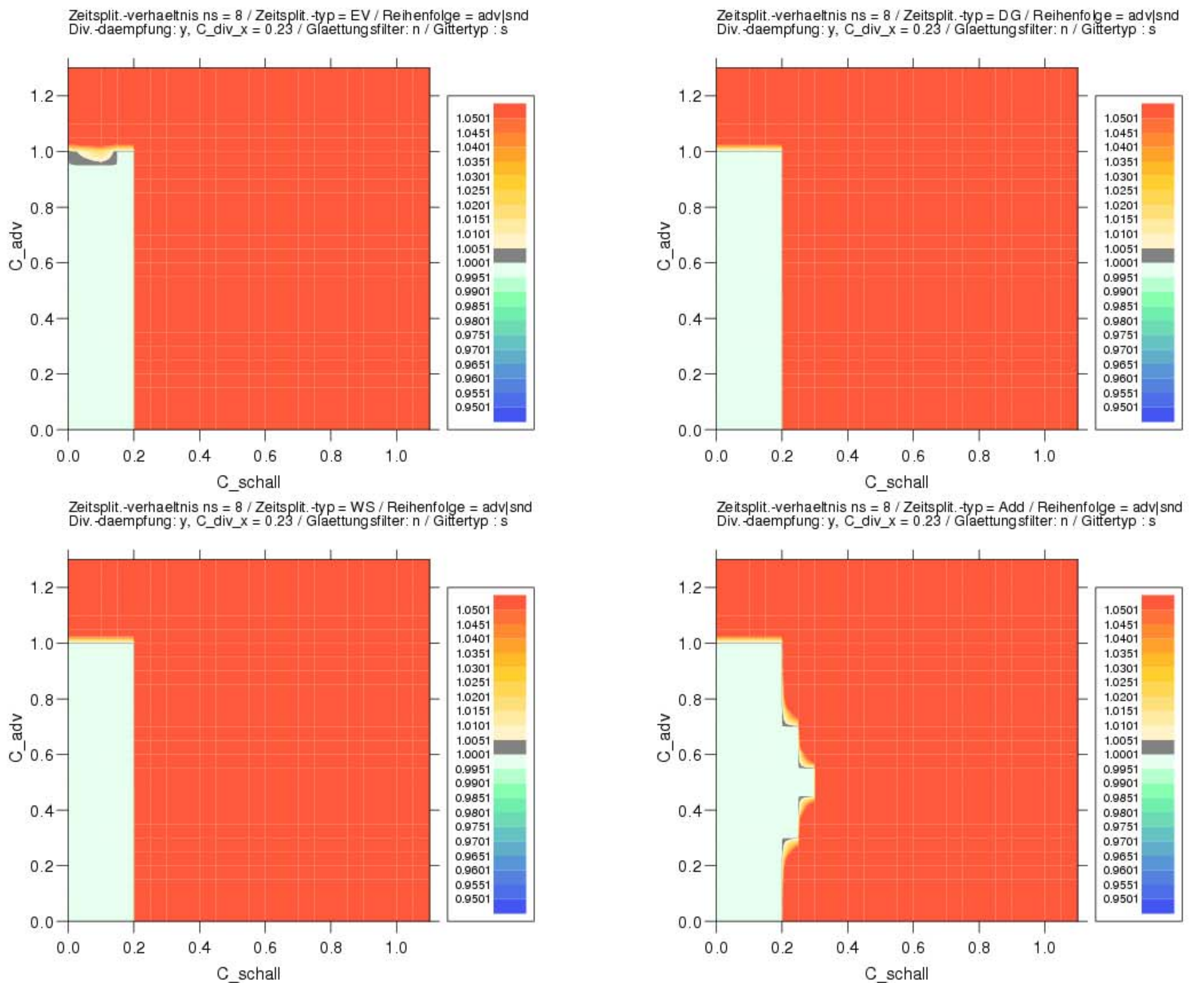
Additiv



no buoyancy, divergence damping $C_{div,i} = 0.23$, $n_s = 8$

KW-EV

KW-RK2-short

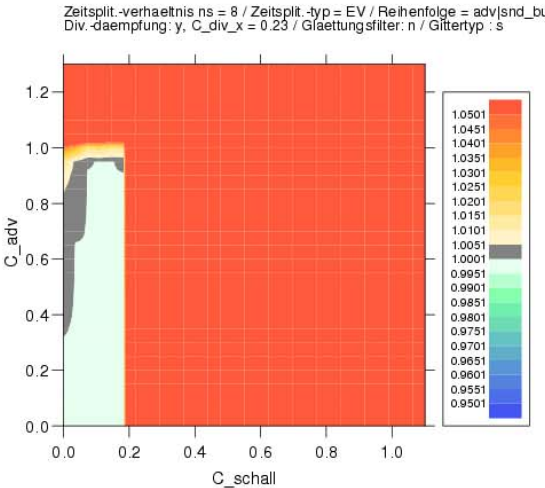


KW-RK2

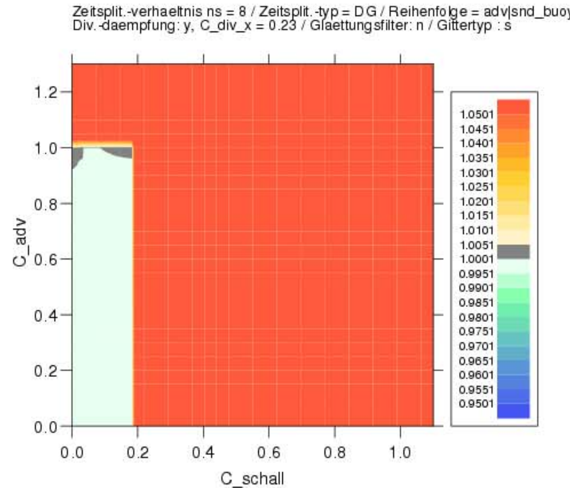
Additiv

buoyancy $C_{buoy} = 0.01$, divergence damping $C_{div,i} = 0.23$, $n_s = 8$

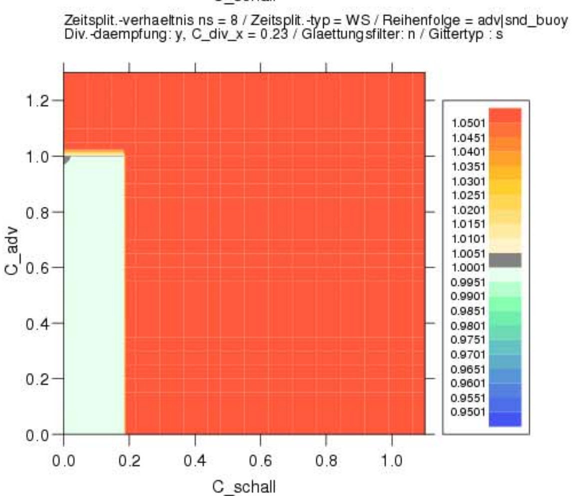
KW-EV



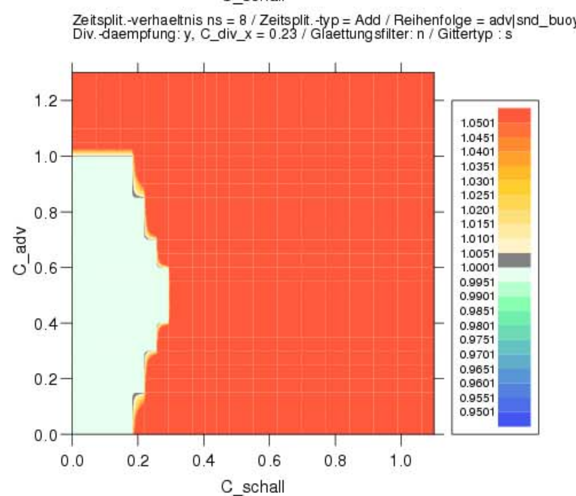
KW-RK2-short



KW-RK2



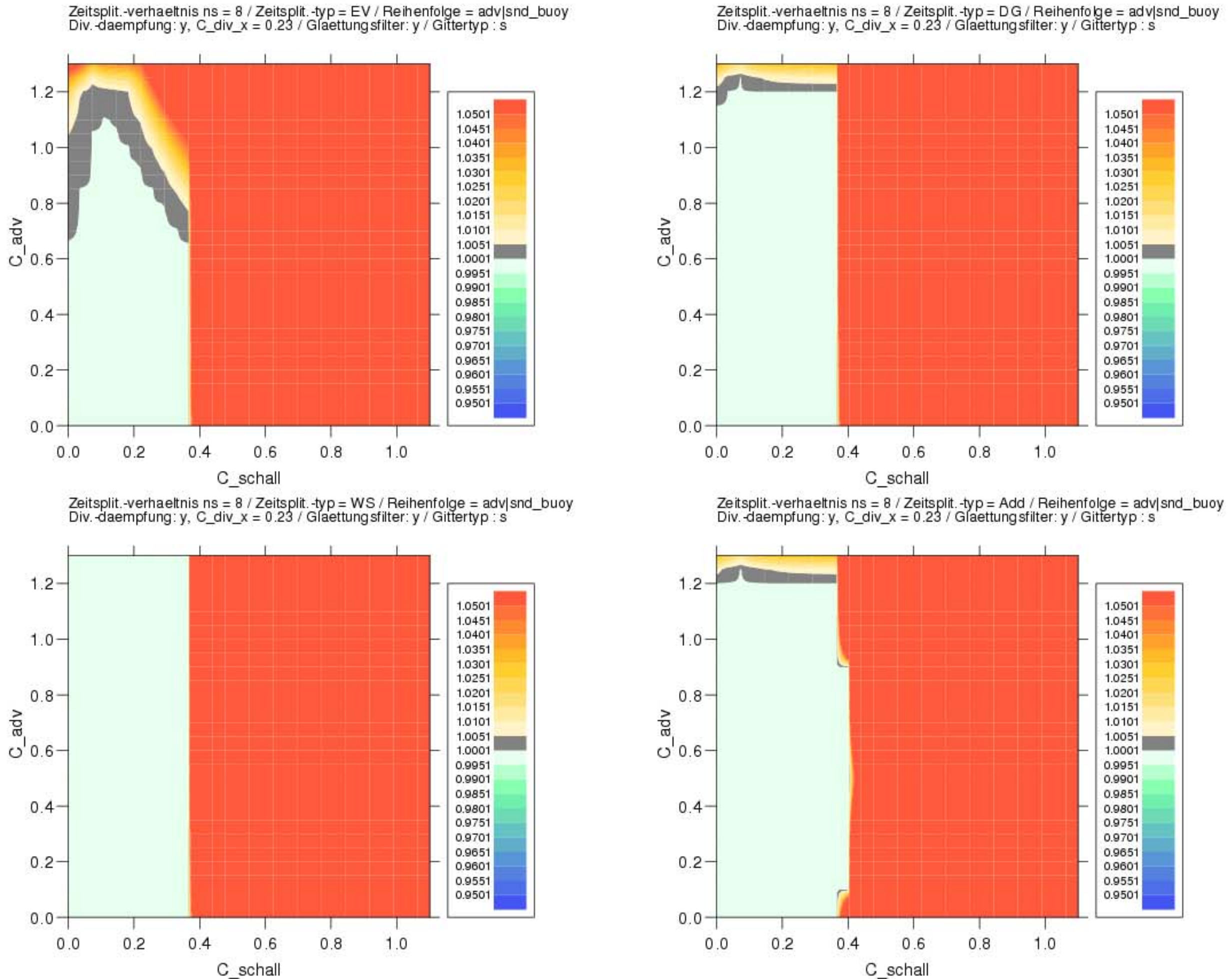
Additiv



buoyancy $C_{buoy} = 0.01$, div. damping $C_{div,i} = 0.23$, with smoothing, $n_s = 8$

KW-EV

KW-RK2-short

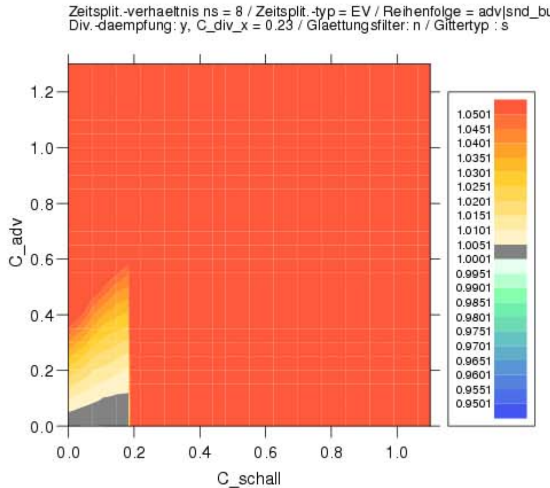


KW-RK2

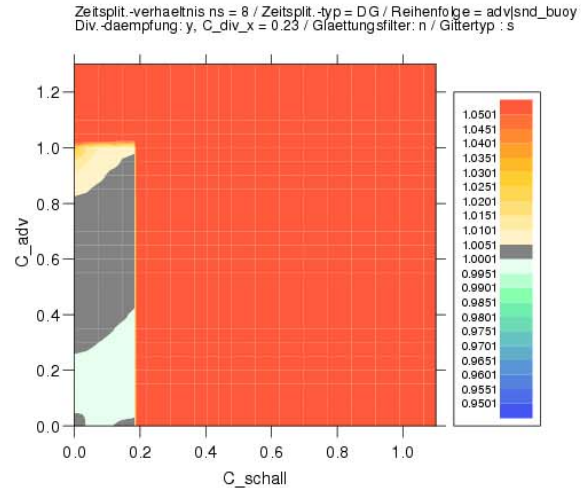
Additiv

buoyancy $C_{buoy} = 0.1$, div. damping $C_{div,i} = 0.23$, $n_s = 8$

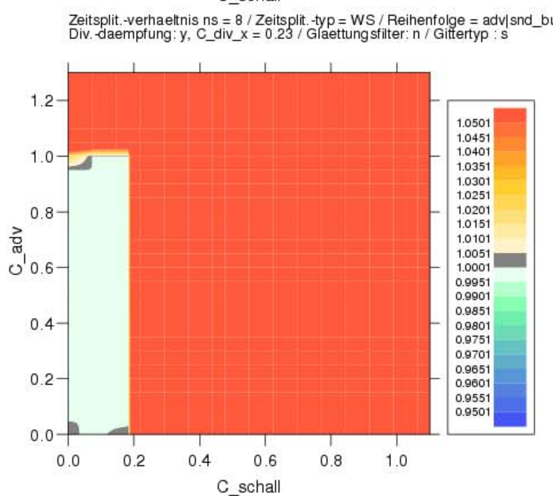
KW-EV



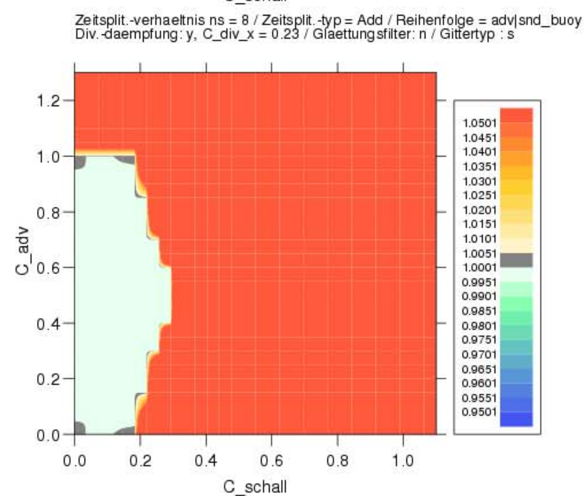
KW-RK2-short



KW-RK2

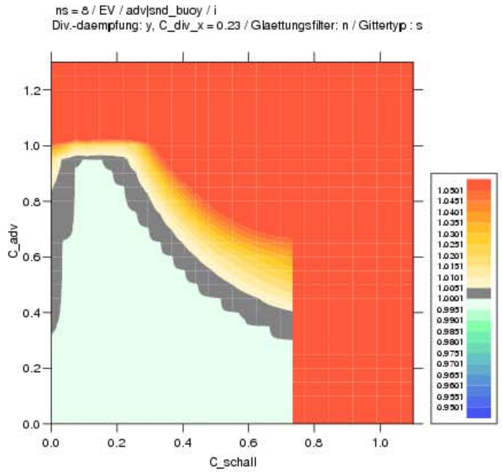


Additiv

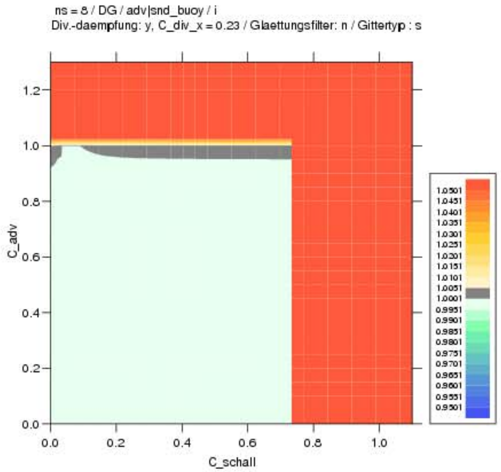


$C_{buoy} = 0.01$, div. damping $C_{div,i} = 0.23$, $n_s = 8$, sound: vertical implicit

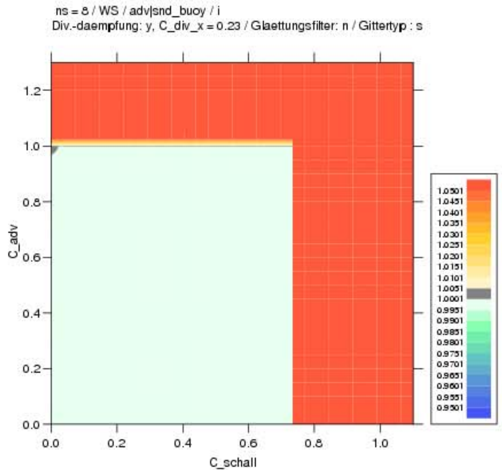
KW-EV



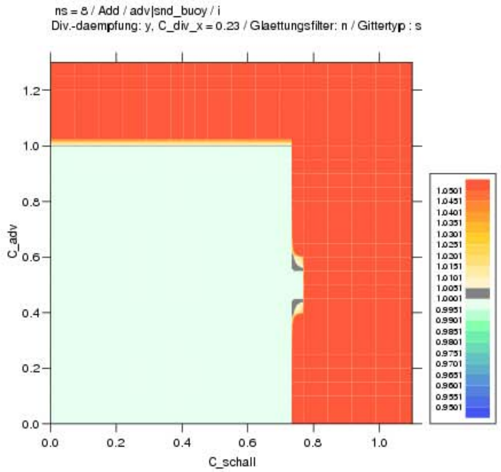
KW-RK2-short



KW-RK2

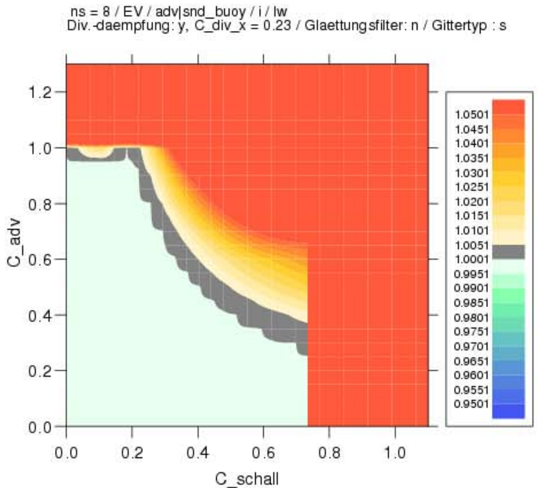


Additiv

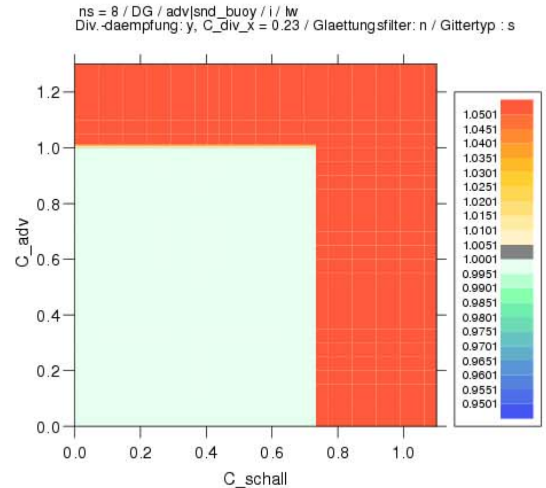


no buoyancy, $C_{div,i} = 0.23$, $n_s = 8$, sound: vert. implicit, Adv.: Lax-Wendroff

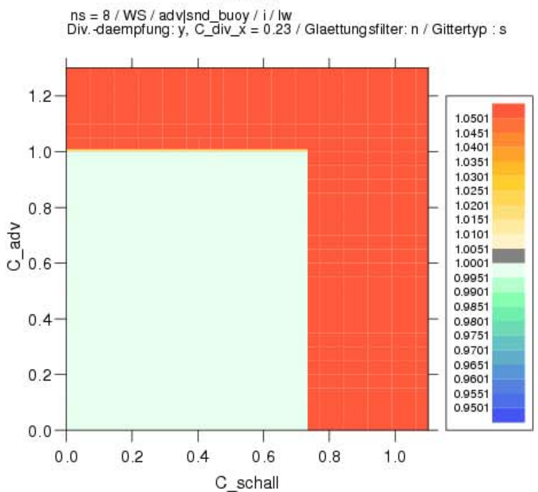
KW-EV



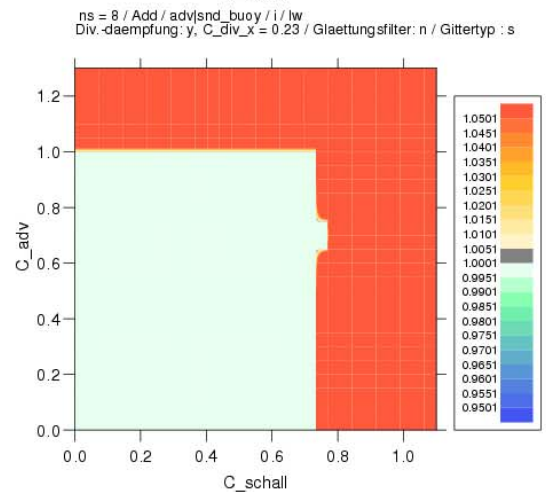
KW-RK2-short



KW-RK2

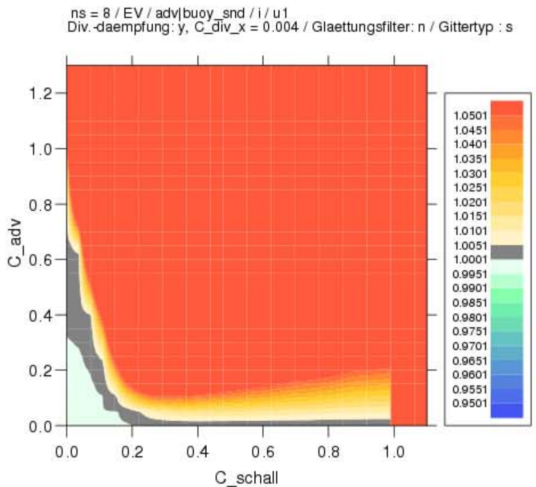


Additiv

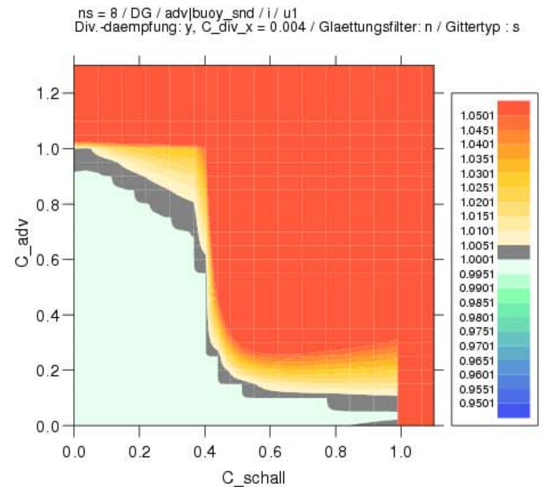


$C_{buoy} = 0.01$, $C_{div,i} = 0.23$, $n_s = 8$, sound: vert. implicit, $\Delta x = 10\,\Delta z$

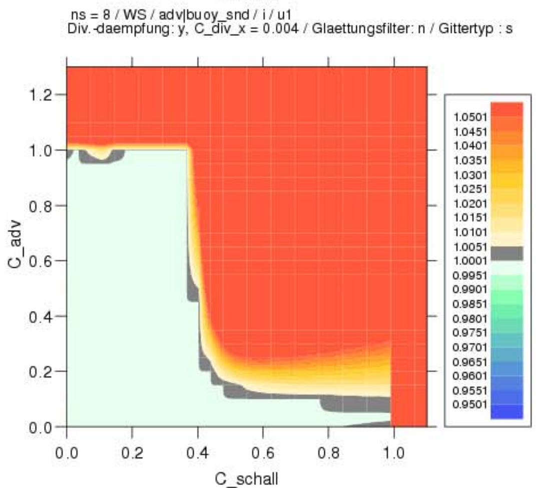
KW-EV



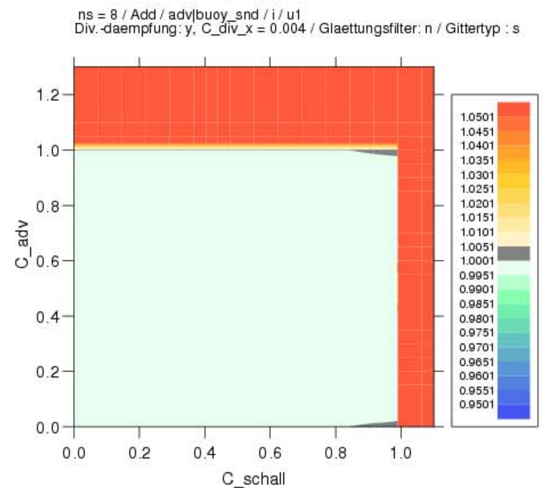
KW-RK2-short



KW-RK2



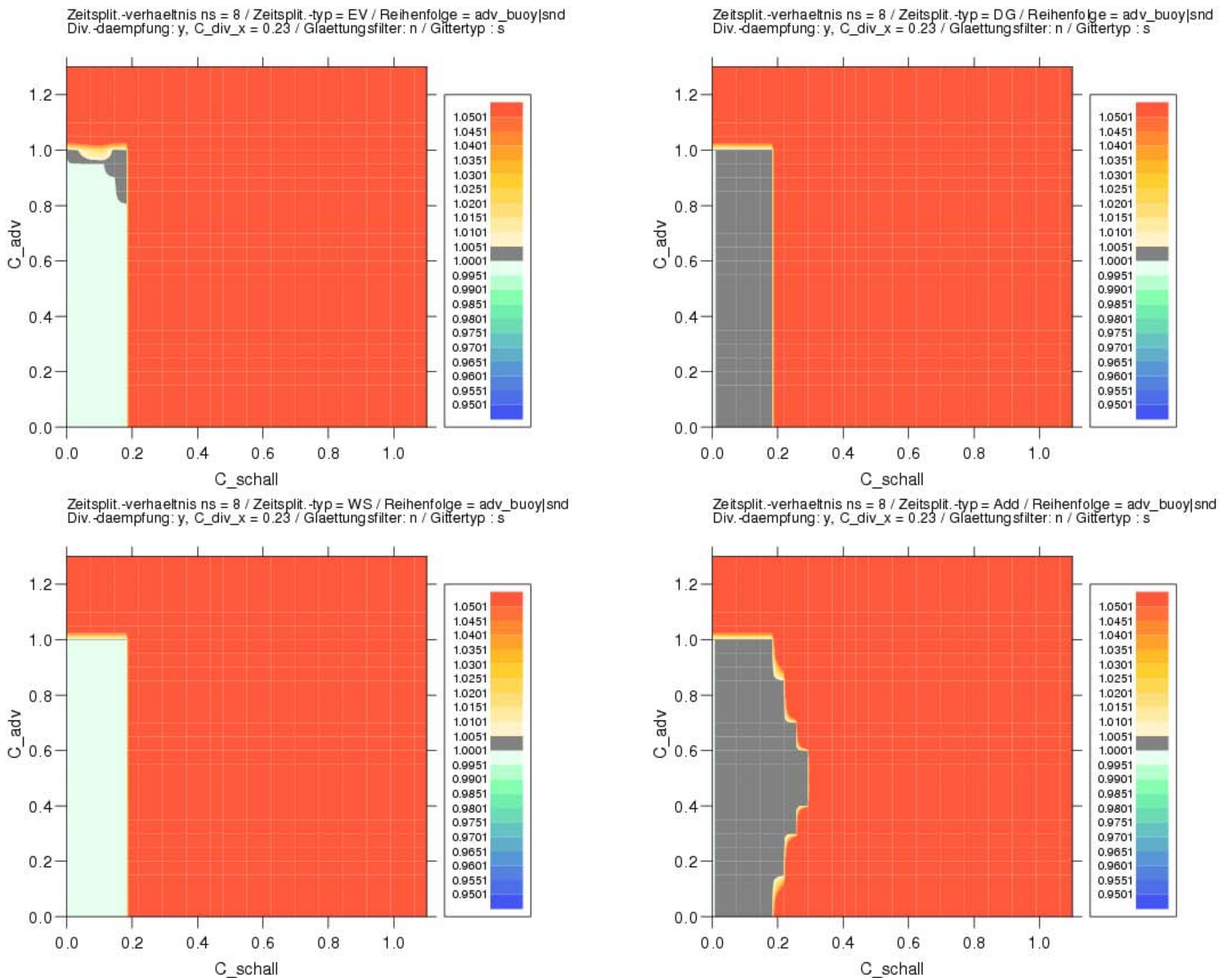
Additiv



$C_{buoy} = 0.1$, $C_{div,i} = 0.23$, $n_s = 8$, buoy in slow proc.

KW-EV

KW-RK2-short

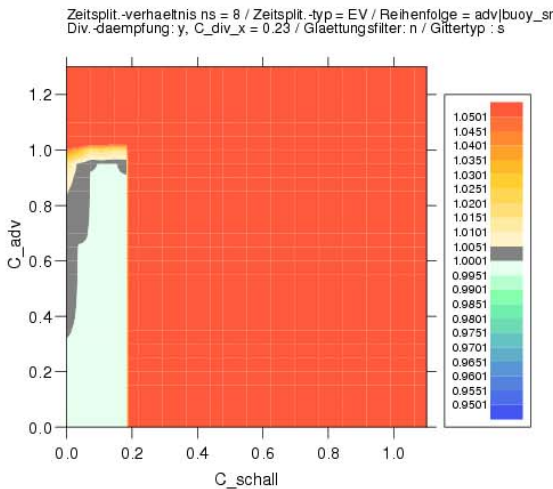


KW-RK2

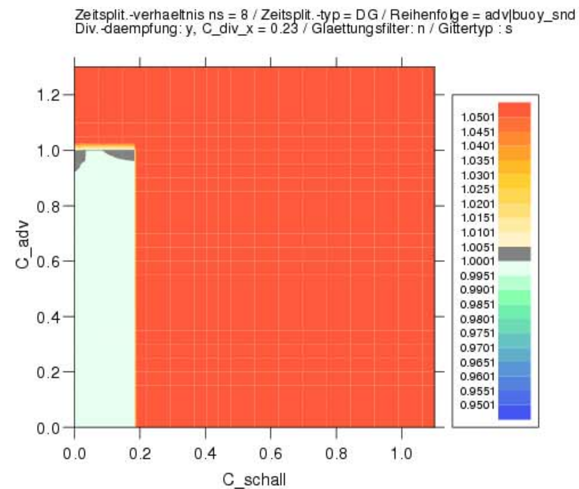
Additiv

buoyancy $C_{buoy} = 0.1$, div.-damp. $C_{div,i} = 0.23$, $n_s = 8$, seq.: Buoy.-Sound-Div.

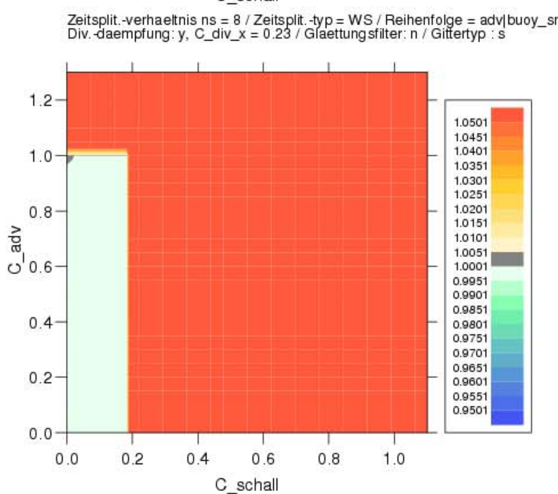
KW-EV



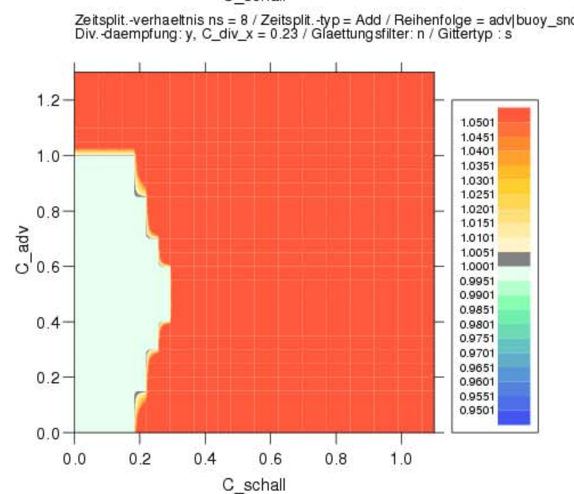
KW-RK2-short



KW-RK2



Additiv



Conclusions from stability analysis of the 2-timelevel splitting schemes

- Additive splitting relatively stable, but in practice too noisy
- KW-EV-splitting efficient, but often too small stable ranges; strong divergence damping needed
- KW-RK2-short version stable, but in practice no good convergence properties
- KW-RK2 scheme: good stability properties

Klemp-Wilhelmson-Runge-Kutta-3.order-splitting

Wicker, Skamarock (2002), MWR

$$\begin{aligned} q^* &= q^n + \frac{\Delta T}{3} f(q^n) \\ q^{**} &= q^n + \frac{\Delta T}{2} f(q^*) \\ q^{n+1} &= q^n + \Delta T\, f(q^{**}) \end{aligned}$$

- is *not* of 3. order in ΔT!
- costs: $3 \times P_s$, $1.833 n_s \times P_f$
- allows higher Courant-numbers (up to 1.42 with upstream 5. order)

RK3b-scheme:
Förstner, Doms (2004), COSMO-News.
Hundsdorfer et al.(1995), JCP

$$\begin{aligned} q^* &= q^n + \Delta T f(q^n) \\ q^{**} &= \frac{3}{4} q^n + \frac{1}{4} q^* + \frac{1}{4} \Delta T\, f(q^*) \\ q^{n+1} &= \frac{1}{3} q^n + \frac{2}{3} q^{**} + \frac{2}{3} \Delta T\, f(q^{**}) \end{aligned}$$

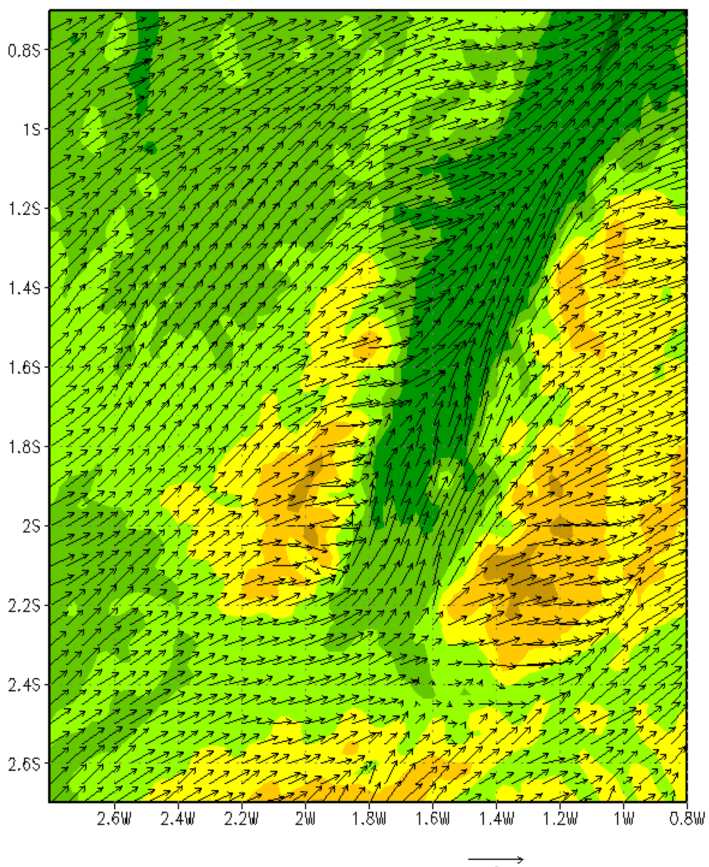
- is of 3. order in ΔT
- costs: $3 \times P_s$, $1.917 n_s \times P_f$
- currently tested in LMK

construction of a timesplitting-method analoguous to KW-RK2-splitting

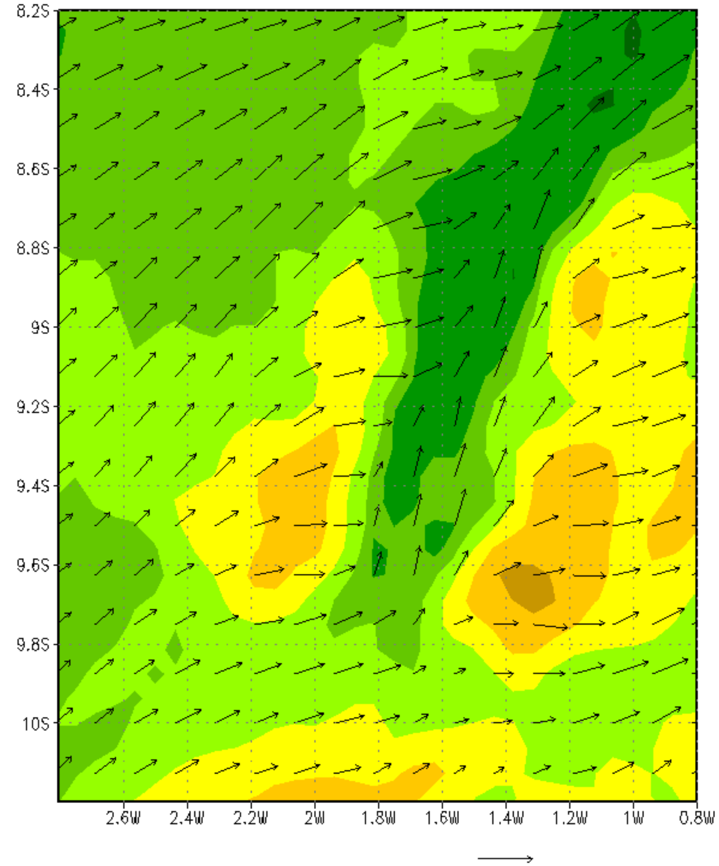
Day 7.5.2004, 06 UTC, Rhine-valley (SW-Germany)

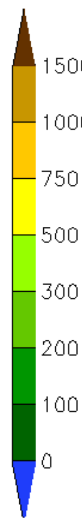
every 2. wind vector in 10 m above ground

LMK, dx=2.8 km, RK3+upstream 5. order



LM, dx=7 km, Leapfrog





LMK (Lokal Modell Kürzestfrist) - Development of a kilometer-scale NWP system for very short range forecasts (2h - 18h)

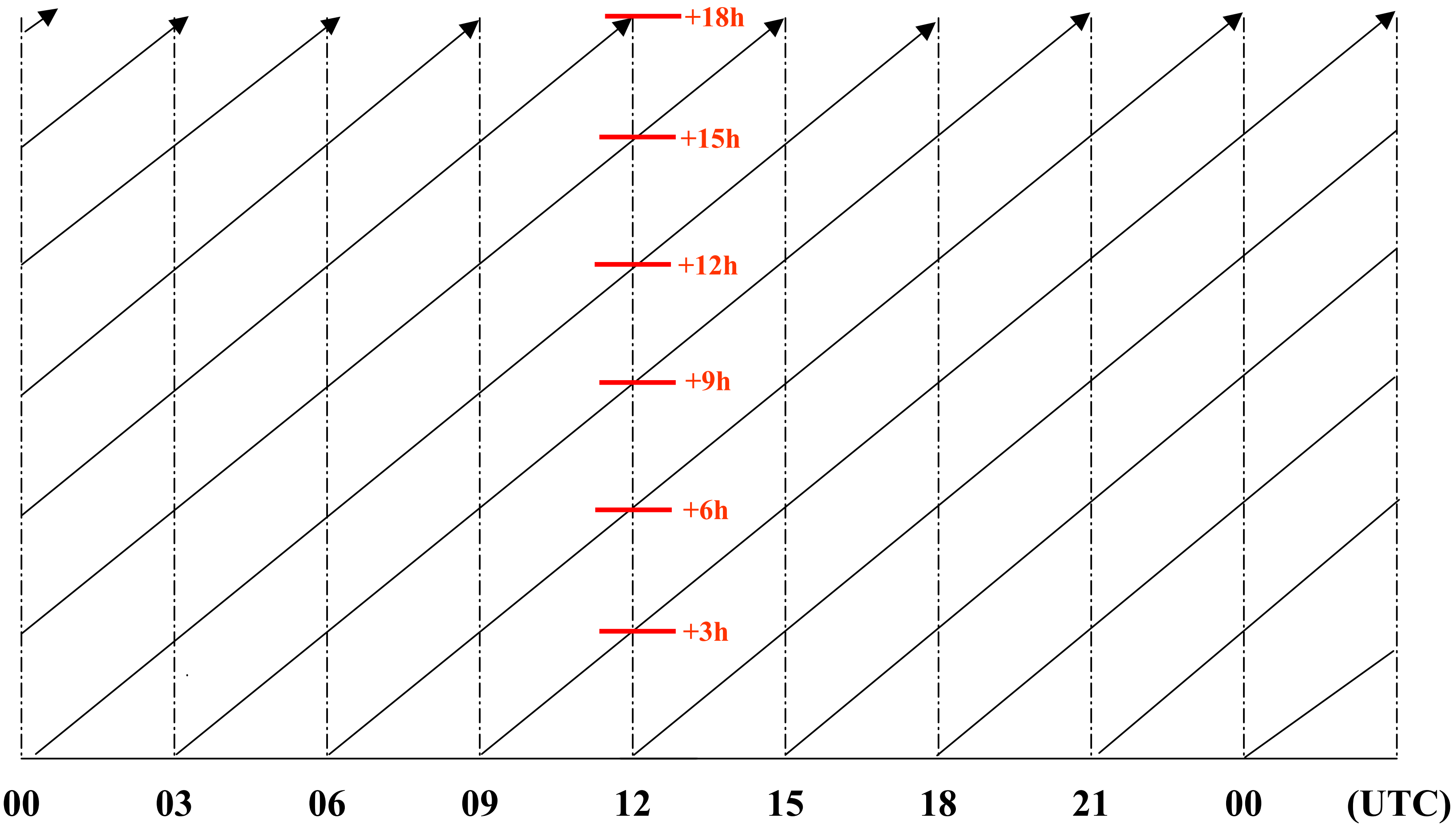
Goals

- Model-based prediction of severe weather events on the meso-gamma scale, especially those related to deep moist convection or to interactions with fine-scale topography
  - super- and multi-cell thunderstorms, squall-lines, rainbands,...
  - severe downslope winds (Föhn-storms), flash floodings, fog, ...

Method

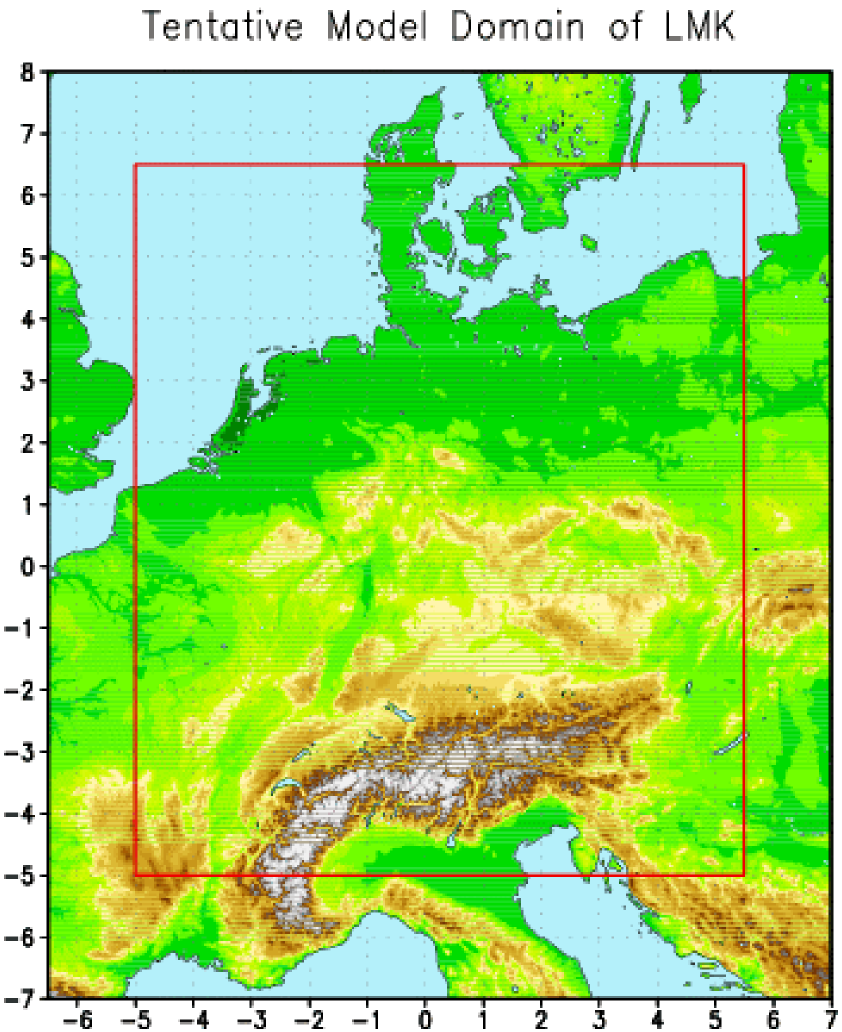
- Application of the LM at a grid-spacing < 3km with about 50 layers allowing for a direct simulation of deep convection
- 18-h forecasts every 3-hours from a rapid-update data assimilation cycle
- Continuous data assimilation based on nudging, short cut-off (< 1h)
- Use of all available data, especially radar reflectivities and winds.

LMK Assimilation and forecast cycle (LAF-Ensemble)



LMK: new forecast every 3 hours

LMK Testsuites



- 2.8 km grid spacing
- 421 x 461 grid points
- 50 layers
- currently 30 sec. time step

Mode configuration as LM, but:
- convection scheme switched off
- no data assimilation

Since 18 Dec 2003 two LMK forecasts per day,
00 UTC, 12 UTC (+18h)

continuous upgrade by new model components

operational use planned end of 2006

Potential benefits from an explicit representation of convection

- no drawbacks from the use of parameterized convection (artificial closure conditions, stationarity, ...)
- more detailed representation of cloud-microphysics
- time evolution (life-cycle) of convective cells is simulated
- transport of convective cells with the wind field (impact of shear)
- formation of new cells along gust-fronts (long-lasting cells, super- and multi-cells development)
- thermodynamic interaction between updraft und downdraft
- scale-interaction and organisation (Squall-Lines)

Related projects:

AROME, UK MetOffice, HIRLAM, WRF (BAMEX)

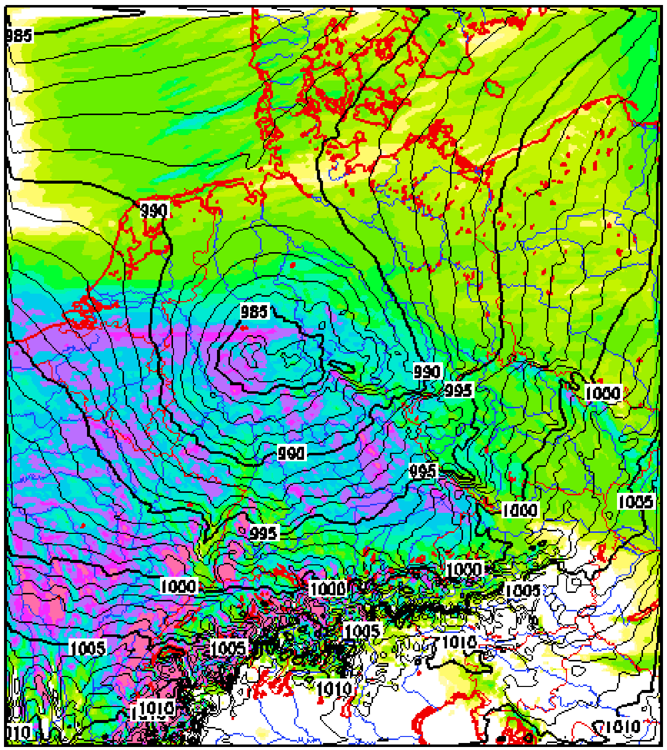
LMK will require not only adjustments of existing schemes but also the development of new components

- Provision of 5-min radar composits (DX-data) including quality control
- Further developments in the method to assimilate radar data (Latent heat nudging, LHN)
- Development of an efficient time-integration scheme allowing for high-order spatial discretization
- Development of new physics schemes for kilometre-scale simulations (graupel and hail microphysics, 3-d turbulence, shallow convection)
- Development of new tools for model validation and verification using remote-sensing data (pseudo-satellite and pseudo-radar products, Lidar, cloud-radars)
- Development of products and methods to interpret model results from deterministic forecasts (on stochastic scales) and to estimate the predictability (probabilistic products)

Deutscher Wetterdienst

LMK 12. JAN 2004 18 UTC

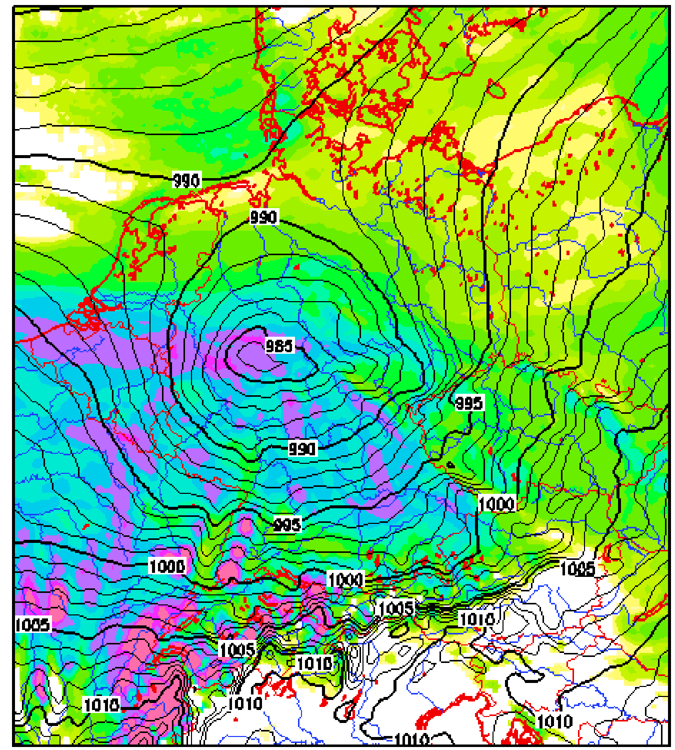
(1) 3h PRECIPITATION (>0.1mm) (2) PMSL



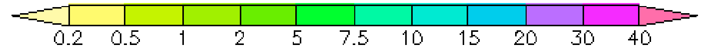
(1) Mean: 7.688 Min: 0 Max: 367.3 Var: 162.6
(2) Mean: 996.5 Min: 982.3 Max: 1015. Var: 0

LM 12. JAN 2004 18 UTC

(1) 3h PRECIPITATION (>0.1mm) (2) PMSL



(1) Mean: 6.974 Min: 0 Max: 187.6 Var: 118.5
(2) Mean: 997.1 Min: 983.3 Max: 1014. Var: 67.23



Contributions by and thanks to

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Detlev Majewski
Thorsten Reinhardt
Jochen Förstner
Uli Schättler
Bernd Dietzer
Thomas Hanisch

the presented work was also partly done at
Forschungszentrum / Universität Karlsruhe (Prof. Fiedler)

Verification

BONIE (<u>Bo</u>den<u>nie</u>derschlag)

• learning strategy derived from theory of artificial intelligence
• derivation of statistical properties of the spatial distribution patterns
• interpolation in analogy to Kriging-method

data base:

- measurements at the stations of DWD and AWGeophysBDBw
- additional about 100 ombrometer measurements in Baden-Württemberg

DWD, Geschäftsbereich VB/HM, Dr. T. Reich
Homepage: http://inet1.dwd.de/vb/hm/BONIE/index.htm

REGNIE (<u>Reg</u>ionalisierung räumlicher <u>Nie</u>derschlagsverteilungen)

• use of regionalised, monthly averaged precipitation values (1961-1990)
• distance dependent interpolation (background field-method)

data base:

- about 600 stations in Germany

DWD, Geschäftsbereich VB/HM 1, Dr. B. Dietzer