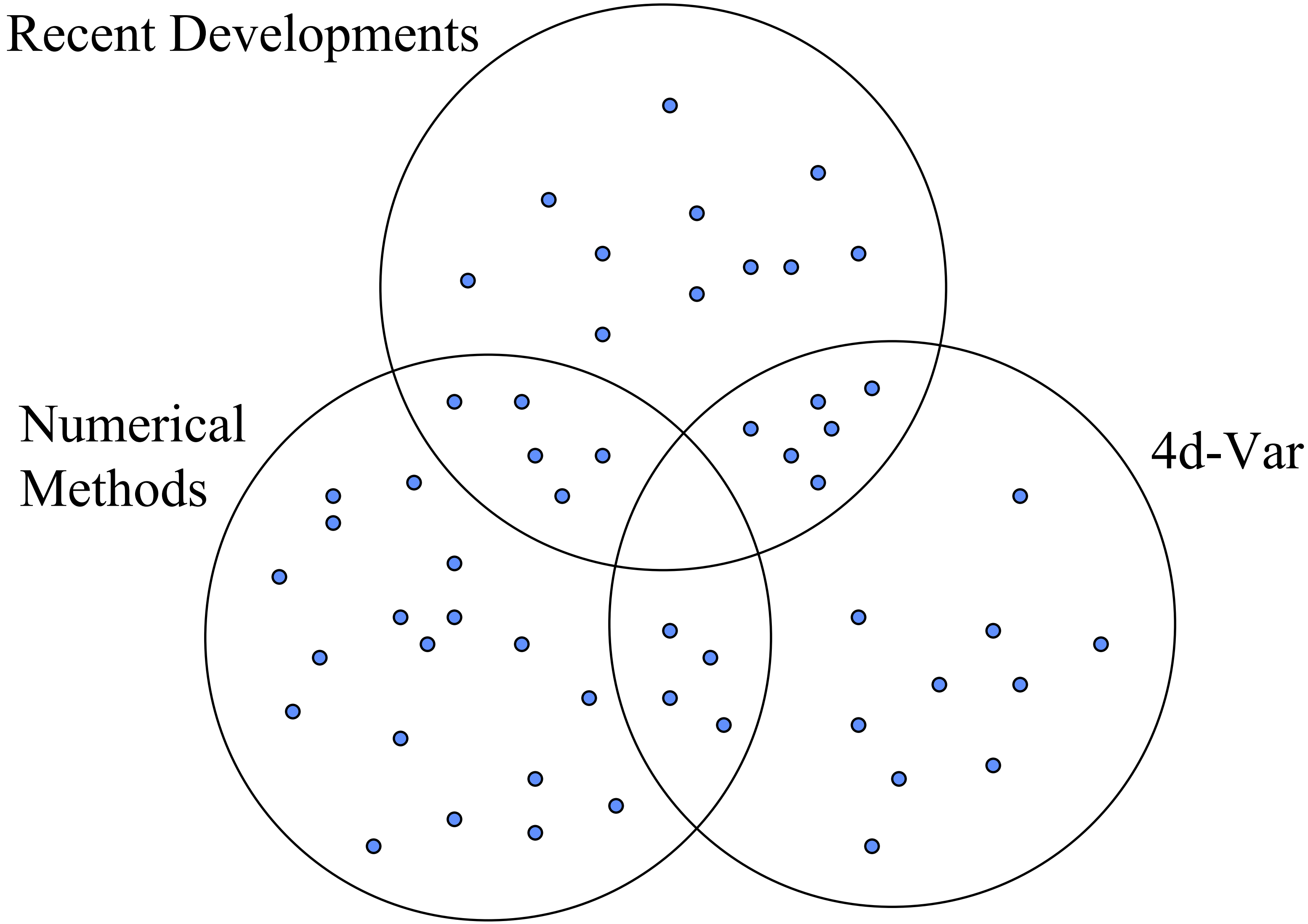
Recent Developments in Numerical Methods for 4d-Var

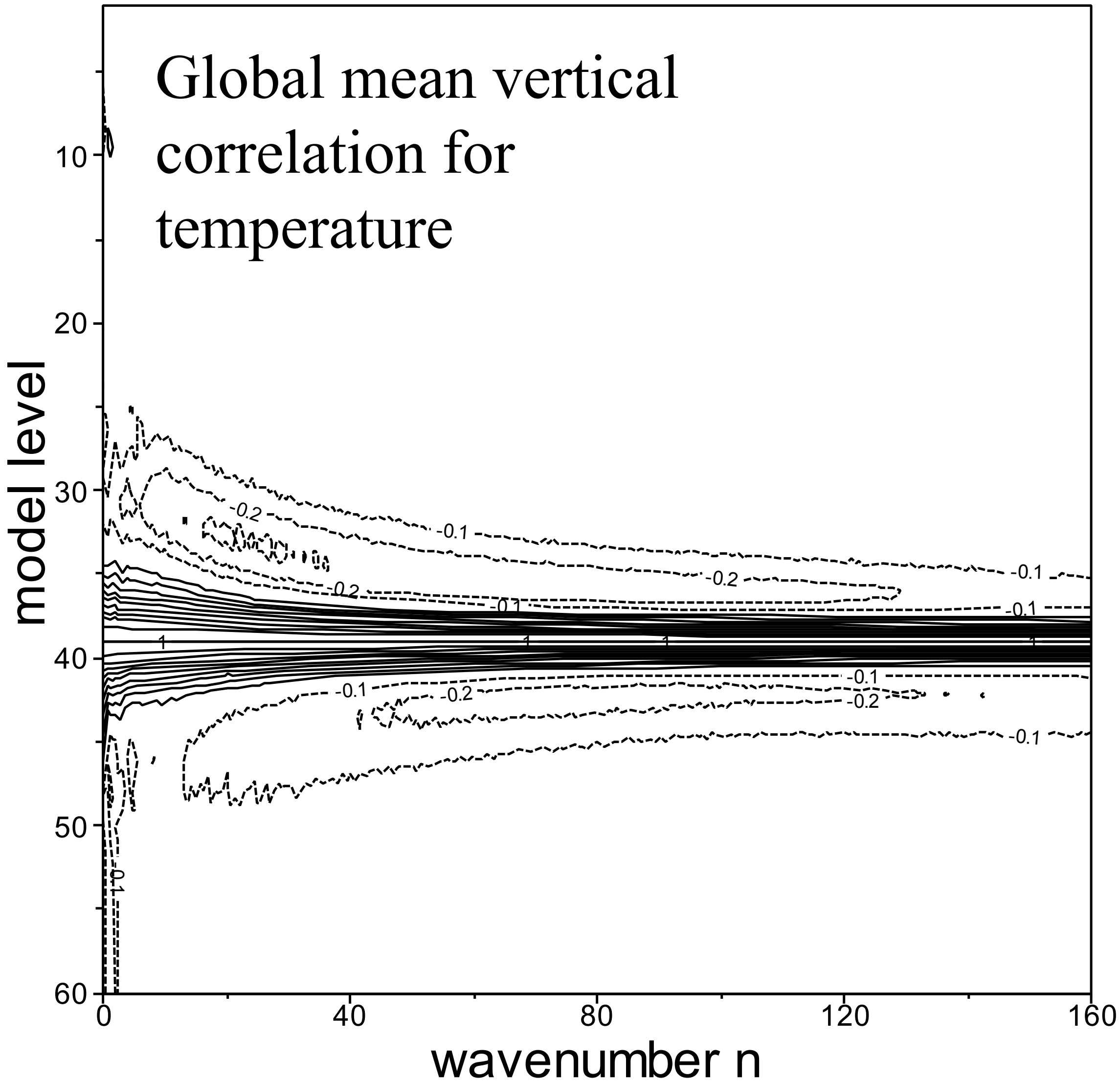
Mike Fisher


Recent Developments
Numerical Methods
4d-Var

Outline

- Non-orthogonal wavelets on the sphere:
  - Motivation: Covariance Modelling
  - Definition: Frames
  - Application: Wavelet J_b

Wavelets on the Sphere – Motivation



Vertical Correlations vary with horizontal spatial scale

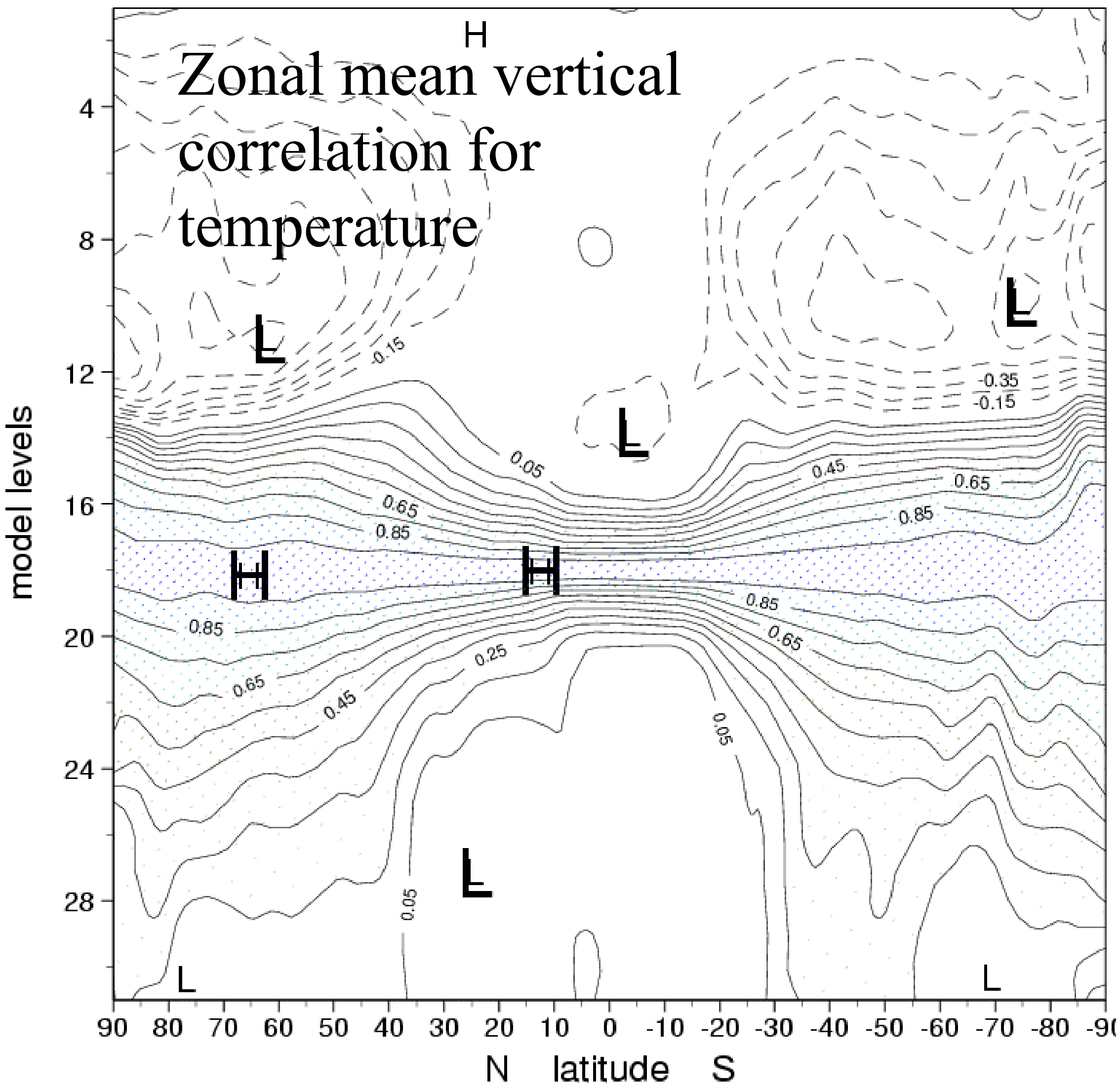
~200hPa

~500hPa

~850hPa

ECMWF

Wavelets on the Sphere – Motivation



~200hPa

~500hPa

~850hPa

Vertical Correlations vary with horizontal location.

Wavelets on the Sphere – Motivation

- **The variation of vertical correlation with location and with horizontal spatial scale are both important features, and both should be included in the covariance model.**
- **However, we are severely limited by the enormous size of the covariance matrix ($\sim 10^7 \times 10^7$).**
- **Essentially, the covariance matrix must be block-diagonal, with block size NLEVS×NLEVS, and with many identical blocks.**
- **Currently, we specify one block per wavenumber, *n*.**
  - variation with scale is modelled, variation with location is not.
- **Alternatively, we could specify one block per gridpoint.**
  - variation with location is modelled, variation with scale is not.
- **Wavelets provide a way to do both (and still keep things sparse).**

Wavelets on the Sphere – Frames

- It is possible to define orthogonal wavelets on the sphere by first gridding the sphere.
- Göttelmann (1997, citeseer.ist.psu.edu/227230.html) defined spherical wavelets using splines on a quasi-uniform latitude-longitude grid.
- Schröder and Sweldens (1995, ACM SIGGRAPH, 161-172) defined them for a triangulation of the sphere.
- However, these approaches necessarily have special points (poles or vertices). They do not retain rotational symmetry for finite truncations of the wavelet expansion.
- If we wish to retain rotational symmetry, we must give up on orthogonality.
- I.e. we must consider <u>frames</u>.

Wavelets on the Sphere – Frames

- **Definitions**
  - A family of functions, { ψ_j ; $j \in J$} in a Hilbert space is called a <u>frame</u> if there exist A>0 and B<∞ such that for all f in the space:

$$A\|f\|^2 \le \sum_{j\in J} \left|\left\langle f, \psi_j \right\rangle\right|^2 \le B\|f\|^2$$

  - The condition is sufficient to ensure the existence of a <u>dual frame</u>, { $\tilde{\psi}_j$; $j \in J$} with the property:

$$\frac{1}{A}\sum_{j\in J}\left\langle f, \psi_j \right\rangle \tilde{\psi}_j = f = \frac{1}{A}\sum_{j\in J}\left\langle f, \tilde{\psi}_j \right\rangle \psi_j$$

  - A particularly interesting case occurs when A=B. This is called a <u>tight frame</u>. Tight frames are self-dual:

$$f = \frac{1}{A}\sum_{j\in J}\left\langle f, \psi_j \right\rangle \psi_j$$

Wavelets on the Sphere – Frames

- **Tight frames share many of the properties of orthogonal bases. (An orthogonal basis is a tight frame with** $\left\|\psi_j\right\|^2 = 1$ **and A=1).**
- **Tight frames define a "transform", since we may write:**

$$f = \frac{1}{A} \sum_{j \in J} \left\langle f, \psi_j \right\rangle \psi_j$$

as: $c_j = \left\langle f, \psi_j \right\rangle \quad , \quad f = \frac{1}{A} \sum_{j \in J} c_j \psi_j$

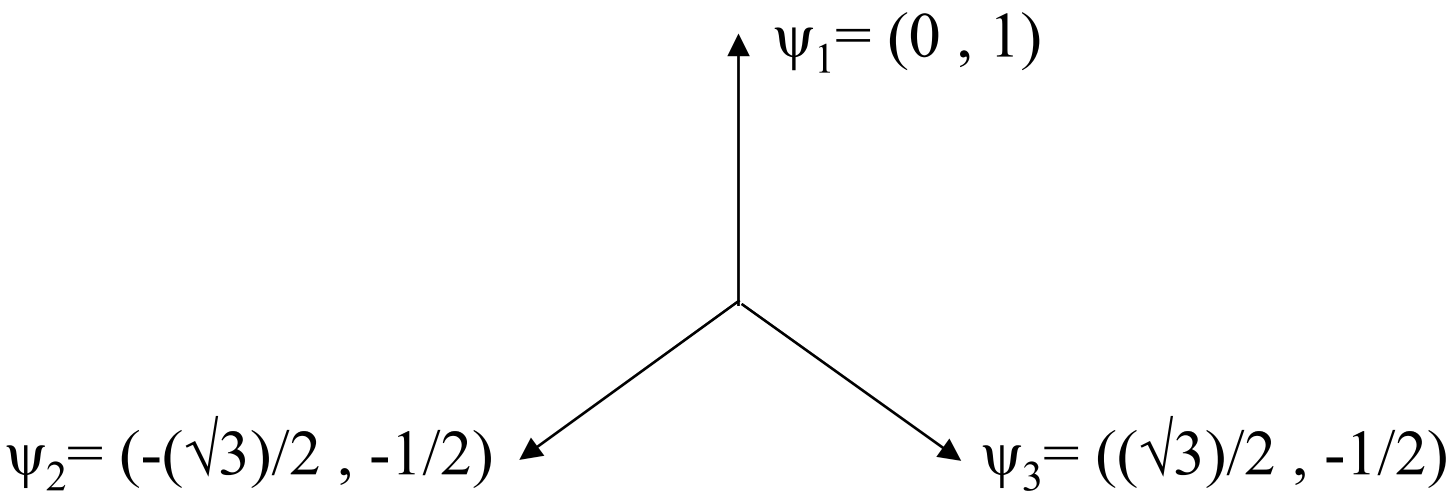
- **c.f. Fourier series:**

$$\hat{f}_m = \left\langle f, e^{2\pi imt/(b-a)} \right\rangle = \frac{1}{b-a} \int_a^b f(t)\, e^{-2\pi imt/(b-a)}\, \mathrm{d}t$$

$$f(t) = \sum_{m=-\infty}^{\infty} \hat{f}_m e^{2\pi imt/(b-a)}$$

Wavelets on the Sphere – Frames

- **Example: The Mercedes-Benz Frame**
  - **Daubechies, 1992: "Ten Lectures on Wavelets"**



- **Tight frame:** $$\sum_{j=1}^{3}\left|\left\langle f, \psi_j \right\rangle\right|^2 = \left|f_y\right|^2 + \left|-\frac{\sqrt{3}}{2} f_x - \frac{1}{2} f_y\right|^2 + \left|\frac{\sqrt{3}}{2} f_x - \frac{1}{2} f_y\right|^2 = \frac{3}{2}\left|f\right|^2$$

- **Hence, for any f:** $$\frac{2}{3}\sum_{j=1}^{3}\left\langle f, \psi_j \right\rangle \psi_j = f$$

Wavelets on the Sphere – Frames

- Example: The discrete spherical transform.
- Consider functions $\hat{f}(m,n)$ where 0≤n≤N, -n≤m≤n.
- Let $\hat{\psi}_j(m,n)$ be the (m,n)th spherical harmonic, evaluated at the jth gridpoint of the Gaussian grid, and multiplied by sqrt(Gaussian integration weight):

$$\hat{\psi}_j(m,n) = \sqrt{w(\phi_j)}Y_{m,n}(\lambda_j,\phi_j)$$

- Then: $\left\langle \hat{f},\hat{\psi}_j \right\rangle = \sum_{m,n} \hat{f}(m,n)\sqrt{w(\phi_j)}Y_{m,n}(\lambda_j,\phi_j) = \sqrt{w_j(\phi_j)}f(\lambda_j,\phi_j)$

$$\hat{f}(m,n) = \sum_j \left\langle f,\hat{\psi}_j \right\rangle \hat{\psi}_j(m,n) = \sum_j w_j(\phi_j)f(\lambda_j,\phi_j)Y_{m,n}(\lambda_j,\phi_j)$$

- NB: This is a tight frame, not an orthogonal transform. There are more gridpoints than spectral coefficients.

Wavelets on the Sphere – Generalized Frames

- The concept of a frame can be generalized to the case where the number of basis functions, ψ_j is uncountable.
- The sum in the frame condition becomes an integral:

$$A\|f\|^2 \le \int_D \left|\langle f, \psi_x \rangle\right|^2 \mathrm{d}x \le B\|f\|^2$$

- Most of the properties carry over from the discrete case.
- In particular, for a tight frame, we have:

$$f = \frac{1}{A}\int_D \langle f, \psi_x \rangle \psi_x \, \mathrm{d}x$$

  - See Kaiser, 1994 “A Friendly Guide to Wavelets”

Wavelets on the Sphere – Generalized Frames

- **We will consider a specific, semi-discrete case.**
- **The basis functions, $\psi_{j,\lambda,\phi}$ are labelled by 3 indices:**
  - j **(discrete) indicates "scale"**
  - λ **(continuous) is longitude**
  - ϕ **(continuous) is latitude**

- **We define:**

$$\psi_{j,\lambda,\phi}(\lambda',\phi') = \Psi_j(r(\lambda',\phi',\lambda,\phi))$$

  - **where** $r(\lambda',\phi',\lambda,\phi)$ **is the great-circle distance between** (λ',ϕ') **and** (λ,ϕ)**.**

Wavelets on the Sphere – Generalized Frames

- **The inner product is:**

$$\left\langle f,\psi_{j,\lambda,\phi}\right\rangle = \int_{\Omega} f(\lambda',\phi')\psi_{j,\lambda,\phi}(\lambda',\phi')\cos(\phi')\mathrm{d}\lambda'\mathrm{d}\phi'$$

- **Let us write:**

$$f_j(\lambda,\phi) = \left\langle f,\psi_{j,\lambda,\phi}\right\rangle$$

- **Then, substituting for $\psi_{j,\lambda,\phi}(\lambda',\phi')$ we see that the inner product corresponds to a convolution on the sphere:**

$$f_j(\lambda,\phi) = \int_{\Omega} f(\lambda',\phi')\Psi_j(r(\lambda',\phi',\lambda,\phi))\cos(\phi')\mathrm{d}\lambda'\mathrm{d}\phi'$$

I.e. $\quad f_j = f \otimes \Psi_j$

Wavelets on the Sphere – Generalized Frames

- **We seek a tight frame. The condition is:**

$$\sum_j \int_\Omega \left| \left\langle f , \psi_{j,\lambda,\phi} \right\rangle \right|^2 \cos(\phi)\mathrm{d}\lambda\mathrm{d}\phi = A\left\| f \right\|^2$$

- **That is:**

$$\sum_j \int_\Omega \left| f_j(\lambda,\phi) \right|^2 \cos(\phi)\mathrm{d}\lambda\mathrm{d}\phi = A\left\| f \right\|^2$$

- **I.e.**

$$\sum_j \left\| f_j \right\|^2 = A\left\| f \right\|^2$$

- **Evaluating the norms in terms of spherical harmonic coefficients, we have:**

$$\sum_{j,m,n} \left| \hat{f}_j(m,n) \right|^2 = A\sum_{m,n} \left| \hat{f}(m,n) \right|^2$$

Wavelets on the Sphere – Generalized Frames

$$\sum_{j,m,n} \left| \hat{f}_j(m,n) \right|^2 = A \sum_{m,n} \left| \hat{f}(m,n) \right|^2$$

- **But, remember that** $f_j = f \otimes \Psi_j$ **, where** Ψ_j **is a function of great-circle distance.**

- **Hence:** $\hat{f}_j(m,n) = \hat{f}(m,n)\hat{\Psi}_j(n)$

  - **where** $\hat{\Psi}_j(n)$ **are Legendre transform coefficients.**

- **The condition for a tight frame is thus:**

$$\sum_{j,m,n} \left| \hat{f}(m,n)\hat{\Psi}_j(n) \right|^2 = A \sum_{m,n} \left| \hat{f}(m,n) \right|^2$$

Wavelets on the Sphere – Generalized Frames

$$\sum_{j,m,n} \left| \hat{f}(m,n) \hat{\Psi}_j(n) \right|^2 = A \sum_{m,n} \left| \hat{f}(m,n) \right|^2$$

- I.e.

$$\sum_{m,n} \left(\sum_{j} \hat{\Psi}_j^2(n) \right) \left| \hat{f}(m,n) \right|^2 = A \sum_{m,n} \left| \hat{f}(m,n) \right|^2$$

$$\Rightarrow \sum_{j} \hat{\Psi}_j^2(n) = A$$

- For convenience, we will scale the basis functions appropriately, so that A=1:

$$\sum_{j} \hat{\Psi}_j^2(n) = 1 \quad \forall n$$

Wavelets on the Sphere – Generalized Frames

- If we have a tight frame, we have the transform property:

$$f = \sum_j \int_\Omega \langle f, \psi_{j,\lambda,\phi} \rangle \psi_{j,\lambda,\phi} \cos(\phi')\mathrm{d}\lambda'\mathrm{d}\phi'$$

$$= \sum_j \int_\Omega f_j(\lambda',\phi')\Psi_j(r(\lambda',\phi',\lambda,\phi)) \cos(\phi')\mathrm{d}\lambda'\mathrm{d}\phi'$$

- But, the right hand side is just another convolution.
- Hence, the "transform pair" is just:

$$f_j = \Psi_j \otimes f$$

$$f = \sum_j \Psi_j \otimes f_j$$

Wavelets on the Sphere – Generalized Frames

$$f_j = \Psi_j \otimes f, \quad f = \sum_j \Psi_j \otimes f_j$$

- **The first equation defines f_j.**
- **We can easily verify the second equation:**

$$\left(\widehat{\sum_j \Psi_j \otimes f_j} \right)(m,n) = \sum_j \hat{\Psi}_j(n) \hat{f}_j(m,n)$$

$$= \sum_j \hat{\Psi}_j^2(n) f(m,n)$$

$$= \hat{f}(m,n)$$

Wavelets on the Sphere – Summary

- **A set of functions of great-circle distance, {** $\Psi_j(r)$**; j=0,1,2,...} whose Legendre transform coefficients satisfy:**

$$\sum_j \hat{\Psi}_j^2(n) = 1 \quad \forall n$$

define a tight generalized frame.

- **The functions define a "transform pair":**

$$f_j = \Psi_j \otimes f$$

$$f = \sum_j \Psi_j \otimes f_j$$

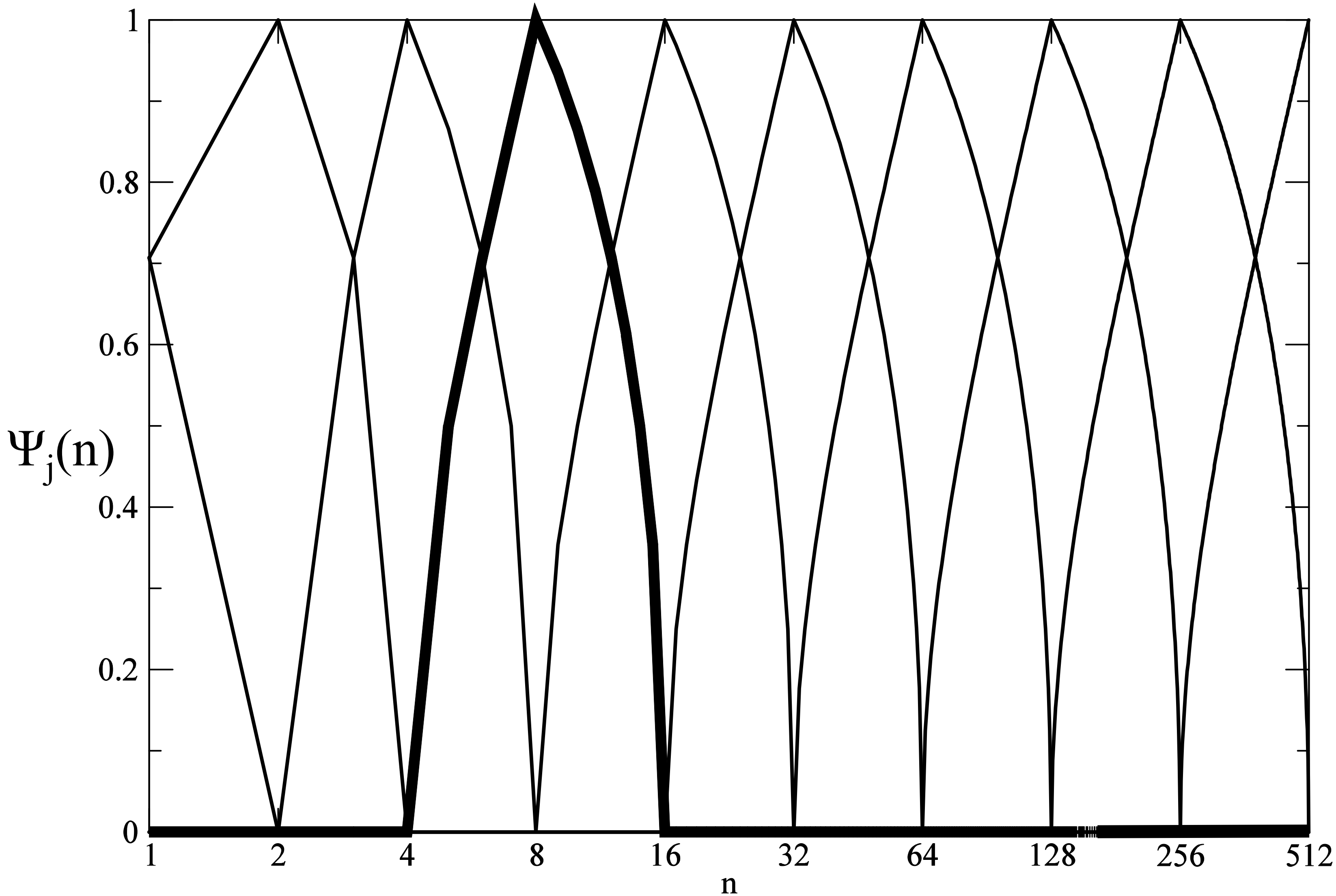
Wavelets on the Sphere – Example

- **The condition:** $\sum_j \hat{\Psi}_j^2(n) = 1 \quad \forall n$

  suggests we define the functions $\hat{\Psi}_j^2(n)$ **to be B-splines.**
- **For example, linear B-splines are triangle functions:**

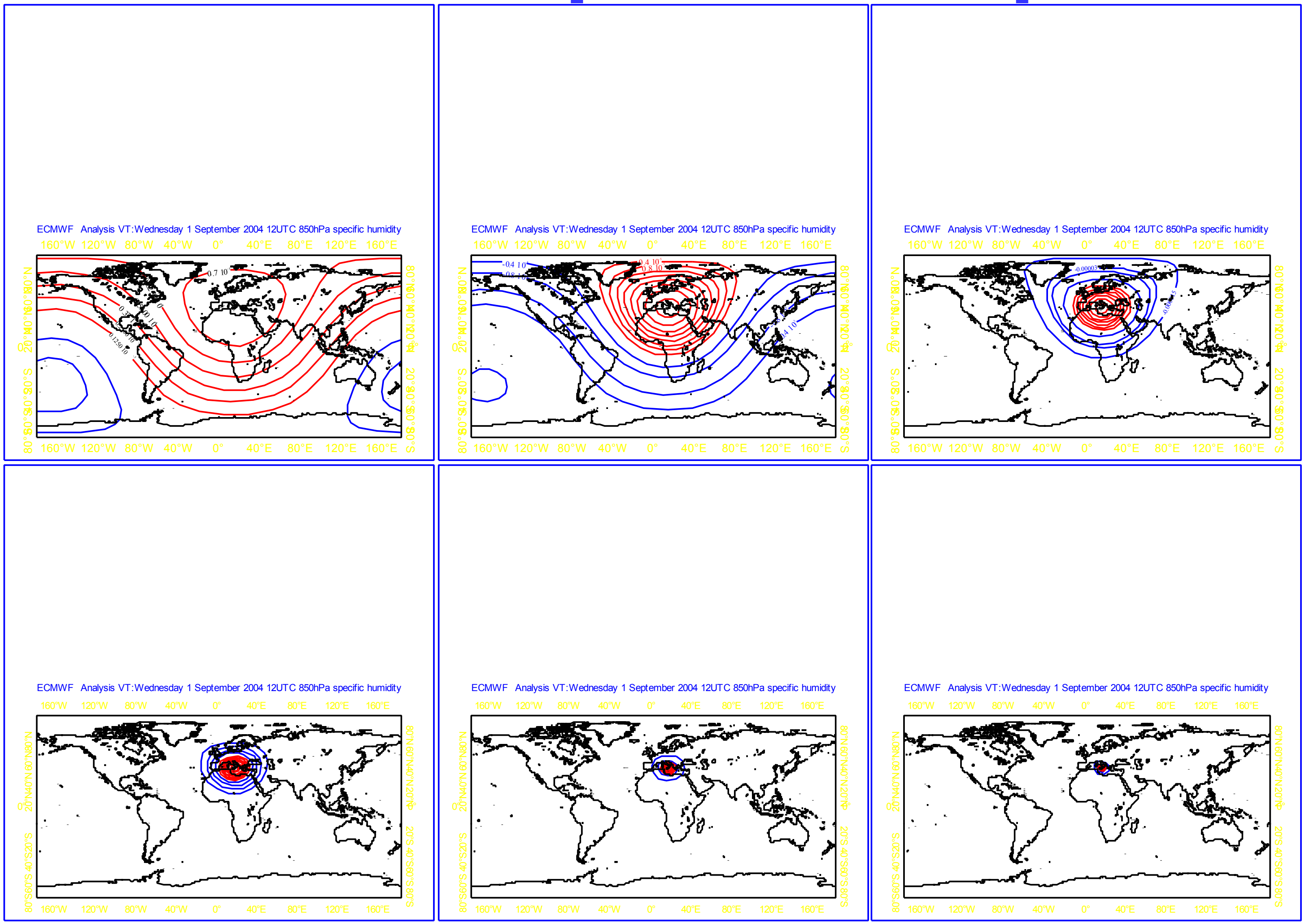
$$\hat{\Psi}_j^2(n) = \begin{cases} \dfrac{n - N_{j-1}}{N_j - N_{j-1}} & N_{j-1} < n \le N_j \\ \dfrac{n - N_{j+1}}{N_j - N_{j+1}} & N_j < n < N_{j+1} \\ 0 & \text{otherwise} \end{cases}$$

Wavelets on the Sphere – Example

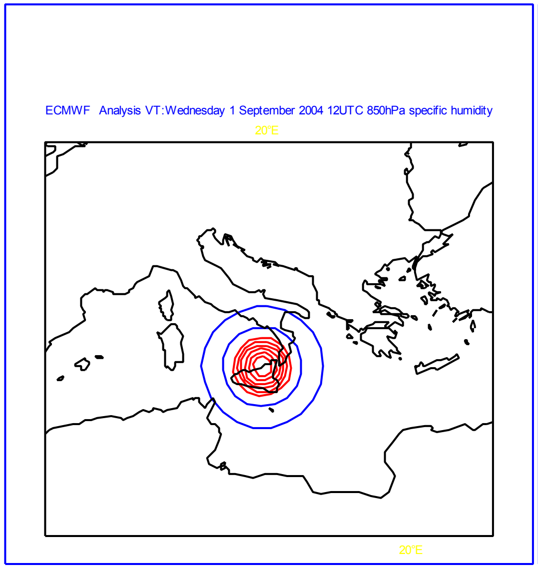


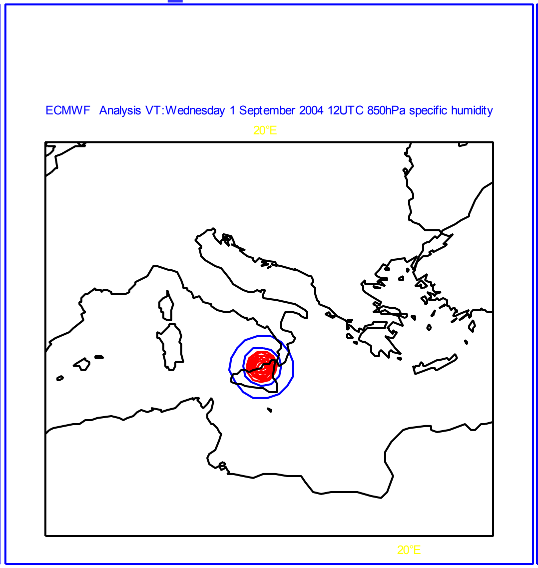
ECMWF

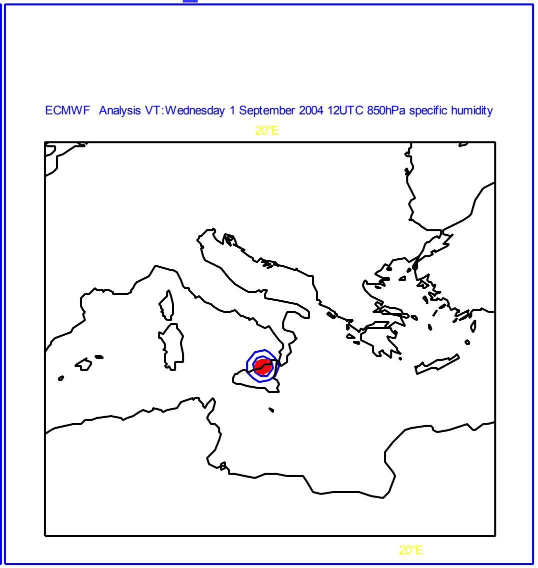
Wavelets on the Sphere – Example

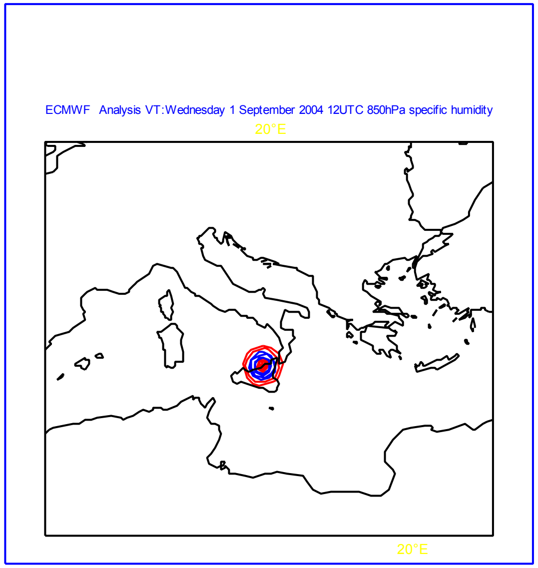


Wavelets on the Sphere – Example



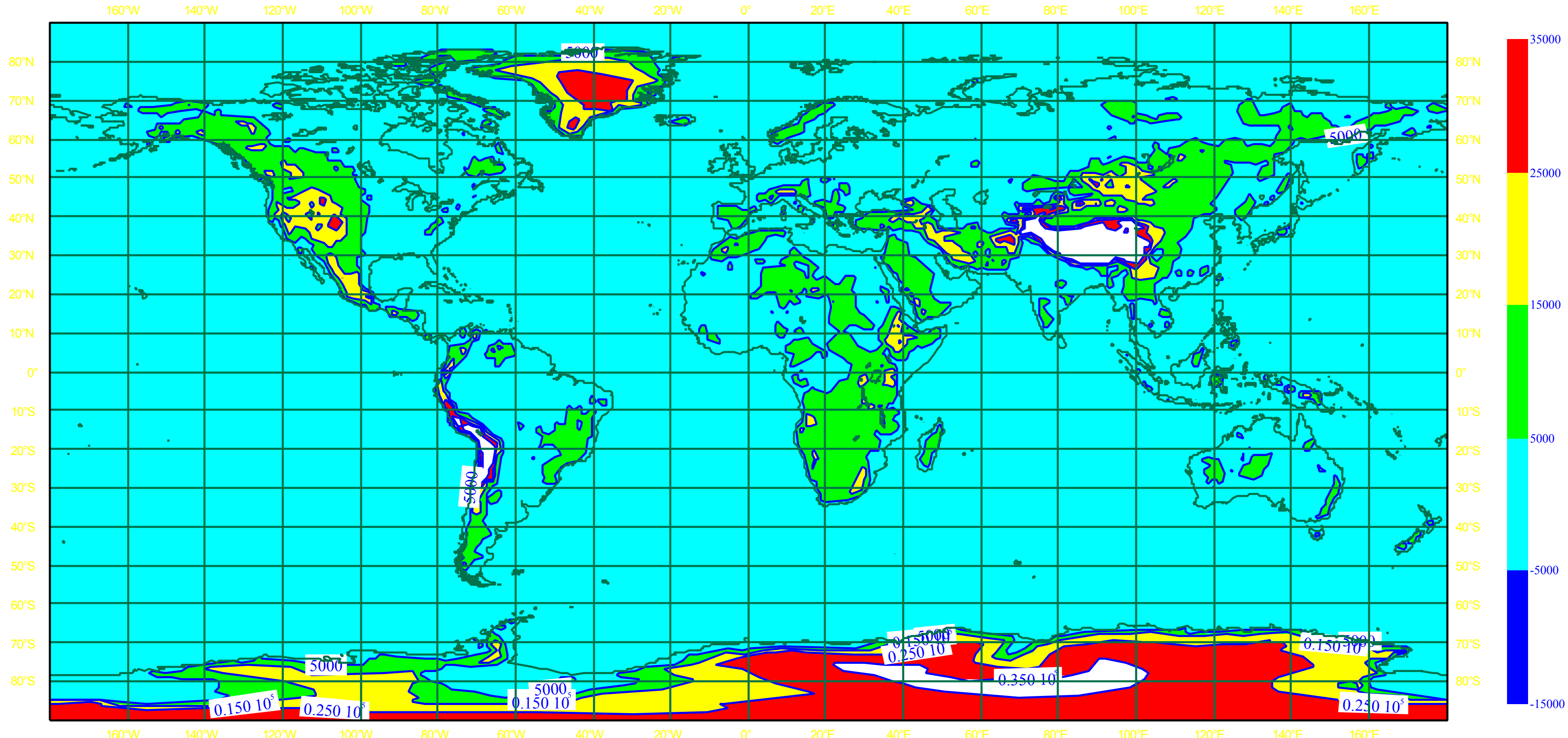






Spherical Wavelets – Example

- **T511 Model Orography:**

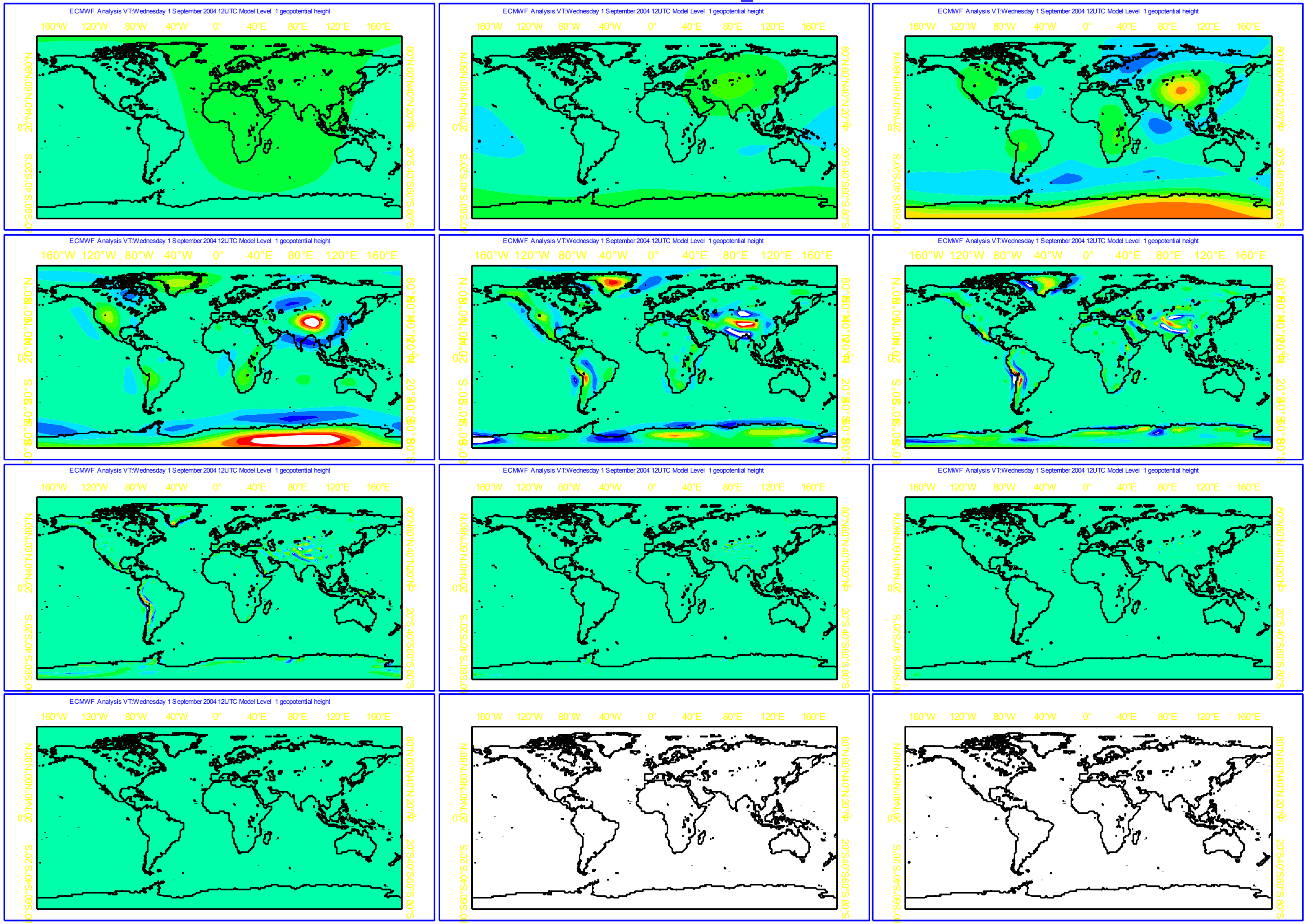


ECMWF

Spherical Wavelets – Example

Convolve with Ψ_j:

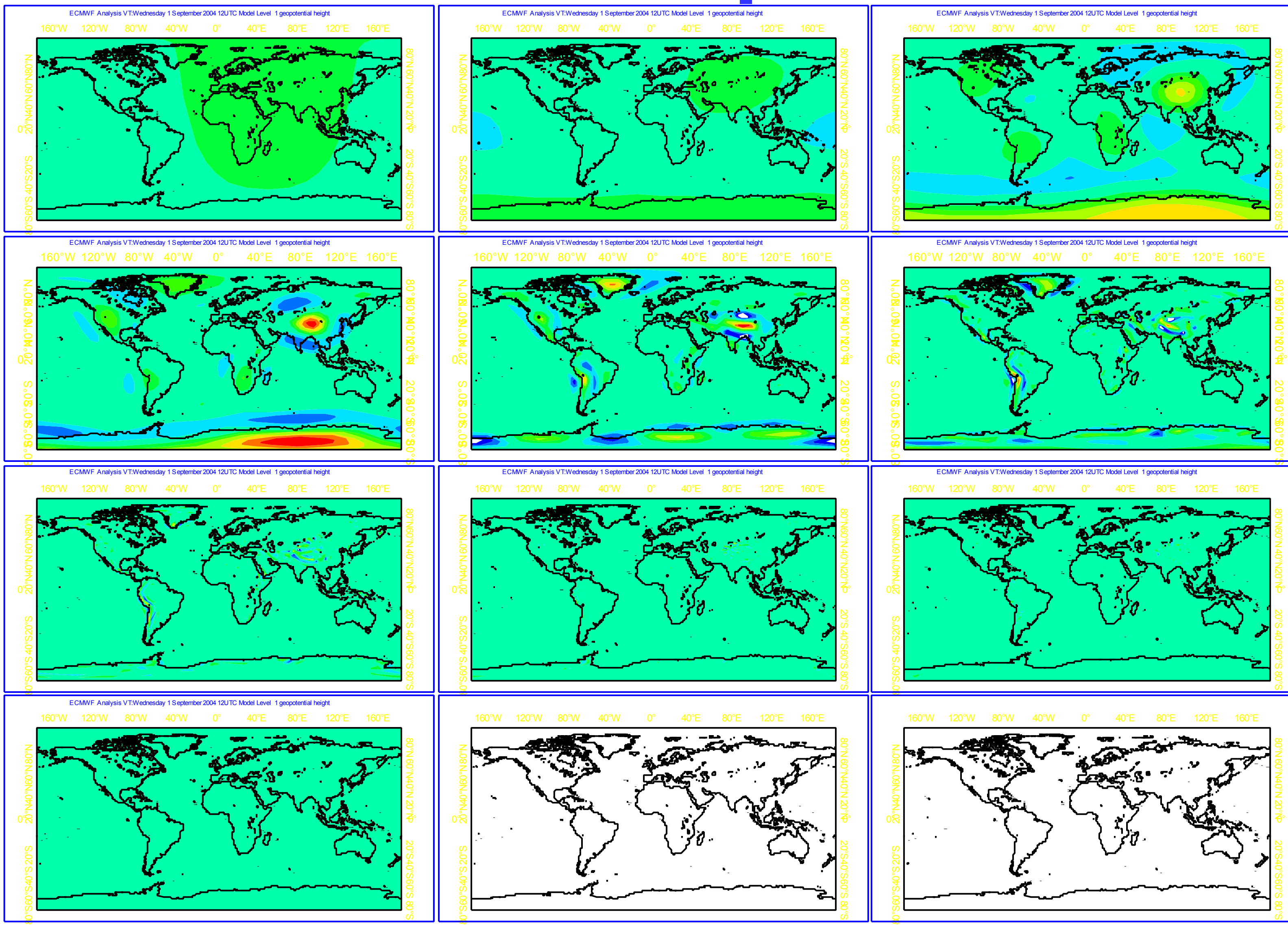
$$f_j = \Psi_j \otimes f$$



Spherical Wavelets – Example

Convolve again...

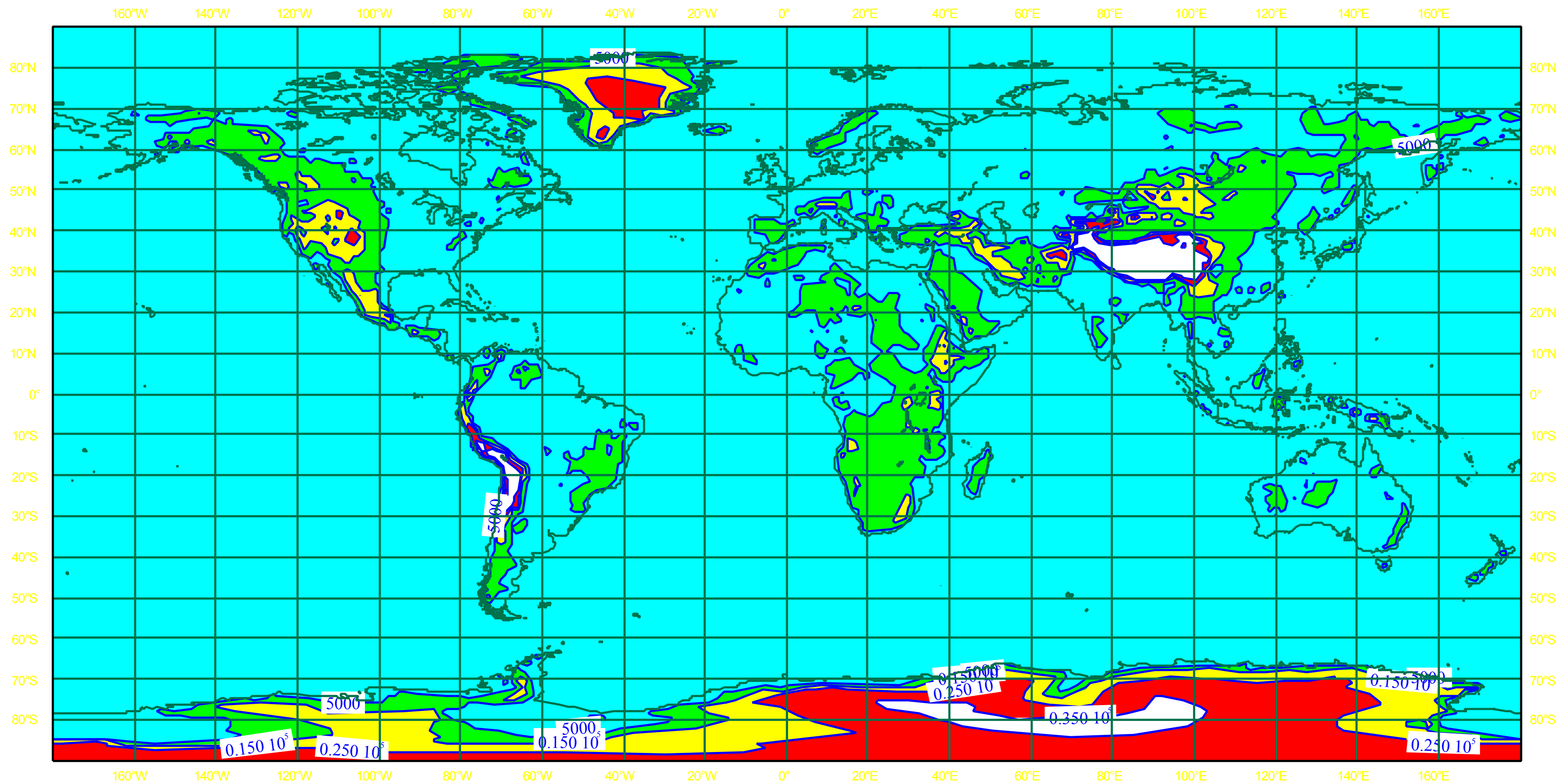
$$\Psi_j \otimes f_j$$



ECMWF

Spherical Wavelets – Example

- ... and add, to retrieve the original field: $f = \sum_j \Psi_j \otimes f_j$



Spherical Wavelets – Other Approaches

- The wavelets we have derived are similar to those of Freeden and Windheuser (1996, Adv. Comp. Math. 51-94.
  - They are a special case of the very broad range of spherical wavelet decompositions described by Freeden et al. (1998, “Constructive Approximation on the Sphere, OUP).
- A different approach, using group theory, is taken by Antoine and Vandergheynst (1999, Appl. and Comput. Harm. Anal. 262-291).
  - They define spherical wavelets as coherent states of the product group of rotations on the sphere, and dilations on the polar-stereographic tangent plane.
- Mhaskar et al. (2000, Adv. Comput. Math.) describe polynomial wavelet frames on the sphere.
  - and cite 11 papers, each defining a different approach to the construction of wavelets on the sphere.

Wavelet J_b

- So, how does all this help us formulate a covariance model?
- First, let's review the main idea behind the current J_b.
- 3d/4d-Var determine the analysis by minimizing a cost function:

$$J = \left(\mathbf{x}-\mathbf{x}_b\right)^{\mathrm{T}} \mathbf{B}^{-1}\left(\mathbf{x}-\mathbf{x}_b\right) + \left(\mathbf{d}-\mathbf{H}(\mathbf{x}-\mathbf{x}_b)\right)^{\mathrm{T}} \mathbf{R}^{-1}\left(\mathbf{d}-\mathbf{H}(\mathbf{x}-\mathbf{x}_b)\right)$$

- Usually, we do not minimize directly in terms of x, but formulate the problem as:

$$J = \chi^{\mathrm{T}}\chi + \left(\mathbf{d}-\mathbf{HL}\chi\right)^{\mathrm{T}} \mathbf{R}^{-1}\left(\mathbf{d}-\mathbf{HL}\chi\right)$$

$$\mathbf{x} = \mathbf{x}_b + \mathbf{L}\chi$$

- NB: L defines the background covariance matrix: $B=LL^T$.

Wavelet J_b

- L defines the background covariance matrix: $B=LL^T$.
- To keep things simple, consider a 2d, univariate model.
- The current ECMWF covariance model boils down to:

$$\mathbf{L} = \mathbf{\Sigma D}$$

- Σ is diagonal in grid space, and corresponds to multiplying each gridpoint by a standard deviation.
  - It accounts for the spatial variation of background error.
- D is diagonal in spectral space, and corresponds to multiplying each wavenumber, n, by a standard deviation, which is a function of n, only.
  - It accounts for the variation of background error with scale.

Wavelet J_b

$$L = \Sigma D$$

- The current covariance model separates the spatial and spectral variation of background error.
- In particular, D corresponds to a convolution.
- D defines the horizontal correlation of background error for the covariance model.
- Because D is a convolution, the horizontal correlations are the same everywhere.
- This is a major shortcoming of the covariance model.

Wavelet J_b

- To define a covariance model using wavelets, note first that there is no requirement for L to be square.
- A rectangular matrix L still defines a valid covariance model, $B=LL^T$.
- We define the control variable as:

$$\chi = \begin{pmatrix} \chi_1 \\ \chi_2 \\ \vdots \end{pmatrix}$$

where the χ_j correspond to different scales.

- The change of variable matrix is defined by:

$$\mathbf{x} - \mathbf{x}_b = \mathbf{L}\chi = \sum_j \hat{\Psi}_j \otimes \mathbf{\Sigma}_j \chi_j$$

Wavelet J_b

$$\mathbf{x} - \mathbf{x}_b = \mathbf{L}\chi = \sum_j \hat{\Psi}_j \otimes \boldsymbol{\Sigma}_j \chi_j$$

- The matrices Σ_j are diagonal in grid space, and account for the spatial variation of background error <u>for each scale</u>.
- Let us write the convolution with $\hat{\Psi}_j$ explicitly as a matrix operator $\mathbf{S}^{-1}\hat{\boldsymbol{\Psi}}_j\mathbf{S}$, where S is the spherical transform, and $\hat{\boldsymbol{\Psi}}_j$ is diagonal.
- Using the symmetry of $\hat{\boldsymbol{\Psi}}_j$, and Σ_j, and the orthogonality of S, we can write the covariance matrix implied by L as:

$$\mathbf{B} = \mathbf{L}\mathbf{L}^{\mathrm{T}} = \sum_j \mathbf{S}^{-1}\boldsymbol{\Psi}_j\mathbf{S}\boldsymbol{\Sigma}_j^2\mathbf{S}^{-1}\boldsymbol{\Psi}_j\mathbf{S}$$

- We will illustrate the covariance structures generated by wavelet Jb by applying this matrix to delta functions.

Wavelet J_b

$$\mathbf{B} = \mathbf{L}\mathbf{L}^{\mathrm{T}} = \sum_j \mathbf{S}^{-1}\boldsymbol{\Psi}_j \mathbf{S}\boldsymbol{\Sigma}_j^2 \mathbf{S}^{-1}\boldsymbol{\Psi}_j \mathbf{S}$$

- **Consider the case where there is no spatial variation in the standard deviations: $\boldsymbol{\Sigma}_j = \sigma_j \mathbf{I}$.**
- **Then, $\boldsymbol{\Psi}_j \mathbf{S}\boldsymbol{\Sigma}_j^2 \mathbf{S}^{-1}\boldsymbol{\Psi}_j$ is diagonal, with elements $\sigma_j^2 \hat{\Psi}_j^2(n)$.**
- **But, $\sum_j \hat{\Psi}_j^2(n) = 1$, so $\sum_j \sigma_j^2 \hat{\Psi}_j^2(n)$ is a weighted average of σ_j's**
- **If we choose $\hat{\Psi}_j^2(n)$ to be B-splines, then the variation of variance with n is an interpolation between the prescribed σ_j's .**

Wavelet J_b

- **Suppose we want approximately Gaussian structure functions, with length scale that is a smoothly-varying function of latitude and longitude.**
- **Weaver+Courtier (2000) give the following expression for the modal variances corresponding to convolution with a quasi-Gaussian function with lengthscale *L*:**

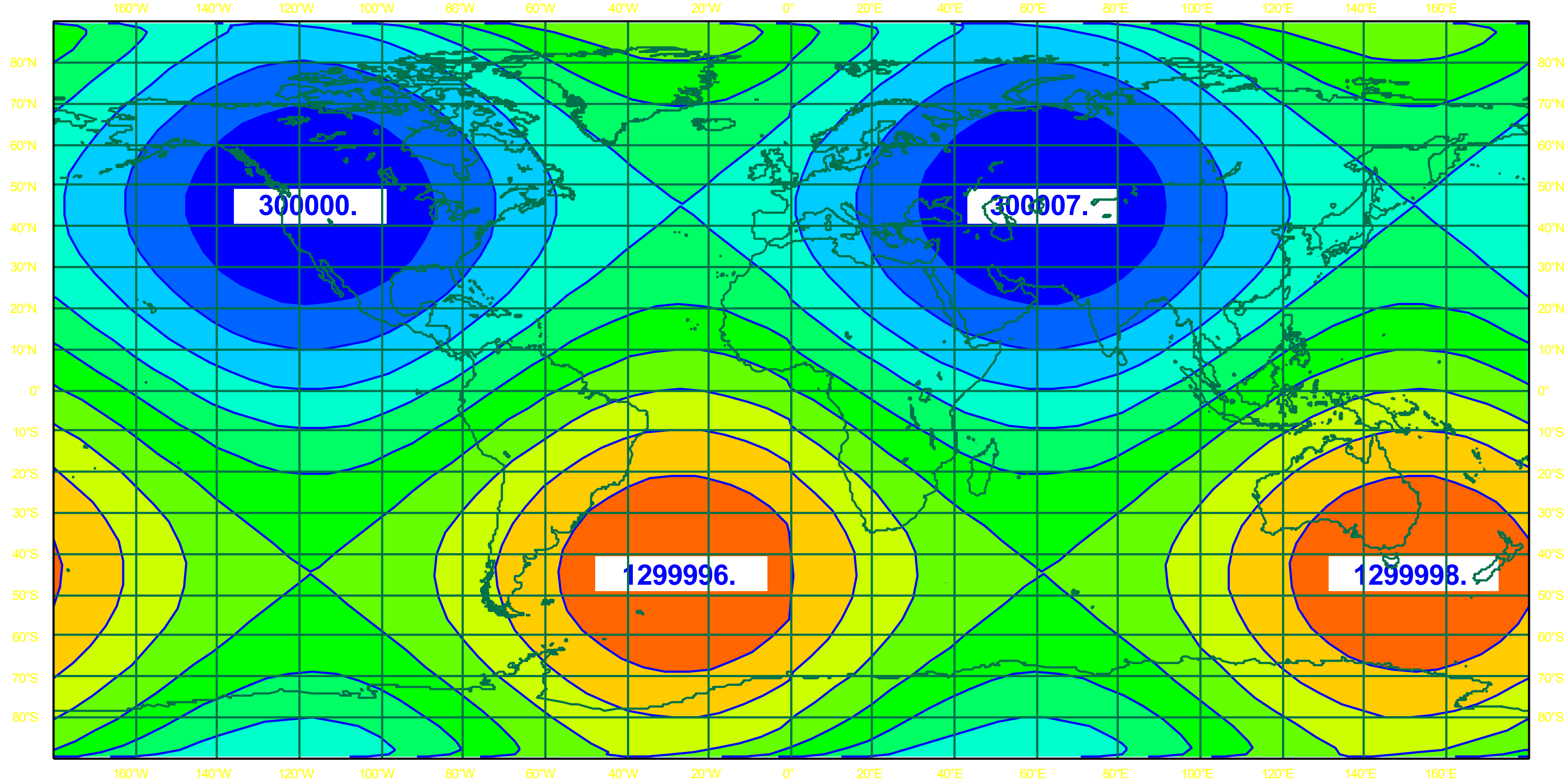
$$b^2(n;L) = \frac{\exp\left[-\left(L^2 n(n+1)\right)/2a^2\right]}{\sum_n (2n+1)\exp\left[-\left(L^2 n(n+1)\right)/2a^2\right]}$$

- **We simply set:** $\sigma_j = (N_{\max}+1)b(N_j;L)$

  but allow *L* (and hence σ_j) to vary with latitude and longitude.

Wavelet J_b

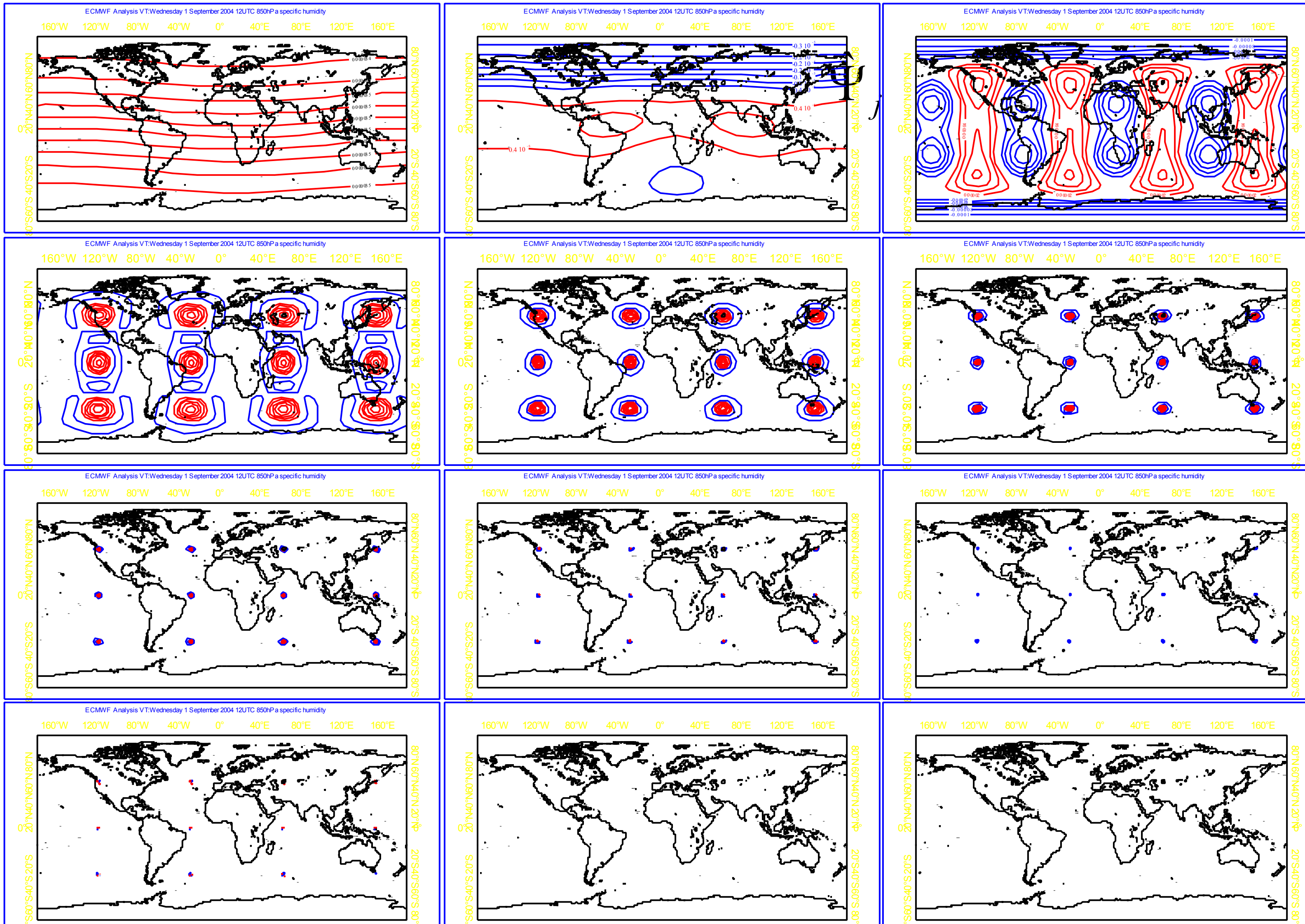
- **Desired Length scale (300km – 1300km):**



Wavelet J_b

$$\Psi_j \otimes \mathbf{x}$$

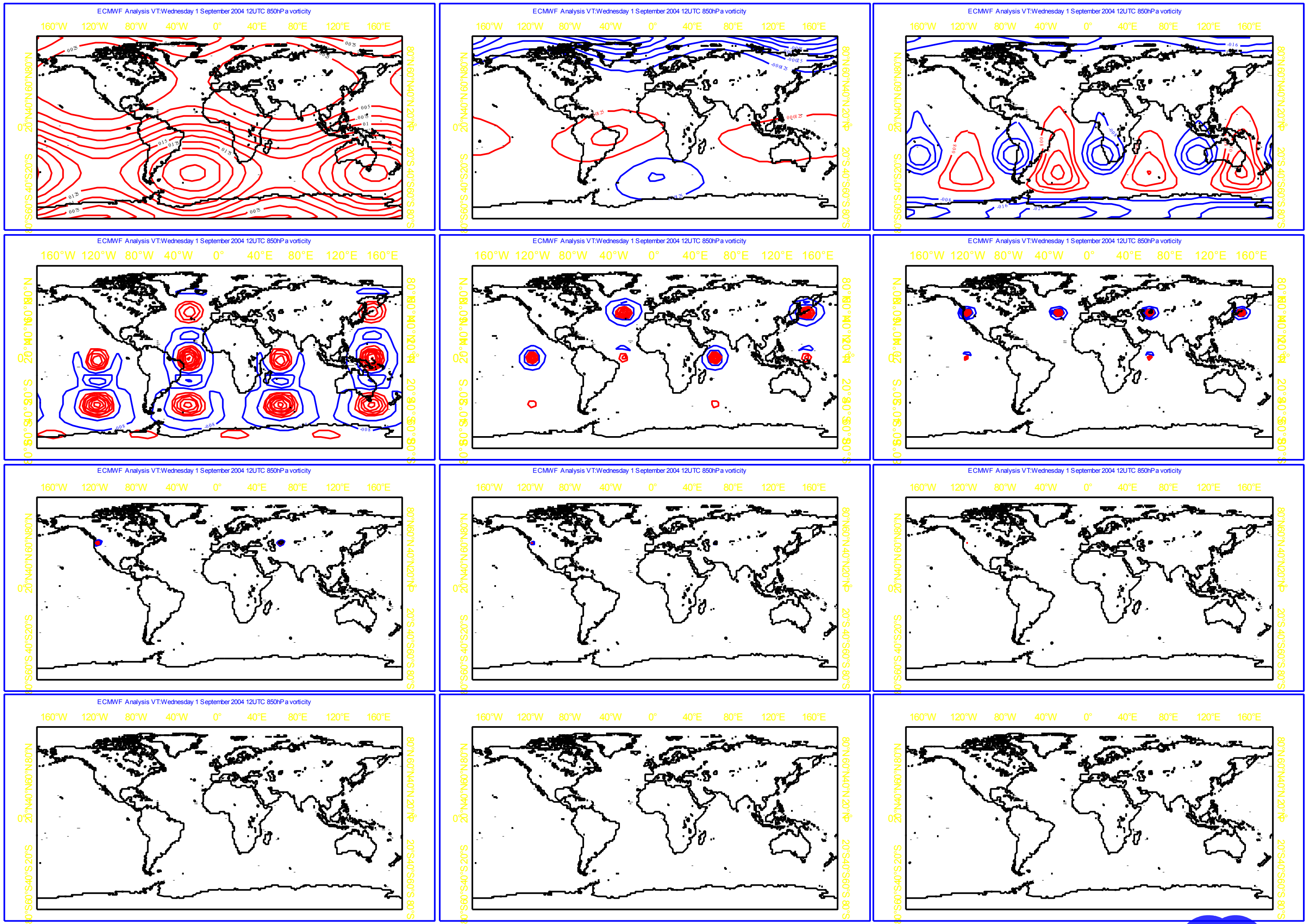
Where **x** is a set of delta functions



ECMWF

Wavelet J_b

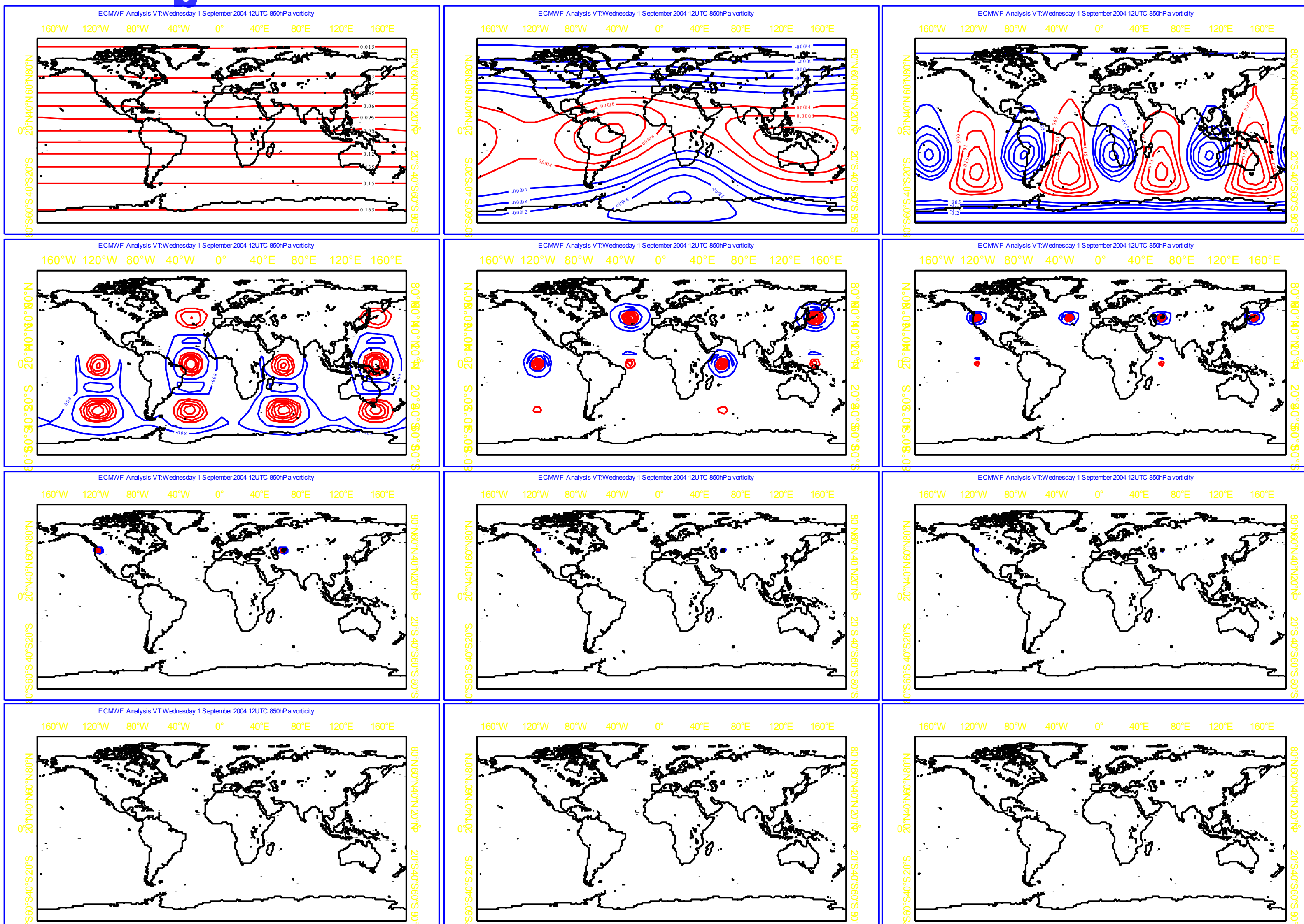
Scale with σ_j^2 at each gridpoint, and for each scale.



ECMWF

Wavelet J_b

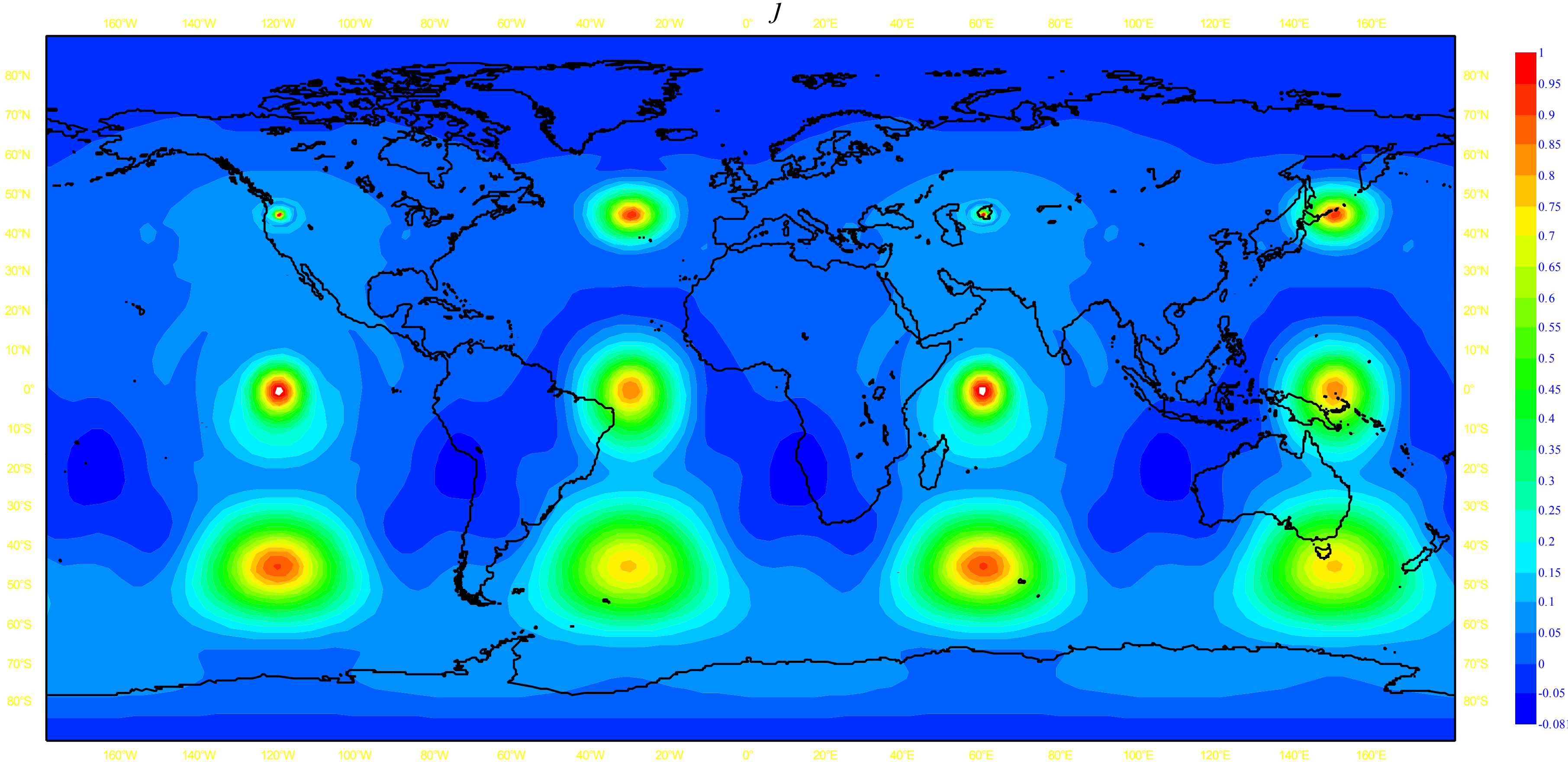
Convolve again with $\hat{\Psi}_j(n)$



ECMWF

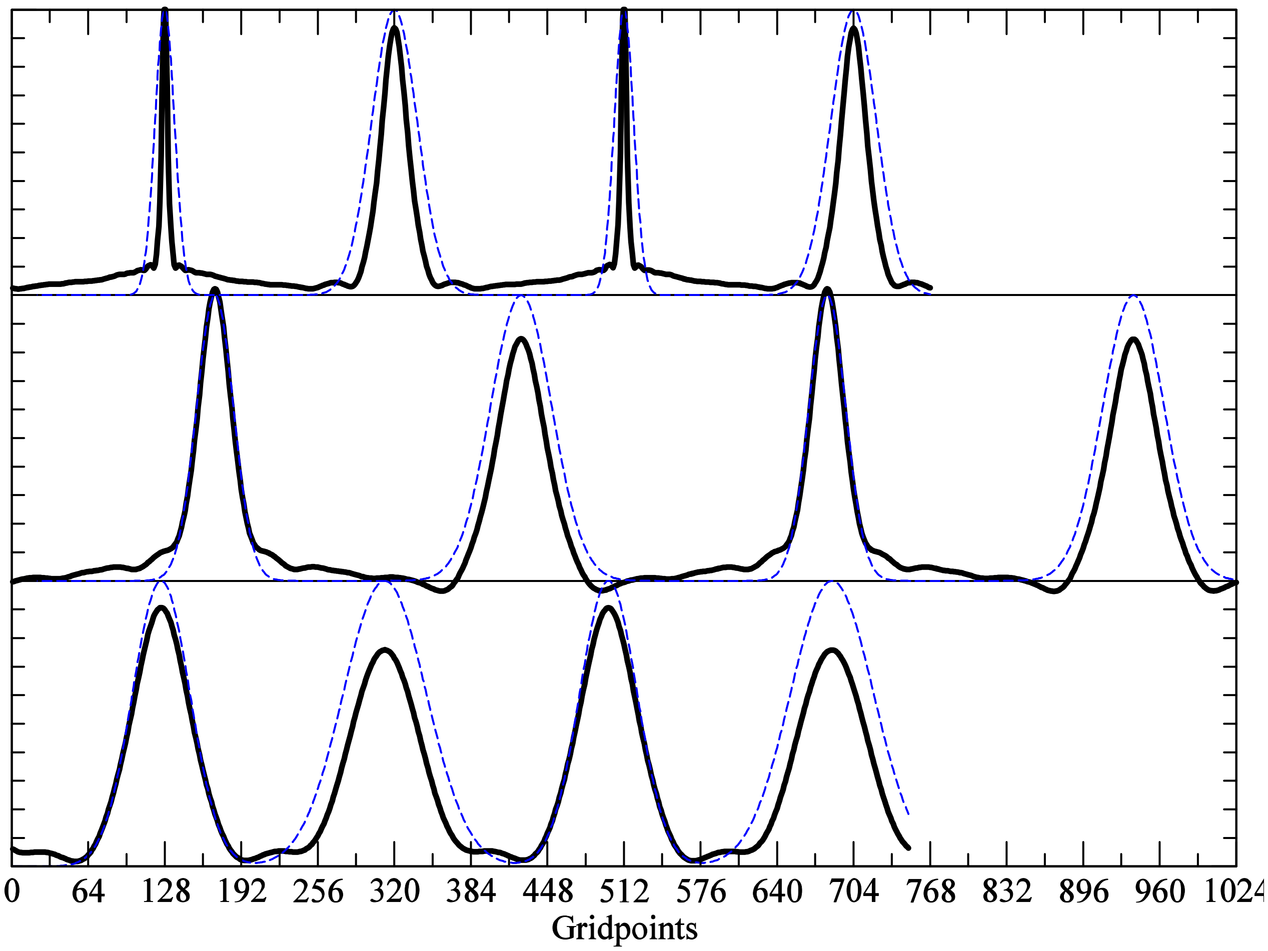
Wavelet J_b

- **Add together to give:** $\mathbf{Bx} = \sum_j \mathbf{S}^{-1}\mathbf{\Psi}_j \mathbf{S}\mathbf{\Sigma}_j^2 \mathbf{S}^{-1}\mathbf{\Psi}_j \mathbf{S}\mathbf{x}$



Wavelet J_b

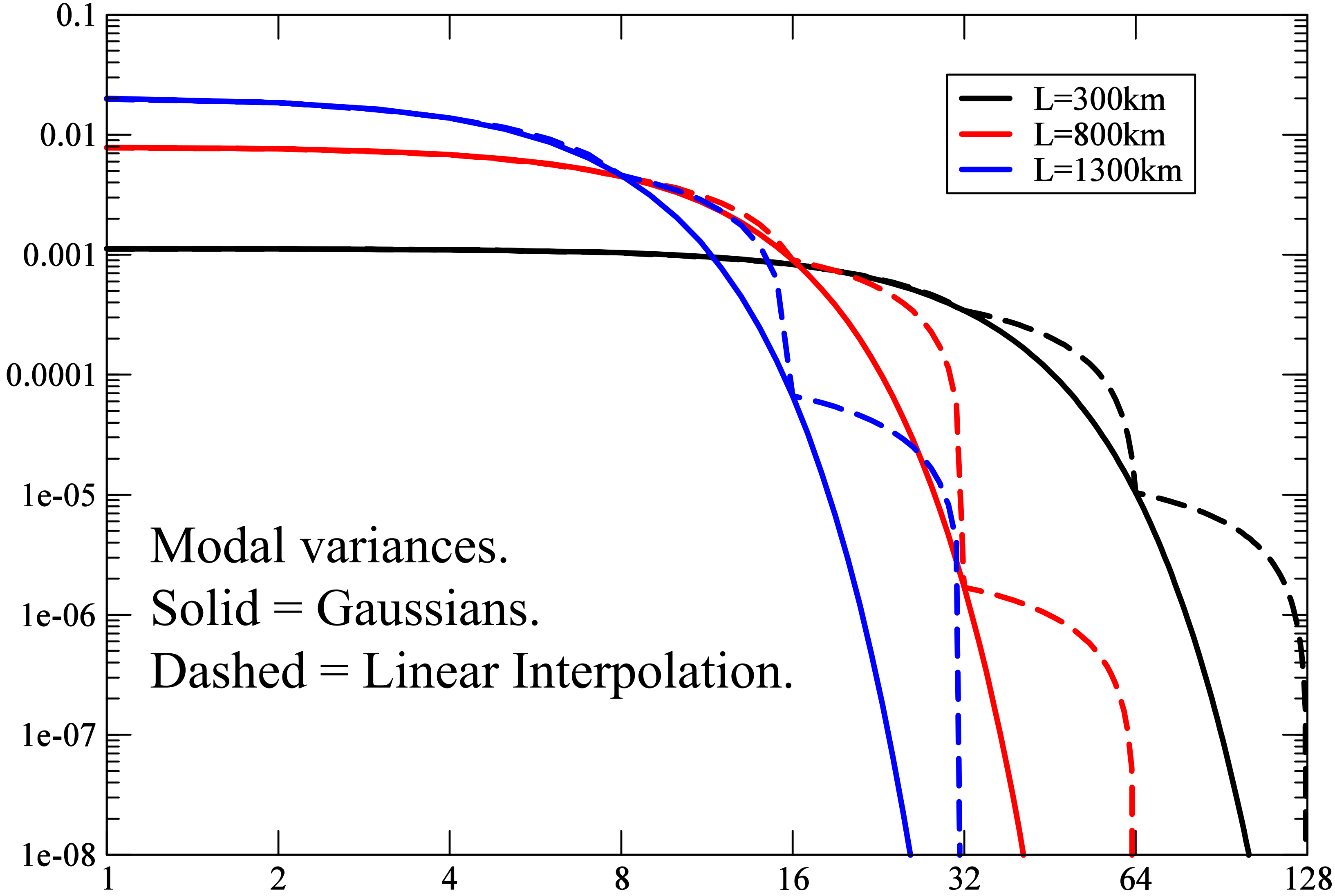
- E-W cross-sections. and the Gaussians we wanted:



Wavelet J_b

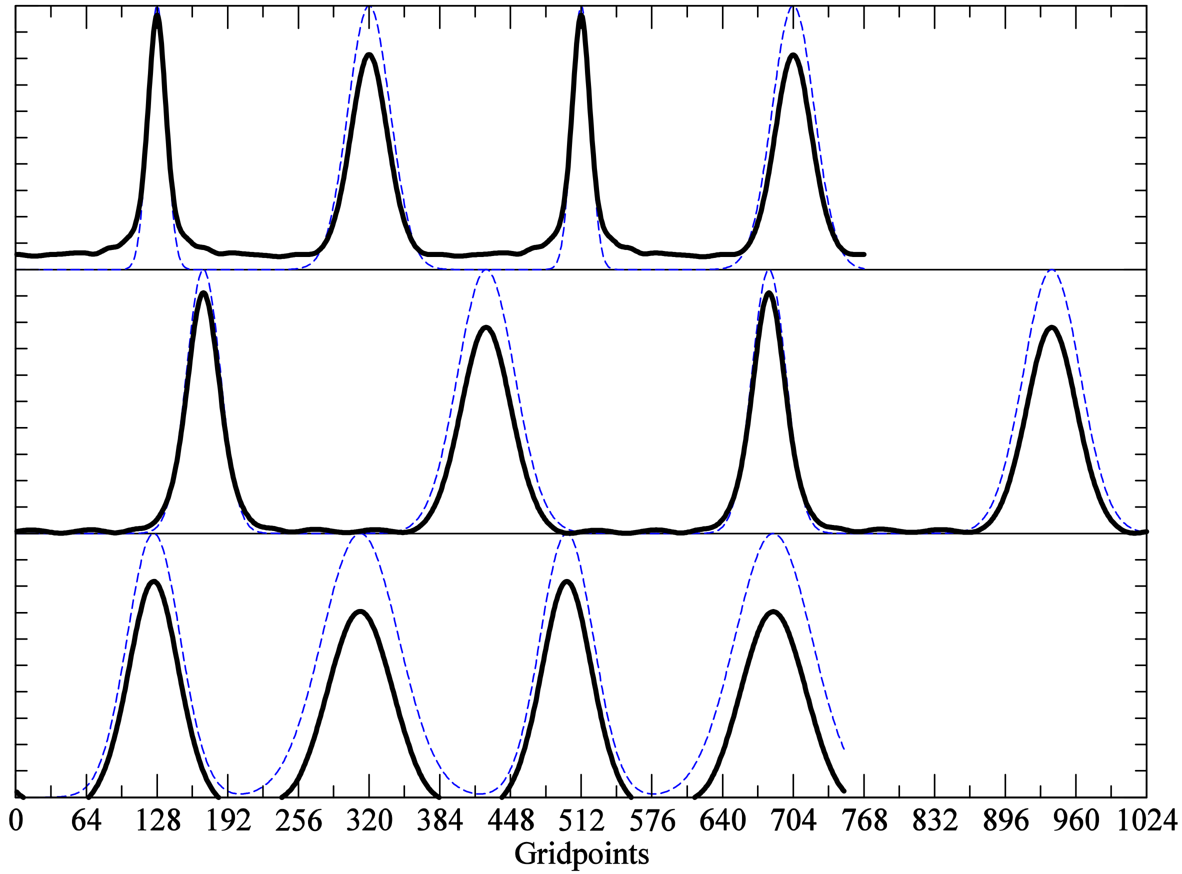
- The agreement between the Gaussians we wanted, and the functions we got is not perfect!
- But note:
  - No attempt was made to tune the cut-off wavenumbers to improve the accuracy with which the Gaussian structure functions were modelled.
  - The implied modal variances are effectively linearly interpolated between those for wavenumbers 0,2,4,8,16,...
  - For large lengthscale (1300km), there is little variance beyond about n=10, so there are very few nodes in the interpolation.
  - More spectral resolution at large scales may improve the approximation.
  - Higher order B-splines would also help.

Wavelet J_b


0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
1
2
4
8
16
32
64
128
L=300km
L=800km
L=1300km
Modal variances.
Solid = Gaussians.
Dashed = Linear Interpolation.

Wavelet J_b

- **E-W cross-sections for a different choice of spectral bands (0,1,3,6,10,16,25,39,60,91,138,208,313,471,511):**



Wavelet J_b

- We demonstrated Wavelet J_b by producing structure functions of a given analytic form.
- However, a significant advantage of Wavelet J_b over other approaches to covariance modelling on the sphere (digital filters, diffusion operators, etc.) is that we can calculate the coefficients of the covariance model directly from data.
- The covariance model is: $\mathbf{x} - \mathbf{x}_b = \sum_j \hat{\Psi}_j \otimes \Sigma_j \chi_j$
- The transform property gives us: $\Sigma_j \chi_j = \hat{\Psi}_j \otimes \left(\mathbf{x} - \mathbf{x}_b \right)$
- But χ_j has covariance matrix = I, so (in 2d), Σ_j is simply the matrix of gridpoint standard deviations of $\hat{\Psi}_j \otimes \left(\mathbf{x} - \mathbf{x}_b \right)$
- This is easily generated, given a sample of bg errors.

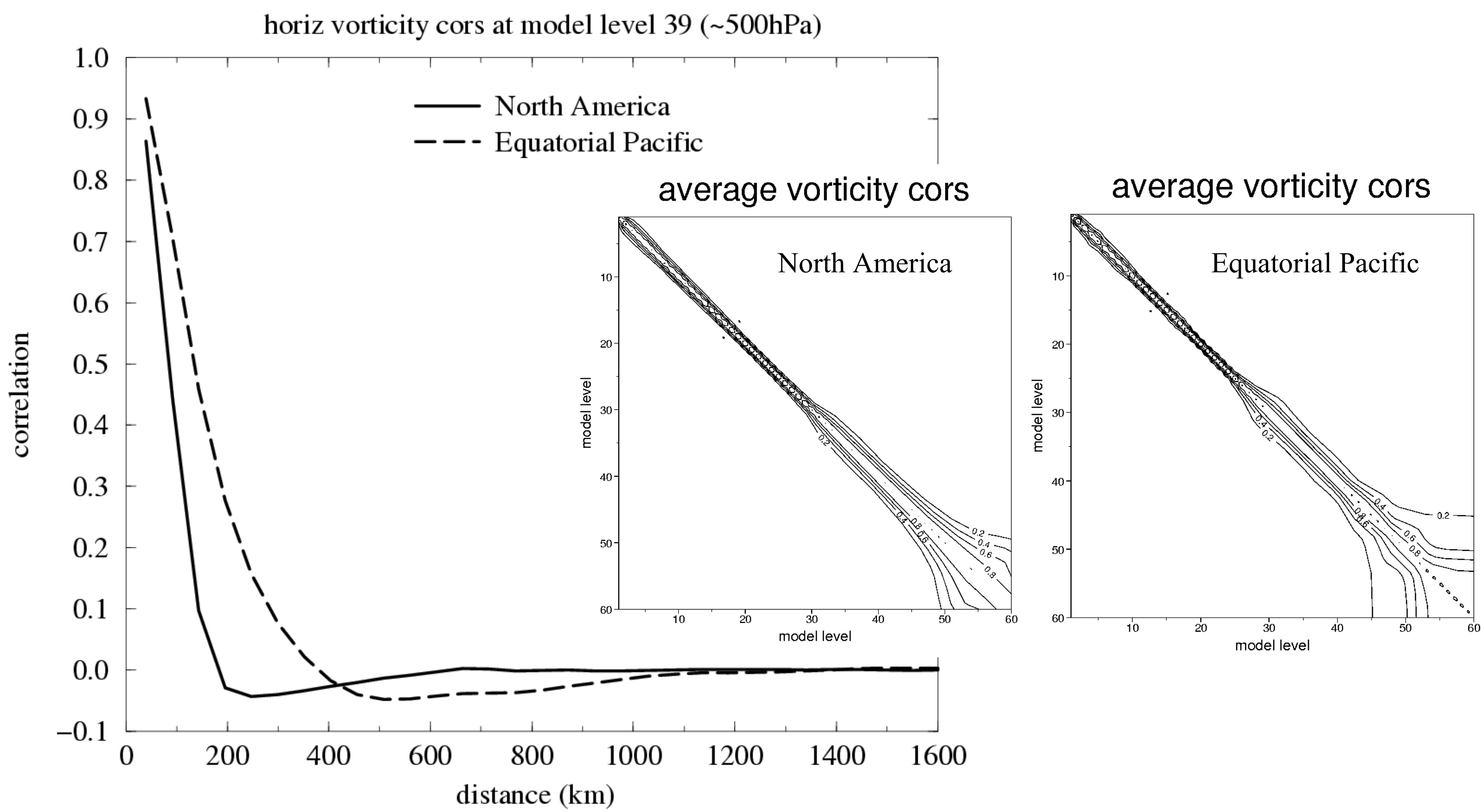
Wavelet J_b

- Extension of Wavelet J_b to 3 spatial dimensions is straightforward.
- In 2 dimensions, we have: $$\mathbf{x} - \mathbf{x}_b = \sum_j \hat{\Psi}_j \otimes \Sigma_j \chi_j$$

  where Σ_j is diagonal. The diagonal elements are standard deviations.
- In 3 dimensions, Σ_j becomes block-diagonal, with blocks of dimension NLEVS×NLEVS. The diagonal blocks are symmetric square-roots of vertical covariance matrices.
- This is not fully general, but is sufficient to capture the variation of vertical correlation with horizontal scale, and with location.

Wavelet J_b

- **Example: Horizontal and vertical Vorticity Correlations.**



Wavelet J_b – Memory and Cost

- **At first sight, the memory requirements for Wavelet J_b appear high.**
- **However, if** $\hat{\Psi}_j(n) = 0 \text{ for } n > N_{j+p}$ **then the** χ_j **are strictly band-limited, and may be represented on Gaussian grids of appropriate resolution.**
- **If we arrange for:** $N_j \le \frac{N_{j+1}}{\sqrt{2}}$

  then the memory requirement for the control vector is at most *p*+2 full model grids, where *p* is the order of the B-splines.
- **Only *p*+1 full-resolution spherical transforms are required.**

Wavelet J_b – Memory and Cost

- Storage for the vertical covariance matrices is potentially enormous. But, can be reduced to manageable levels by reducing their spatial resolution (e.g. one matrix for every 10 gridpoints).
- The main CPU requirement is in handling the increased-length control vector.
- The bottom line is that Wavelet J_b adds about 5% to the cost of a 4d-Var analysis.
- I think it's worth it!

Summary

- Tight frames provide a useful mathematical construct.
- They share many of the desirable properties of orthogonal bases, but allow considerable flexibility.
- Using tight frames, a flexible family of wavelets may be defined on the sphere.
- Unlike grid-based wavelets, there is no pole problem.
- There is a lot of scope for tuning of the spectral and spatial resolution. (This remains to be explored.)
- Wavelet J_b is expected to be implemented operationally in the ECMWF 4d-Var system by the end of this year.