Horizontal representation by Double Fourier series on the sphere

Clive Temperton
ECMWF

Overview

- Motivation
- Useful background information
- History
- Choice of basis functions
- Example: solving Poisson equation via double Fourier series on the sphere
- Pole problem: conjecture/hope
- Current status of project
- Urgency (?)

Motivation

ECMWF's operational (high-resolution) model is spectral, based on spherical harmonics.

Currently T511L60 (~ 40 km)

Coming soon T799L91 (~ 25 km)

Brainstorming about the "future dynamical core"

Are the hounds of Legendre baying in the distance?

Useful background information (1)

Each associated Legendre polynomial (with zonal wavenumber m) can be written as a finite series of sines (m odd) or cosines (m even).

$\Rightarrow$If a function can be expressed as a finite linear combination of associated Legendre polynomials (truncated at total wavenumber N), then it can be expressed instead as a finite sine or cosine series (up to N terms).

Useful background information (2)

Unfortunately, the converse is not true.

⇒If a function is expressed as a truncated sine or cosine series, then in general we will have to *filter* it if we want to ensure that it lives in the subspace spanned by the associated Legendre polynomials.

(In other words, we have lost one of the “magic” properties of spherical harmonics and the pole problem may come back to haunt us.)

History

- Merilees (1973) – pseudospectral SWE
- Orszag (1974) – various applications
- Boyd (1978) – elliptic/eigenvalue problems
- Yee (1981) – Poisson equation
- Fornberg (1995) – various applications
- Spotz et al. (1998) – SWE (with spherical harmonic filter)
- …….=>

History (continued)

- Shen (1999) – various applications
- Cheong (2000) – elliptic & vorticity equations
- Cheong (2000) – SWE
- Layton & Spotz (2003) – SWE (semi-Lagrangian, still with spherical harmonic filter)

Choice of basis functions (i)

m odd => $\sin n\theta$ (m = zonal wavenumber,
θ = colatitude)

X, $dX/d\theta$ behave correctly at poles.

Choice of basis functions (ii)

$m = 0 \;\Rightarrow\; \cos n\theta$ (m = zonal wavenumber,
θ = colatitude)

X, $dX/d\theta$ behave correctly at poles.

(But should we use Legendre polynomials just for m=0 ?)

Choice of basis functions (iii)

m even, $m > 0$ => $\cos n\theta$

$dX/d\theta$ behaves correctly at poles, but not X.

Or $\sin n\theta \sin\theta$ (Cheong)

then X, $dX/d\theta$ behave correctly at poles

BUT: $X / \sin\theta$ is represented by a sine series – is this OK ?

Choice of truncation: rectangular? elliptic?

Example: Poisson equation (1)

$$\nabla^2 u = f$$

Fourier series in longitude =>

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left\{\sin\theta\frac{d}{d\theta}u_m(\theta)\right\} -$$

$$\frac{m^2}{\sin^2\theta}u_m(\theta) = f_m(\theta)$$

Example: Poisson equation (2)

Set $u_m(\theta) = \sum_{l=0}^{L} u_{l,m} \sin l\theta$ (m odd)

Similarly $f_m(\theta)$

$$\Rightarrow \quad (l-2)(l-1)u_{l-2,m} - \left(2l^2 + 4m^2\right)u_{l,m}$$
$$+ (l+1)(l+2)u_{l+2,m} =$$
$$- f_{l-2,m} + 2f_{l,m} - f_{l+2,m}$$

Example: Poisson equation (3)

Similarly for m even.

$\Rightarrow$For each zonal wavenumber m we get two tridiagonal systems to solve (one for odd values of l, one for even values of l).

Helmholtz equation (e.g., from semi-implicit scheme) is very similar.

The pole problem: conjecture/hope

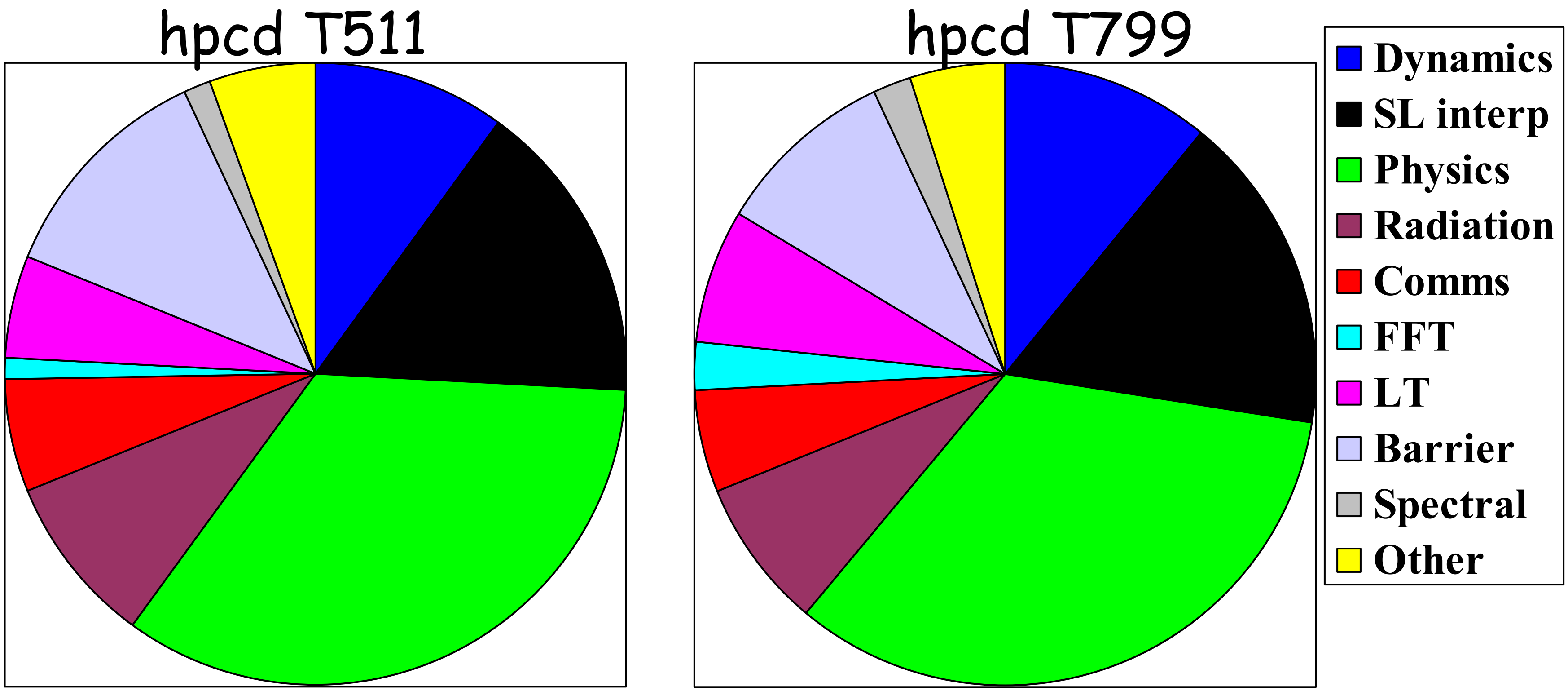
For efficiency, in our current (spherical harmonic) model we use a *reduced* grid (Δx ~ constant) to give approximately uniform resolution over the sphere.

The big question: would the reduced grid be sufficient to control the pole problem when using double Fourier series (since the grid cannot support high zonal wavenumbers near the pole)?

Current status of project

- We have nearly completed the coding to test a double Fourier series formulation of the SWE, within the IFS (including the Williamson et al. tests)
- Some options left open for now
- No results yet (sorry!)
- How close are the hounds of Legendre anyway?

T511- L60 compared with T799 - L91
64 MPI Tasks and 4 OpenMP threads



Extra cost (per timestep) for T799 - L91 = 3.5 times