Inversions of Surface Fluxes from Atmospheric Data

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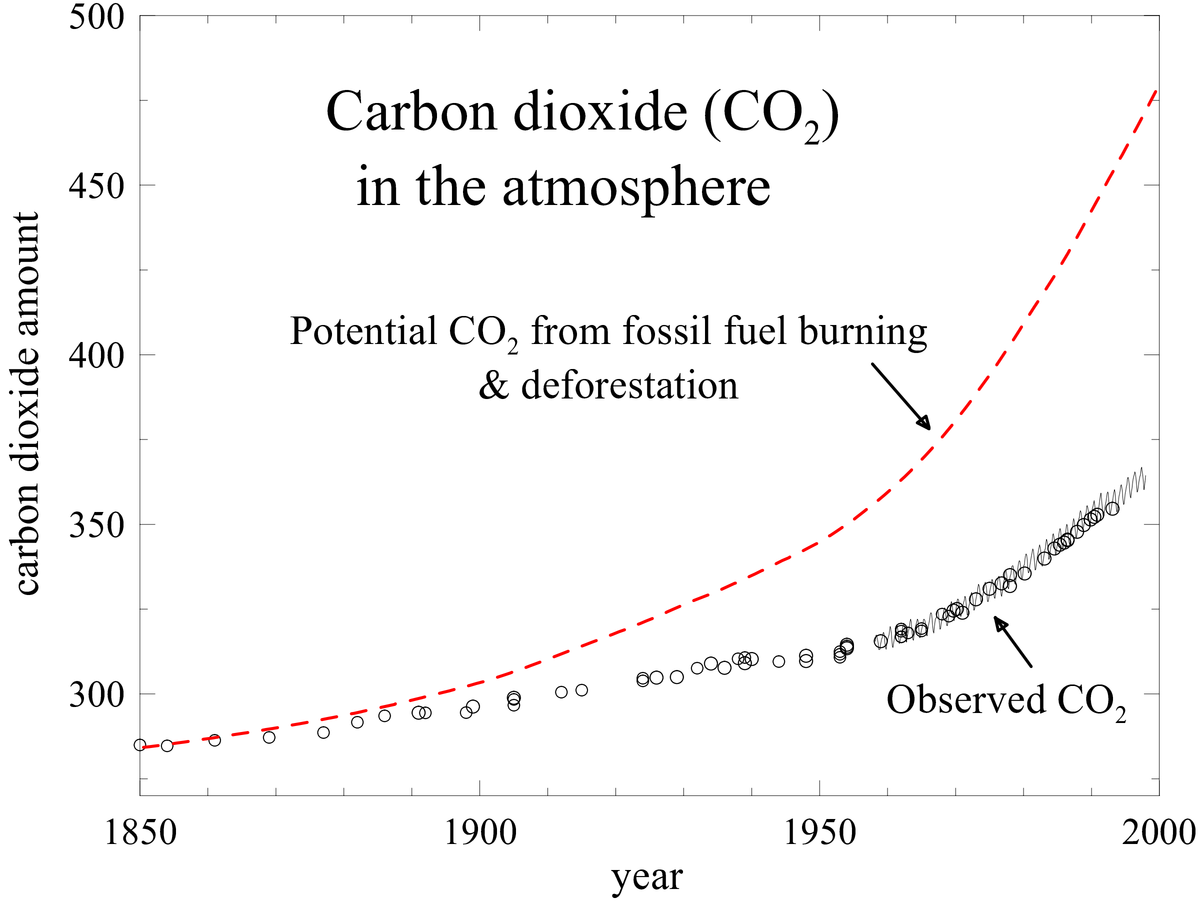
Acknowledgement to Ian Enting

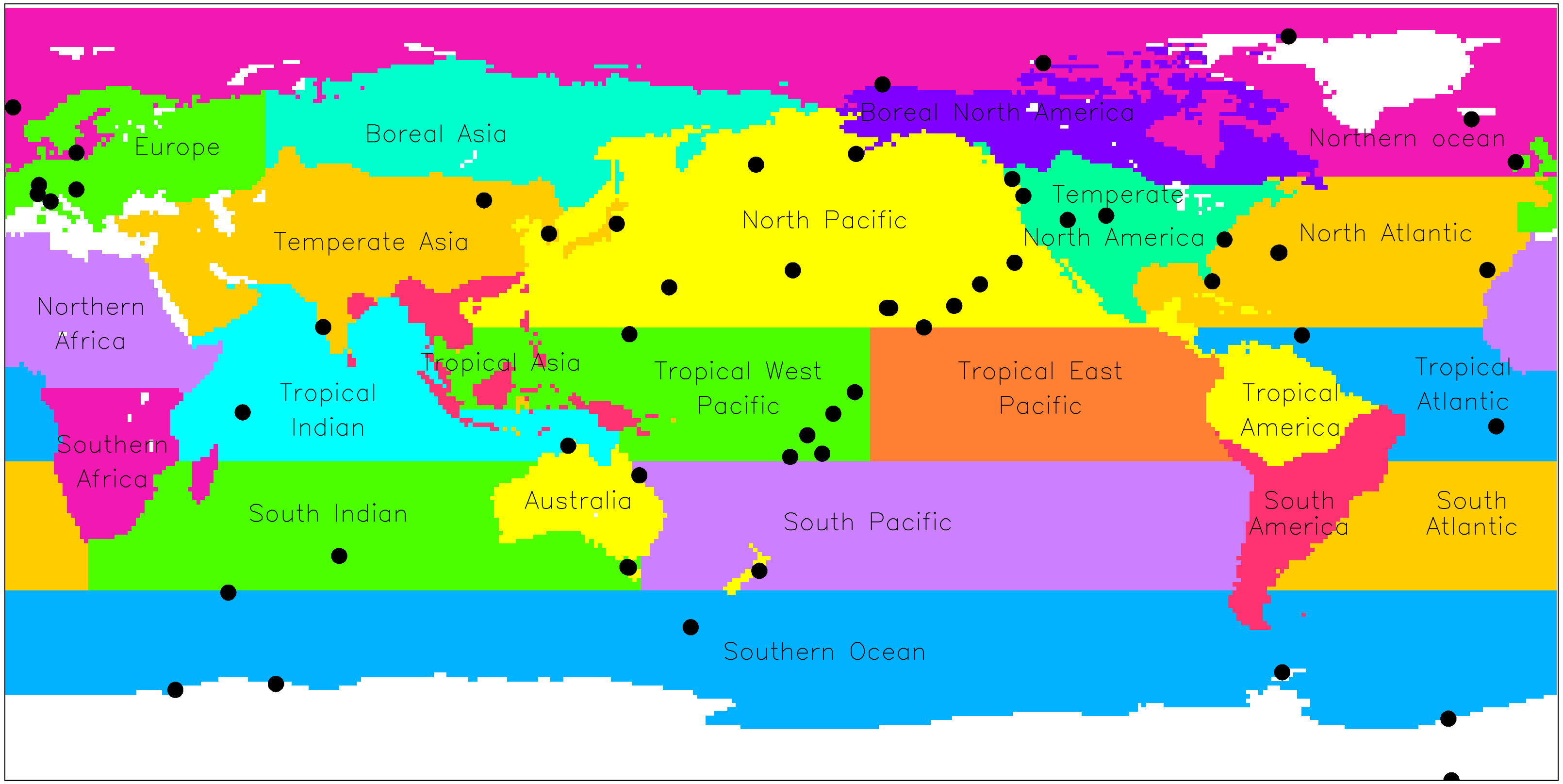
Outline

- Motivation
- Basic Theory
- What has the atmosphere told us
- Current Problems and new directions

History of Atmospheric CO_2



Regions and network



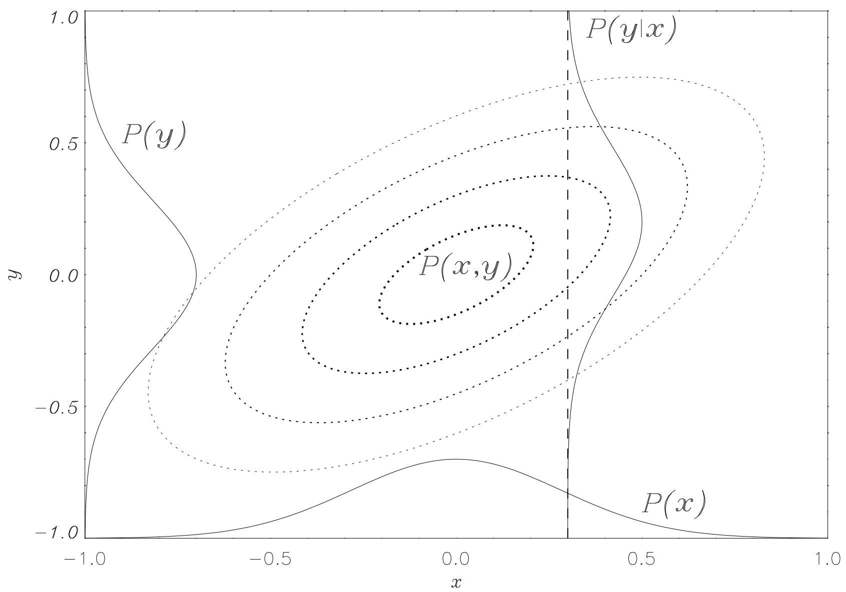
Basic Inverse Principles

- Reverse direction of causality
- Fundamentally a statistical problem
- Bayesian methods combine prior information and data

Why use Bayesian methods

- Risk biasing solution
- Regularization.
- Provide a meaningful norm
- There is other real information

Illustration from Rodgers 2000



- $P(x, y)$, joint PDF of x and y.
- $P(x)$ $P(x, y)$ integrated over y.
- $P(x|y)$, $P(x)$ given y.
- $\mathbf{M}$ mapping from x to y (measurement).

Bayes Theorem

- $P(x|y) = \frac{P(y|x)P(x)}{P(y)}$
- use $\mathbf{M}$ to calculate $P(y|x)$

Linear Gaussian Example

- $\mathbf{s}$ and $\mathbf{d}$ sources and data. Subscript 0 initial value and σ standard deviation.

- $P(\mathbf{s}) \propto e^{-\frac{1}{2}\left(\frac{\mathbf{s}-\mathbf{s}_0}{\sigma_s}\right)^2}$

- $P(\mathbf{d}|\mathbf{s}) \propto e^{-\frac{1}{2}\left(\frac{\mathbf{Ms}-\mathbf{d}_m}{\sigma_d}\right)^2}$

- $P(\mathbf{s}|\mathbf{d}) \propto e^{-\frac{1}{2}\left[\left(\frac{\mathbf{Ms}-\mathbf{d}_m}{\sigma_d}\right)^2+\left(\frac{\mathbf{s}-\mathbf{s}_0}{\sigma_s}\right)^2\right]}$

Parameters of PDF

- Maximum of PDF most likely value. Minimize

$$\chi^2 = \left(\frac{\mathbf{Ms} - \mathbf{d}_m}{\sigma_d}\right)^2 + \left(\frac{\mathbf{s} - \mathbf{s}_0}{\sigma_s}\right)^2$$

- χ^2 cost function
- Means and variances calculated as moments of PDF
- Assumes $\mathbf{M}$ perfect.

The Classical Solution

- Assume $\mathbf{M}$ linear and replace with matrix $\mathbf{J}$
- Generalize uncertainties to covariances $\mathbf{C}$ and discretize $\mathbf{s}$ and $\mathbf{d}$ to $\vec{S}$ and $\vec{D}$

$$\vec{S} = \vec{S}_0 + \mathbf{C}(\vec{S}_0)\mathbf{J}^T \left(\mathbf{J}\mathbf{C}(\vec{S}_0)\mathbf{J}^T + \mathbf{C}(\vec{D})\right)^{-1} \left(\vec{D} - \mathbf{J}\vec{S}_0\right)$$

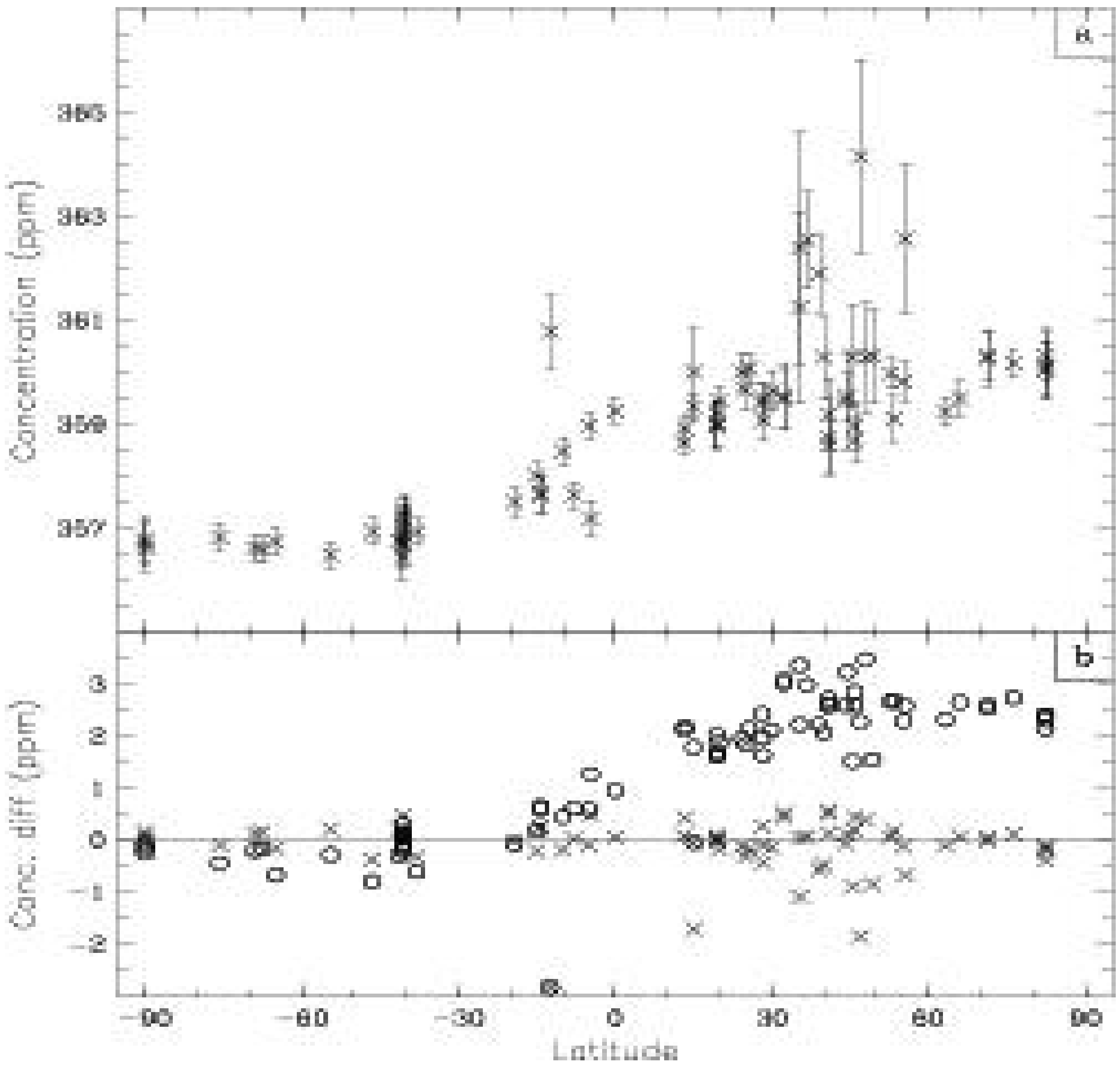
$$\mathbf{C}(\vec{S})^{-1} = \mathbf{C}(\vec{S}_0)^{-1} + \mathbf{J}^T\mathbf{C}(\vec{D})^{-1}\mathbf{J}$$

Calculating Jacobian

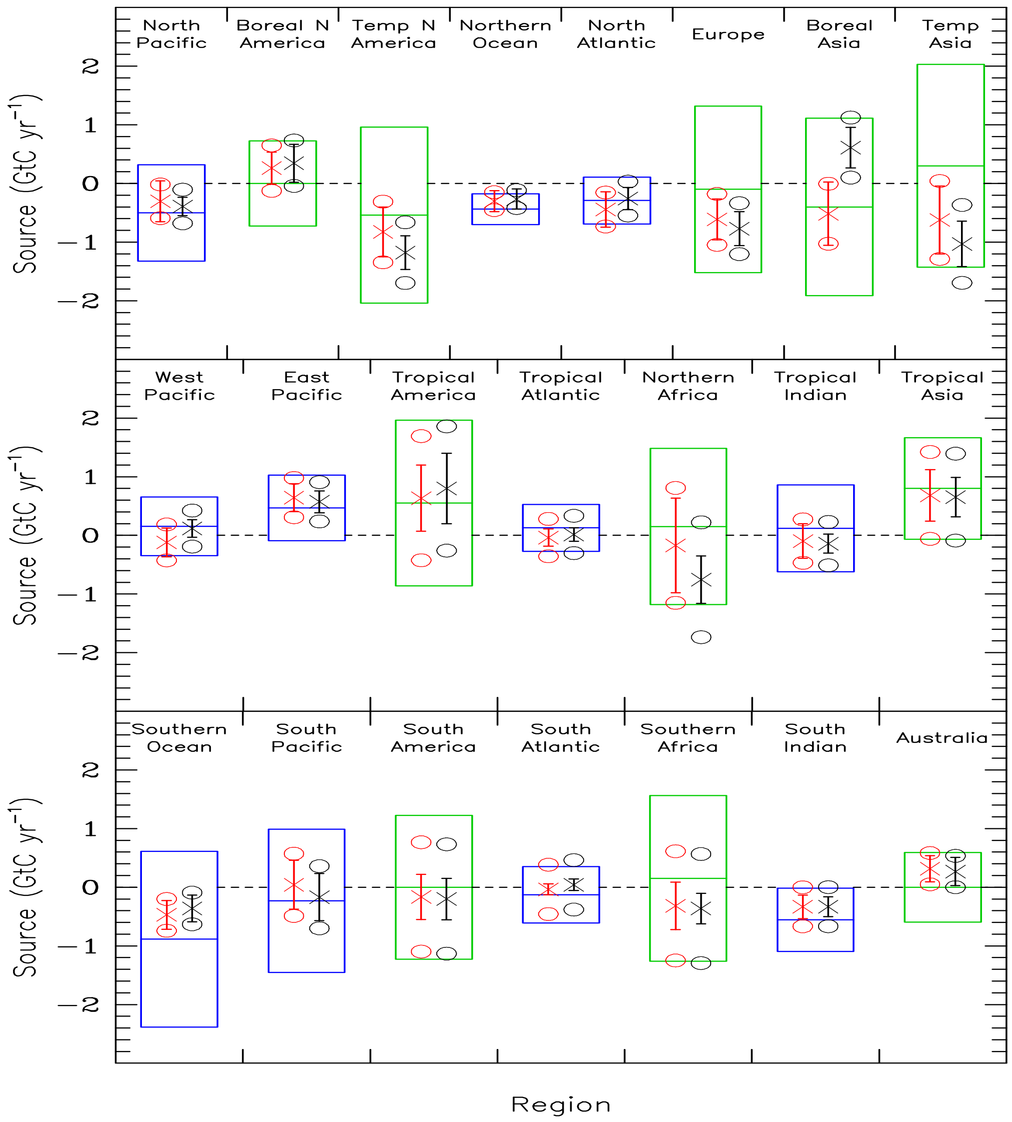
Forward Mode	Adjoint mode
Divide earth into regions	Create adjoint model
Run source for each region	Run tracer backwards for each observation.
Limits source resolution	Freezes choice of observations
Enables network design	Allows coupling

Matrix Free Methods

- Only practical way forward
- Variational method directly minimizes χ^2
- Kalman Filter sequentially assimilates data
- Avoid linearity assumption
- Only low-order approximation of covariance

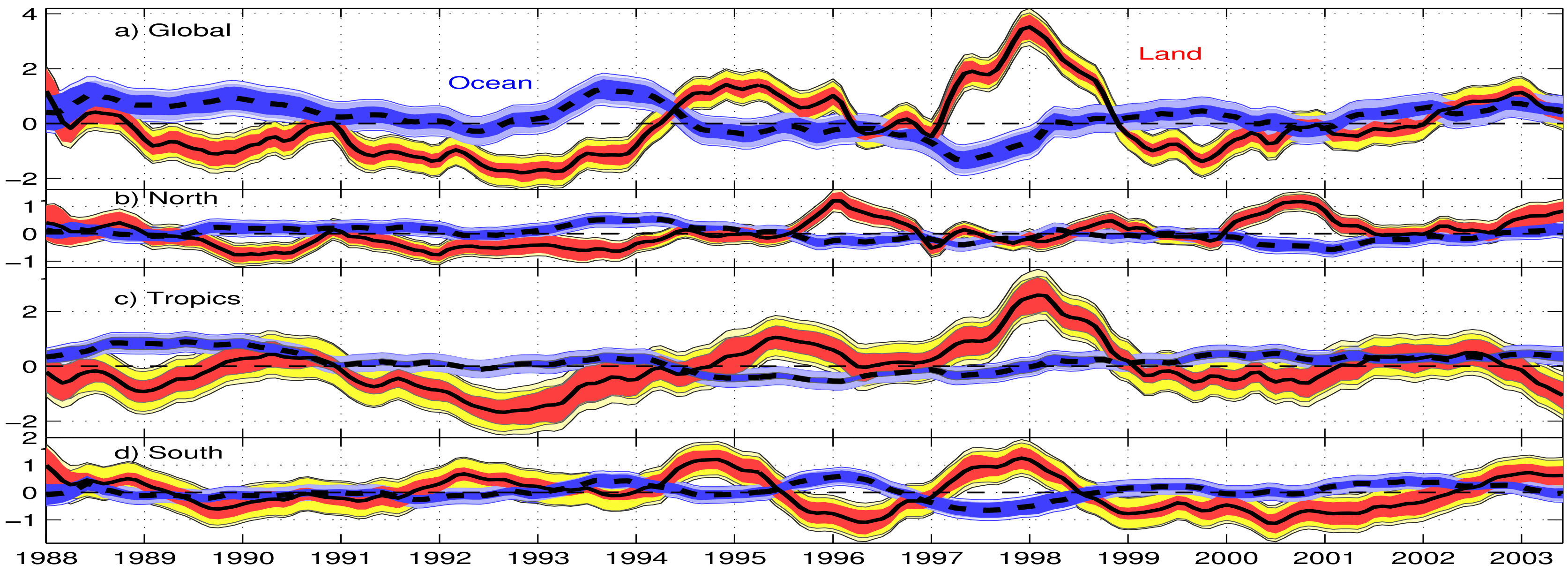


Meridional gradient of model mean concentrations for the sum of the three background fluxes ('+'), the observational CO2 values ('x') and the uncertainty assigned to them in the inversion (error bars), and the model mean concentrations after inverting for regional fluxes ('o'). Concentrations are means over 1992-1996. for all models.



Mean inversion results for the control inversion (red symbols in each box) and an inversion without the background seasonal biosphere flux (black symbols in each box). Mean estimated fluxes are shown by the 'X' and include all background fluxes except fossil fuel. Positive values indicate a source to the atmosphere. The prior flux estimates and their uncertainties are indicated by the boxes (land in green, ocean in blue); the central horizontal bar indicates the prior flux estimate and the top and bottom of the box give the prior flux uncertainty range. The mean estimated uncertainty across all models (the "within-model" uncertainty) is indicated by the circles. The standard deviation of the models' estimated fluxes (the "between-model" uncertainty) is indicated by the "error bars".

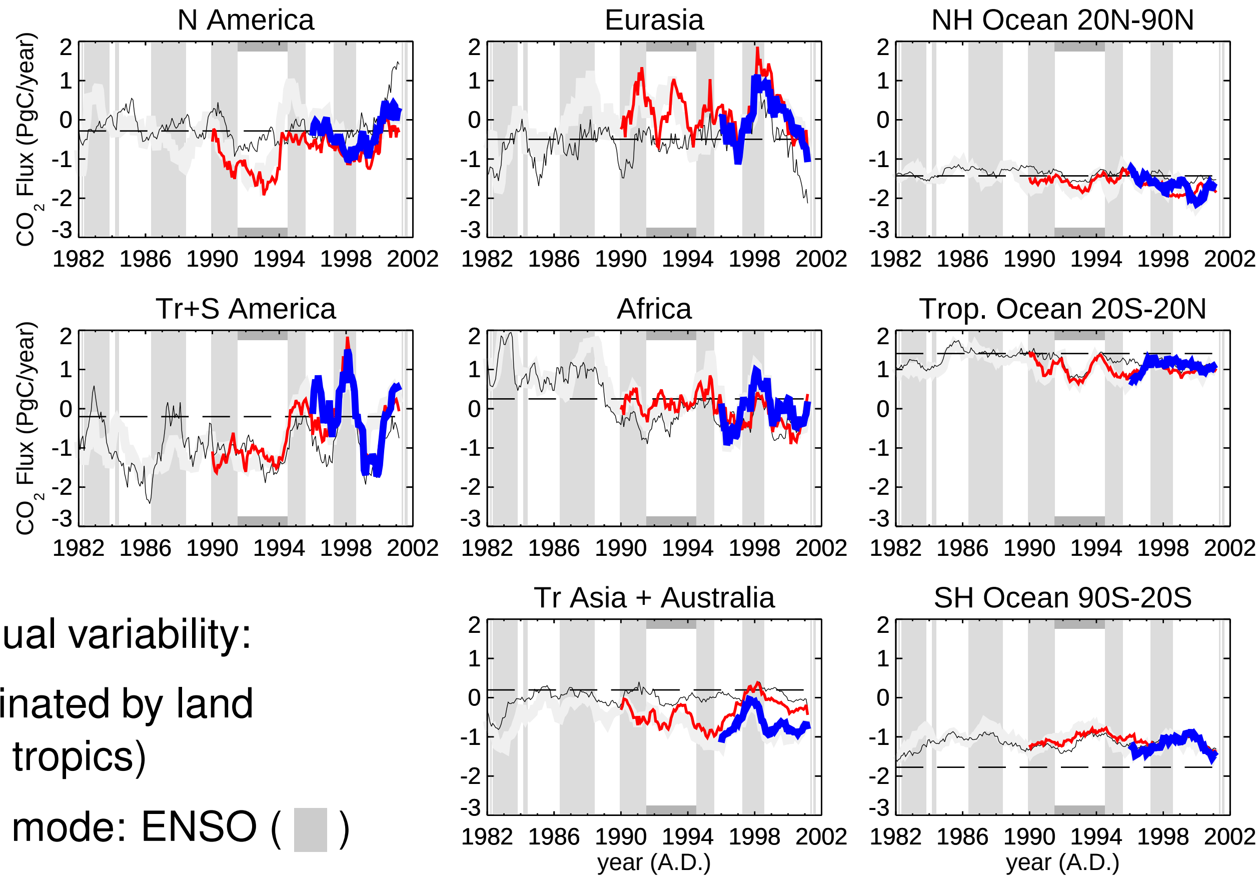
Interannual Variability



The flux interannual variability (PgC yr^{-1}) for the full globe, and for three broad latitude bands and partitioned into land fluxes (solid lines) and ocean fluxes (dashed lines).

CO_2 Flux Estimates

- Regionally integrated • deseasonalized • non-fossil CO_2 only
- 11, 19, or 35 sampling locations • positive: surface → atmosphere



Interannual variability:

- Dominated by land (esp. tropics)
- Main mode: ENSO ( )
- Exception: 'Post-Pinatubo'

[Rödenbeck et al.: Atmos. Chem. Phys. Discuss., 3, 2575-2659, 2003]

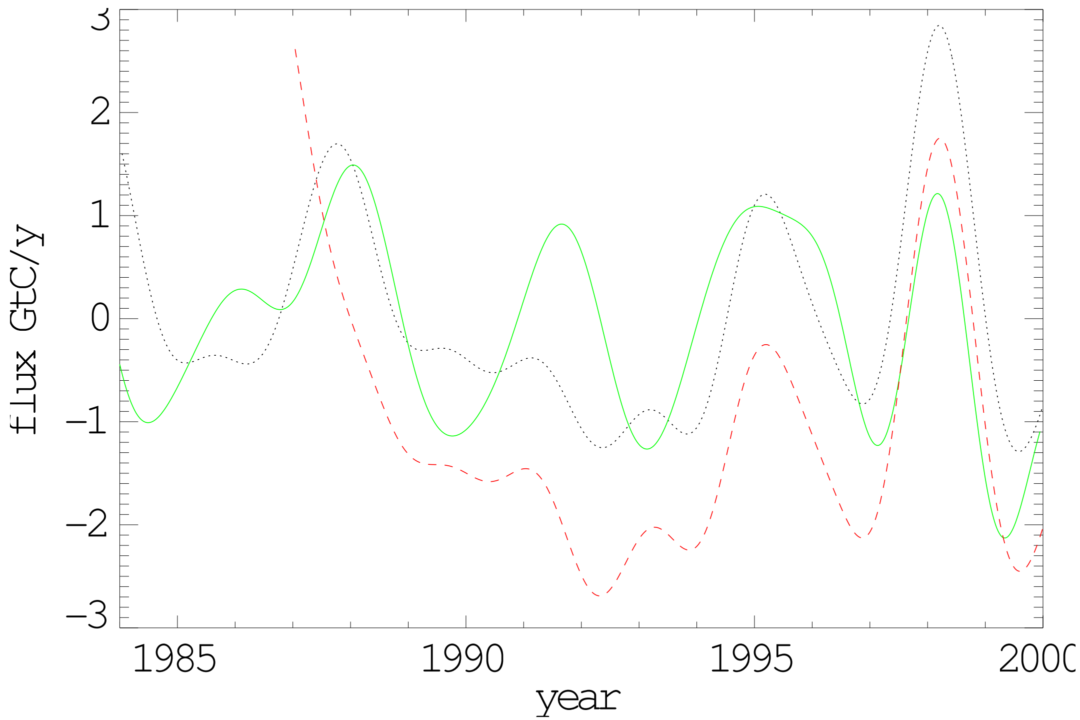
Current Problems

- Not enough data
- Not all sources at surface
- Transport models insufficient

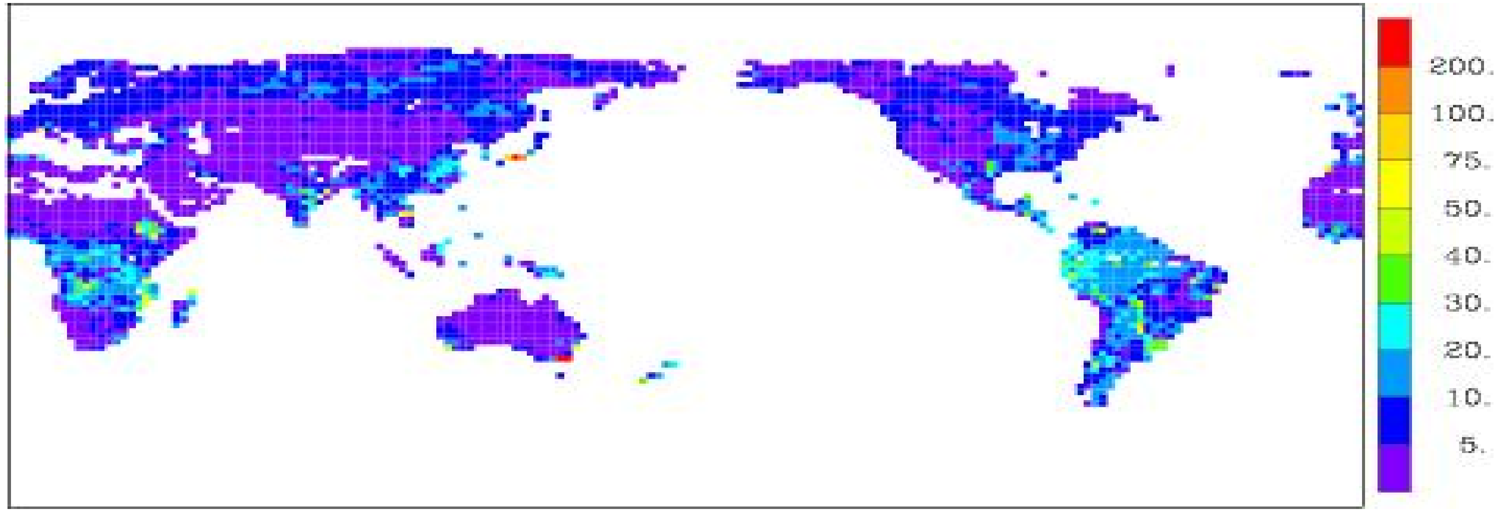
Carbon Cycle Data Assimilation

- Reduce number of unknowns
- Replace flux map by model; unknowns are model parameters and state
- Can learn about processes
- Results biased by underlying model

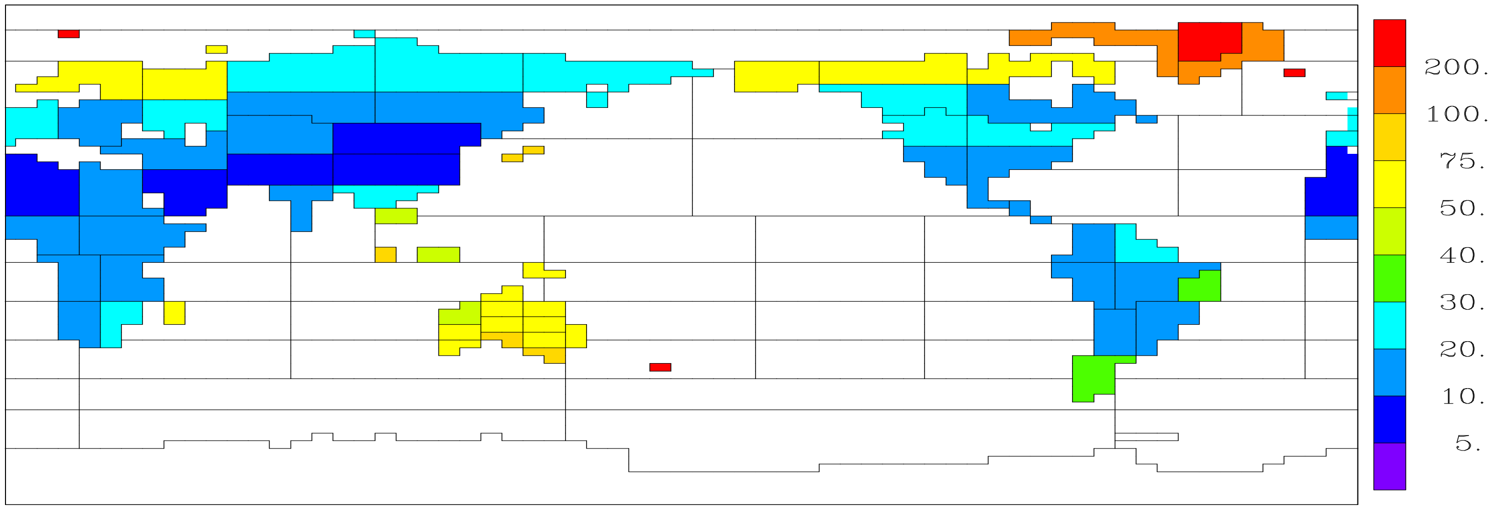
Comparison of Direct and Model-based Inversion



Comparison of our optimized terrestrial flux (solid green) with inversions using 16 stations (dotted black) and 19 stations (dashed red) of Rödenbeck et al. (2003).



Annual mean uncertainty from process model



Annual mean uncertainty from potential satellite

Summary

- Atmospheric inversion is a statistical problem
- Information is limited to large scales by lack of data
- Techniques and models are evolving to cope with large data volumes
- Combination with process models offers an alternative way forward.