Modelling of background error covariances for the analysis of clouds and precipitation

Thibaut Montmerle
CNRM-GAME/GMAP

ECMWF-JCSDA Workshop on Assimilating Satellite Observations of Clouds and Precipitation into NWP models, 15 – 17 June 2010

METEO FRANCE
Toujours un temps d'avance

Outlines

1. Introduction

2. Diagnosis of covariances in clouds and precipitation

3. How to get cloud and precipitation-dependent statistics?

4. Conclusions and Possible strategies

Outlines

1. Introduction

2. Diagnosis of covariances in clouds and precipitation

3. How to get cloud and precipitation-dependent statistics?

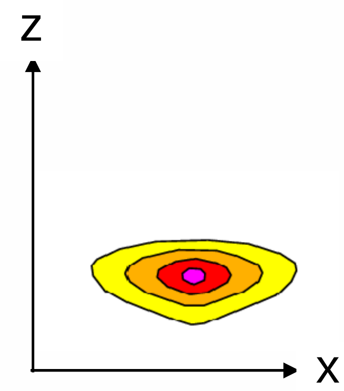
4. Conclusions and Possible strategies

Introduction: B and BLUE

In statistical linear estimation theory, the analysis increment δx^a write:

$$\delta x^a = x^a - x^b = \mathbf{B}\mathbf{H}^T\left(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R}\right)^{-1} d$$

Where $\mathbf{B} = E((x^b - x^t)(x^b - x^t)^{\mathrm{T}})$ and $\mathbf{R} = E(\varepsilon^o \varepsilon^{o\mathrm{T}})$ are the background and the observation error covariance matrices, $d = y - H[x^b]$ the innovation, and $\mathbf{H}$ is the linearized version of the observation operator H



Vertical Cross section of T increment for 1 obs exp.

B has a profound impact on the analysis in VAR:

- Correlations in **B** performs information smoothing and spreading from the observation points
- **B** propagates information to other variables and imposes balance.

Practical difficulties:

- x^t is unknown. Differences between forecasts are commonly used to mimic forecast errors in order to compute **climatological covariances**
- Because of its size, **B** can be neither estimated at full rank nor stored explicitly => **covariances have to be modelled**

Introduction: B modelling unsing CVT

To compute δx^a , **the method of Control Variable Transforms (CVT)** is widely used. This requires to write the cost function in an incremental formulation (Courtier 1997), and to replace the increment δx by a control variable χ in order to simplify the background term:

$$\delta x = \mathbf{B}^{1/2}\chi$$

The challenge is to capture in $\mathbf{B}^{1/2}$ the known important features of $\mathbf{B}$

Following Derber and Bouttier (1999):

$$\delta x = \mathbf{B}^{1/2}\chi = \mathbf{K}\ \mathbf{B}_S^{1/2}\ \delta x$$

$\mathbf{K}$ is called the balance operator

$\mathbf{B}_S^{1/2}$ is a block diagonal matrix called the spatial transform

Introduction: Balance operators

$$\delta x = \mathbf{K}\mathbf{B}_u^{1/2}\chi = \mathbf{K}\tilde{\chi}$$

K aims in taking increments of the model's variable and to output new less correlated parameters on the same grid using balance constraints.

Basic hypothesis for K:

- The control variables χ are thought to be relatively uncorrelated and can be different than increment variables δx

- Errors in geostrophically balanced parameters (associated with Rossby modes) are decoupled from errors in unbalanced parameters (associated with inertio-gravity modes)

No hydrometeors are considered yet in operational configurations

Introduction: Balance operators $\delta x = \mathbf{K}\mathbf{B}_u^{1/2}\chi = \mathbf{K}\tilde{\chi}$

ECMWF/Météo-France parameter transform (similar to what is used at NCEP and JMA) **:** Fields in **spectral representation (allows scale-dependency)**

$$\begin{pmatrix}\delta\zeta\\ \delta\eta\\ (\delta T,\delta P_S)\\ \delta q\end{pmatrix} = \begin{pmatrix} I & 0 & 0 & 0\\ MH & I & 0 & 0\\ NH & P & I & 0\\ QH & R & S & I\end{pmatrix}\begin{pmatrix}\delta\tilde{\zeta}\\ \delta\tilde{\eta}_u\\ (\delta\tilde{T},\delta\tilde{P}_S)_u\\ \delta\tilde{q}_u\end{pmatrix} + \begin{pmatrix}0\\ \delta\eta_b(\delta\zeta,\delta T)_\Omega\\ 0\\ 0\end{pmatrix}$$

M, N, P are regression operators **that adjust couplings with scales**

extension for δq operationally used at MF at regional and convective scale (Berre(2000))

H is an analytical NLBE operator giving the pressure field in geostrophic balance with $\delta\zeta$, this balanced pressure being regressed to $\delta\eta$ via M

Met-Office formulation (Similar to what is used for WRF, MM5 and at CMC) **:** Fields in **grid-point representation**

$$\begin{pmatrix}\delta\zeta\\ \delta\eta\\ \delta P\\ \delta T\\ \delta q\end{pmatrix} = \begin{pmatrix} \nabla_h^2 & 0 & 0 & 0\\ 0 & \nabla_h^2 & 0 & 0\\ H & 0 & I & 0\\ \mathrm{T}H & 0 & \mathrm{T} & 0\\ (\Gamma + Y\mathrm{T})H & 0 & \Gamma + Y\mathrm{T} & \Lambda\end{pmatrix}\begin{pmatrix}\delta\tilde{\psi}\\ \delta\tilde{\chi}\\ \delta\tilde{P}_u\\ \delta\tilde{\mu}\end{pmatrix}$$

- parameters are stream function $\delta\psi$, velocity potential $\delta\chi$, unbalanced pressure δP_u and relative humidity $\delta\mu$
- analytical operators instead of statistical regressions

Introduction: Spatial transforms

$$\chi = \mathbf{B}^{-1/2}\delta x = \mathbf{B}_S^{-1/2}\mathbf{K}^{-1}\delta x = \mathbf{B}_S^{-1/2}\tilde{\chi}$$

The spatial transforms aim in:

- projecting each parameter onto uncorrelated spatial modes
- dividing by the square root of the variance of each mode

ECMWF's like formulation is based on the diagonal spectral hypothesis:

$$\mathbf{B}_S^{-1/2} = \left(\mathbf{D}^{-1/2}\mathbf{E}^T\right)\ \mathbf{W}^{-1/2}\left(\mathbf{S}^{-T}\mathbf{V}^{-1/2}\mathbf{S}^T\right)$$

GP σ_b

spectral σ_b

} Horizontal cov. deduced from calibration proc.

Vertical cov. } Deduced from EOFs

$\Rightarrow$**The resulting correlations are homogeneous, isotropic and non-separable**

In the Met-Office-like formulations vertical correlations depends on horizontal position instead of scale

Outlines

Diagnostics of covariances in clouds and precipitation

3D Error covariances:

In Montmerle and Berre (2010) **B**s are computed separately for rainy and non-rainy areas using geographical masks in an ensemble assimilation based on a Cloud Resolving Models (CRM), in order to:

• capitalized on an explicit microphysical scheme **to document covariances and balances between traditional control variables** in areas that are under-represented in samples of forecast differences used for climatological covariances computation

• **quantify what should be taken into account in B matrix modeling** for VAR in those areas, in terms of balances and flow-dependecy

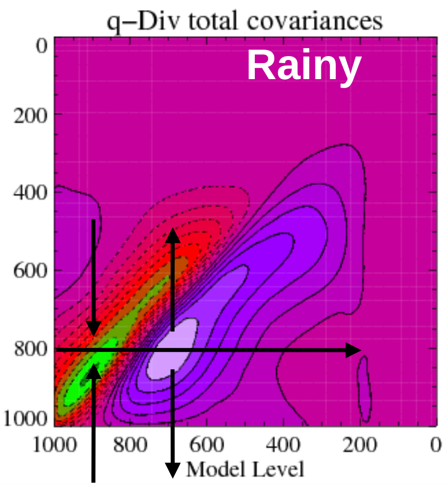
• **give and insight of spatialization lengths** to be applied to filter out sampling noise in ensemble-based DA methods such as EnKF in those areas

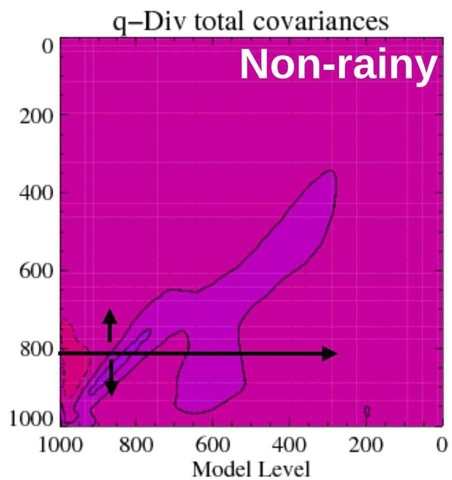
• build set of B matrices that could be further used in the heterogeneous background error covariances formulation

Diagnostics of covariances in clouds and precipitation

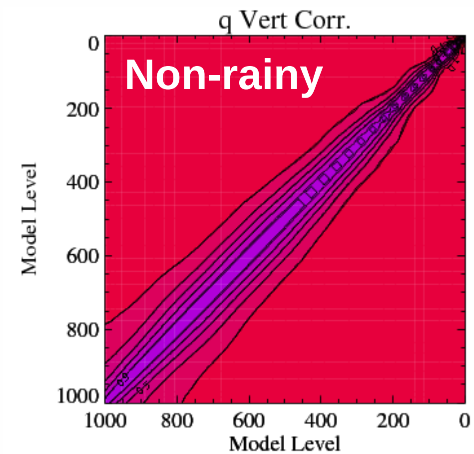
$$\delta q = QH\delta\tilde{\zeta} + R\delta\tilde{\eta}_u + S\left(\delta\tilde{T}, \delta\tilde{P}_S\right)_u + \delta\tilde{q}_u$$

The coupling with moisture in convective clouds allows in particular **σ_b(q) to be mostly explained by $\delta\eta_u$ in precipitating areas at mesoscale**, and to be almost univariate and linked to the mass field in clear air

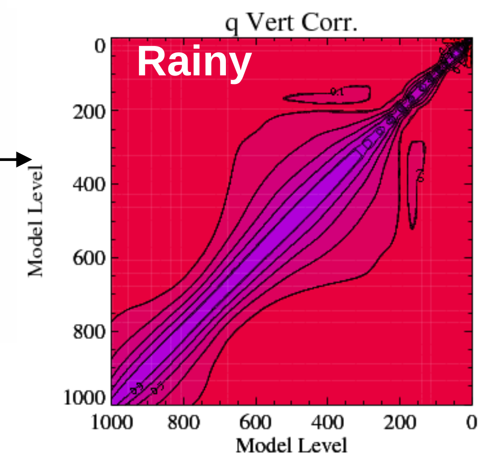


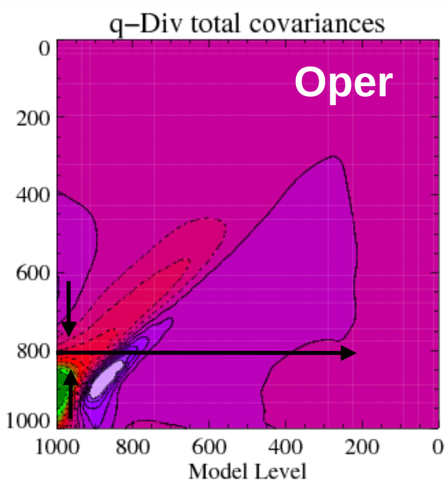


Cov(q,div): A positive increment of δq will enhance convergence below 800 hPa, and divergence above in precipitating areas



Cor(q,q): larger mid-tropospheric mixing in precipitations





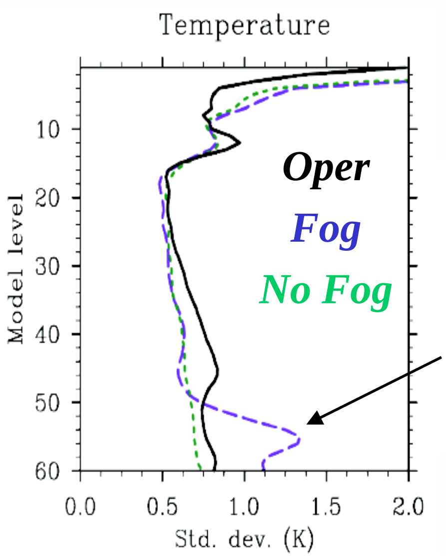
Also:

- ~2 times shorter horizontal correlation lengths for q and T in precipitation
- greater σ_b for $\delta\zeta$ and $\delta\eta$ in precipitation, because of the more intense dynamics

Montmerle and Berre (2010)

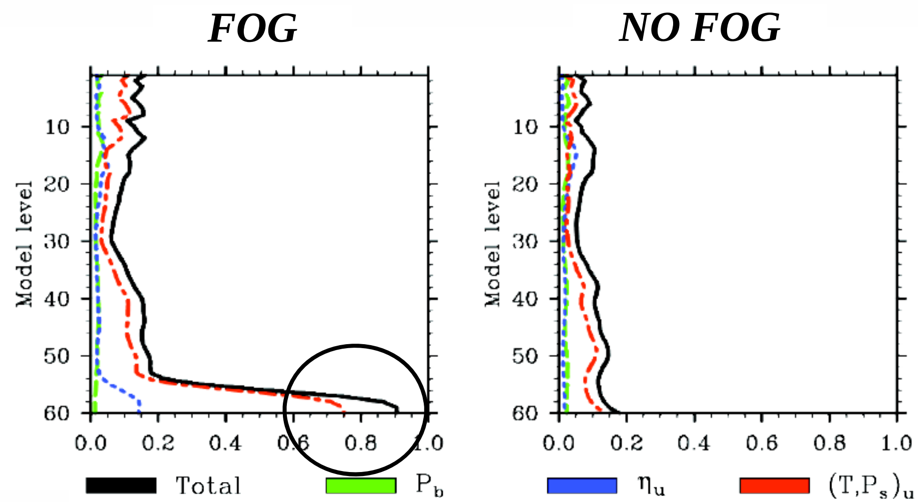
Diagnostics of covariances in clouds and precipitation

Same approach has been recently applied for fog:



$\sigma_b(T)$

Maximum reflecting T inversion above fog



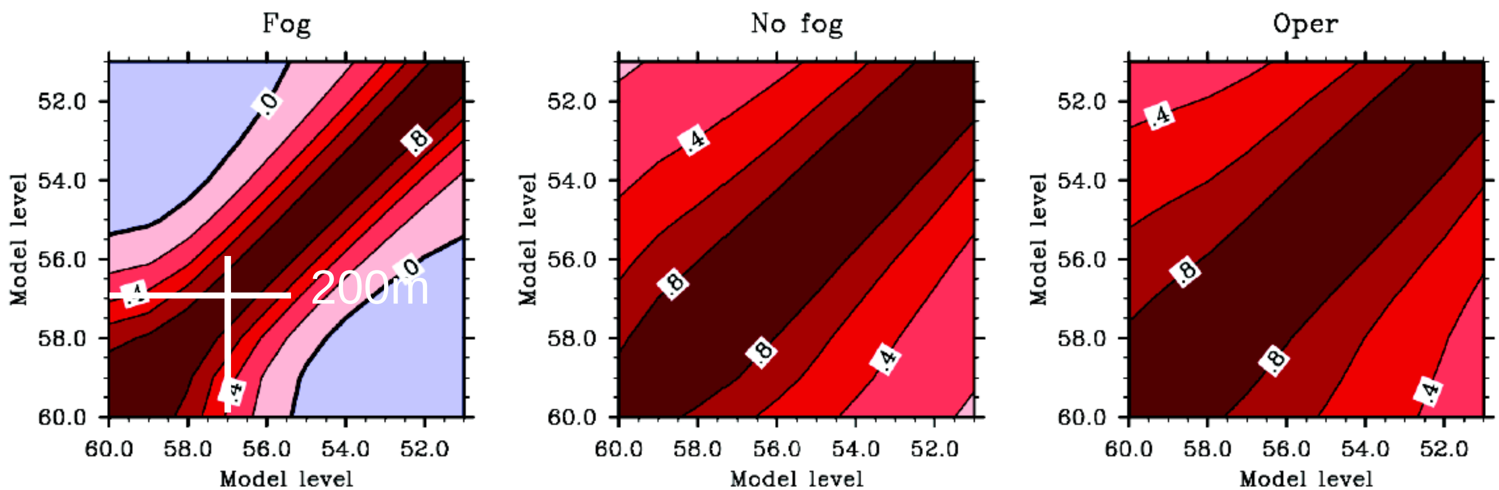
Fraction of explained q variance ratios

Very strong coupling between q and T in fog due to saturation

Vertical correlations for T

(zoom in the first 500m)

Vertical stability of fog

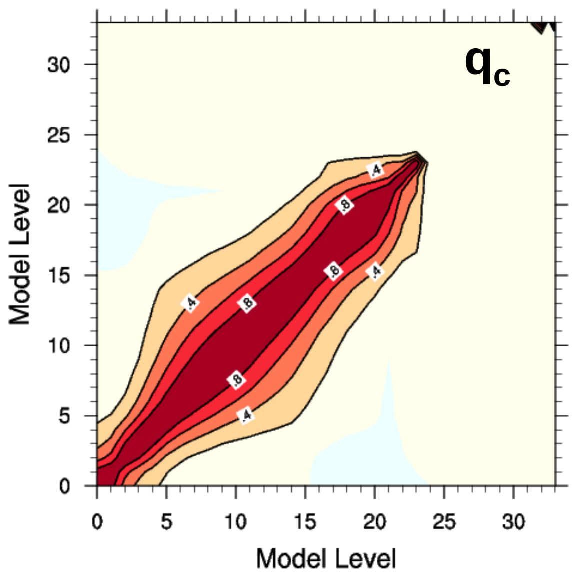


B. Ménétrier

Diagnostics of covariances in clouds and precipitation

Full error covariances for hydrometeors:

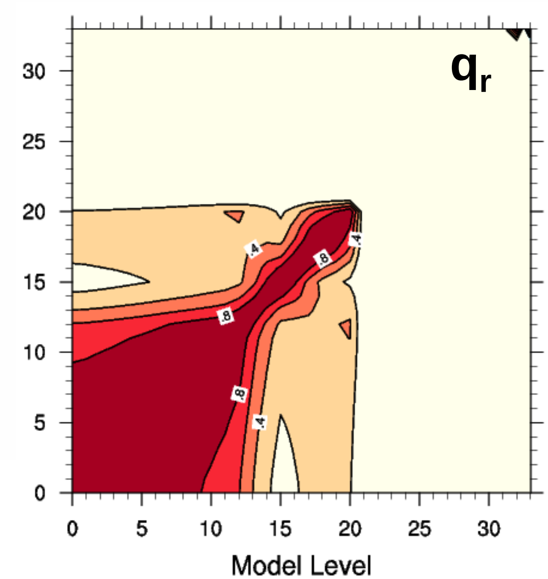
Montmerle and Berre (2010) approach has been also recently extended by Michel et al. (2010) for q_c and q_r using forecast outputs from an EnKF based on WRF, and by extending Berre (2000) multivariate formulations for those variables.



Vertical auto-correlations

Vertical mixing of clouds in convection

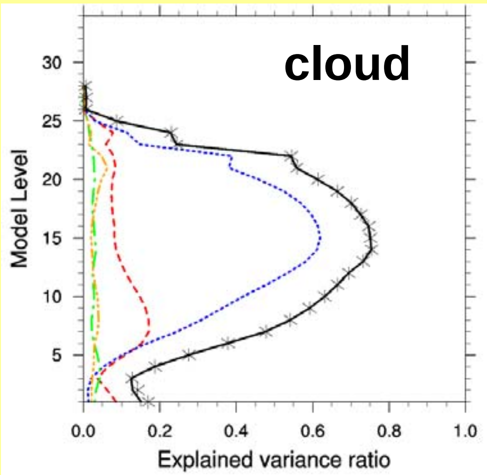
Rain falling bellow the freezing level, some rain in convective towers



Michel, Auligné and Montmerle (2010)

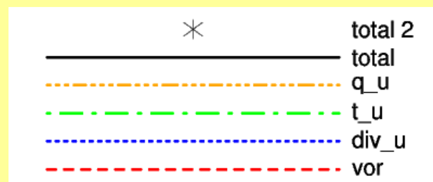
Diagnostics of covariances in clouds and precipitation

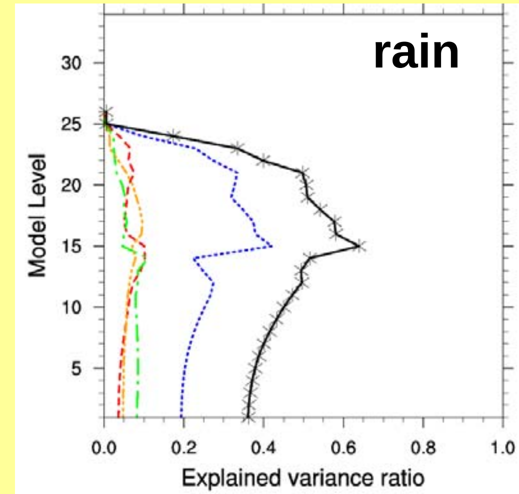
This study highlights the **strong coupling of humidity, cloud and rain content with divergence** (and also the shorter lengthscales in precipitating areas).

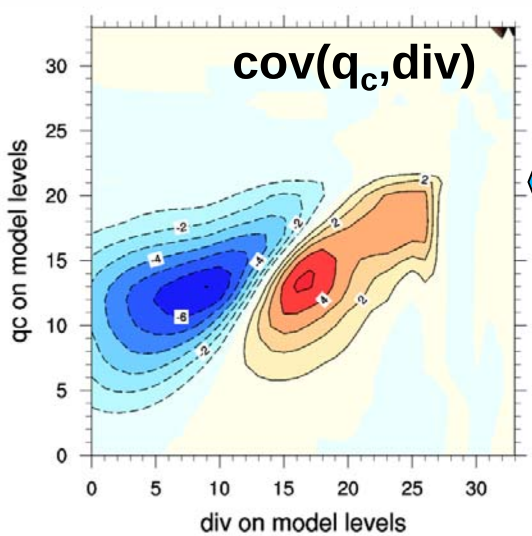


Explained variance ratios

Strong couplings with η_u (but also to q, since a large ratio of q variance is also explained by η_u)



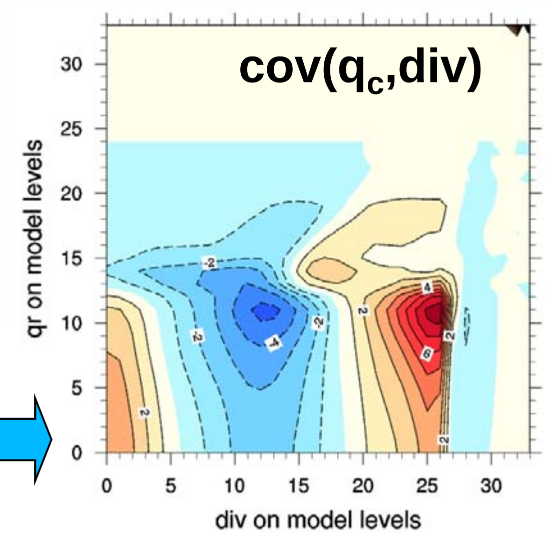




Cross-covariances

Cov(qc,div) shares the same structures than cov (q,div), but translated vertically

Cov(qr,div) is more complex and displays structures that depend on LFC and freezing level hight



Diagnostics of covariances in clouds and precipitation

In clouds and precipitation and for traditional CV, these results suggest that:

• overall, very different statistics between rainy and non-rainy areas were obtained. **Operational formulations of B may thus be far from optimal in clouds and precipitation**

• background error covariances are characterized by balances that are coherent with the model's physic (e.g explicit convection)

• coupling between humidity and divergence is predominant

• horizontal correlation lengths are shorter in precipitation

• vertical correlations reflect the cloud vertical extension due to convection

For liquid cloud and rain, even if hydrometeor errors are probably non-Gaussian distributed, their statistics show :

• reasonable and physically meaningful auto-covariances and statistical couplings with other variables (especially with divergence)

• even shorter horizontal correlation lengths

Outlines

1. Introduction

2. Diagnosis of covariances in clouds and precipitation

3. **How to get cloud and precipitation-dependent statistics?**

4. Conclusions and Possible strategies

How to get cloud and precipitation-dependent statistics?

The structures linked to clouds and precipitations that we want to analyze **are thus likely to be differently balanced, strongly anisotropic and flow dependent.**

Different degrees of weather-dependency in the CVT can be achieved using:

- **4DVar instead of 3DVar:** allows to propagate **B** to the appropriate times of the observations. However, it is reset to its static value at each assimilation cycle.
- **Non linear balance constraints**
- **Ensemble flow-dependent B**
- **Heterogeneous covariances**

How to get cloud and precipitation-dependent statistics?
Non linear balance relationships

$$\delta\eta = MH\delta\tilde{\zeta} + \delta\tilde{\eta}_u + \delta\eta_b(\delta\zeta, \delta T)_\Omega$$

The use of a geostrophic Non-Linear Balance Equation (NLBE) (Barker et al. 2004, Fisher, 2003) **allows to add some flow-dependency** by taking into account cyclostrophic terms that are important in regions of strong curvature (cyclones...)

At regional scale, Carron and Fillon (2010) have shown that **the gap with the geostropic balance increases with precipitation intensities**

⇒ Use of scale-dependent regression coefficients allows however to relax this balance for clouds and precipitation that are rather small scales

Use of Quasi-Geostrophic Omega balance

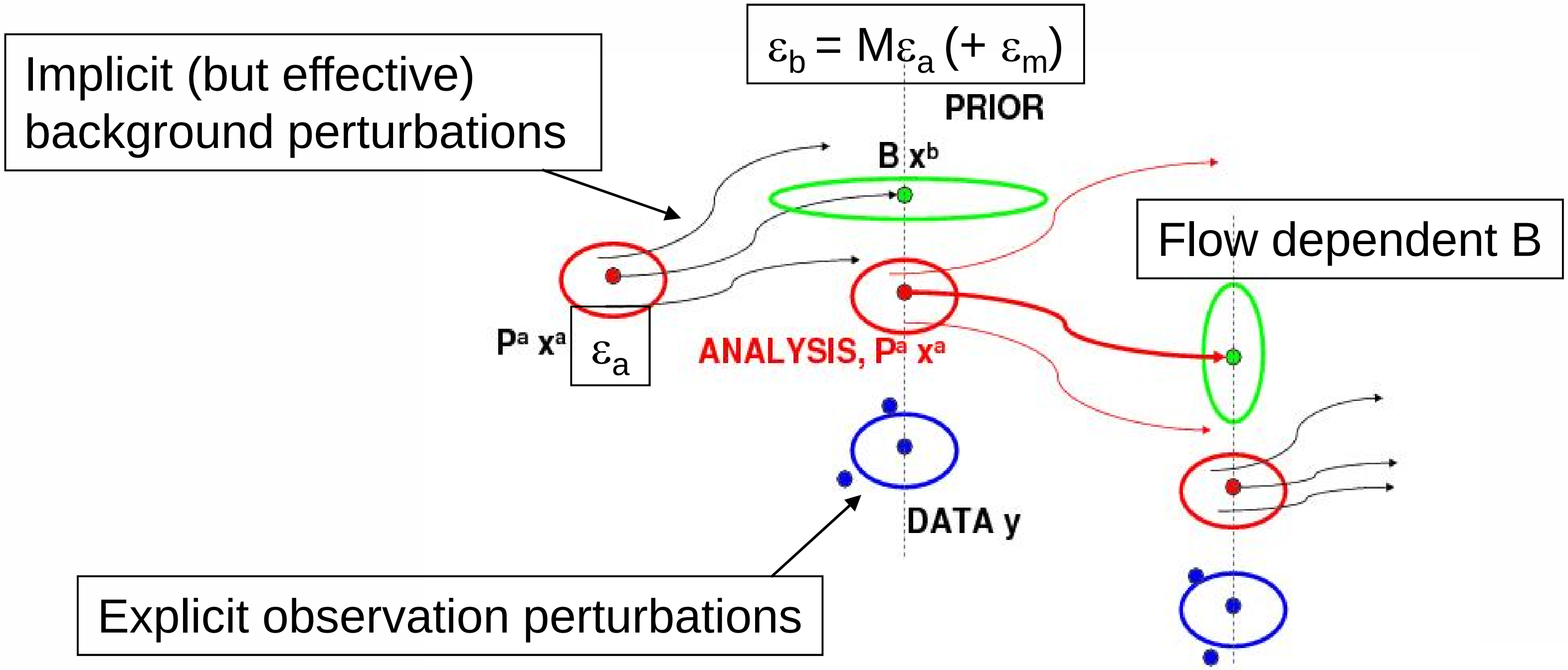
A similar approach allows to add an analytically balanced divergence $\delta\eta_b$ according to the QG omega and continuity equations

⇒ Using a CRM, Pagé et al. (2007) propose to revisit this formulation with **the introduction of diabatic forcing of balanced vertical motion**. No inclusion of these additional terms in CVT formulation have been tried yet.

How to get cloud and precipitation-dependent statistics?
Ensemble-based flow-dependent B

Ensemble-based flow-dependent B:

• **Include partially (hybrid DA) or totally a flow-dependent B matrix**, computed from daily runs of an ensemble assimilation, into 3 or 4D-Var, without significant change of the existing setup of operational VAR system



Houtekamer et al 1996; Fisher 2003 ; Ehrendorfer 2007 ; Berre et al 2006
En4DVar (Liu et al, 2008)

How to get cloud and precipitation-dependent statistics?
Ensemble-based flow-dependent B

Main issue: although very attractive (weather dependent covariances, sharper correlations), **this method is computationally very expensive** (especially for CRMs!).

A solution is to consider an ensemble with few members and use **optimized filtering techniques to reduce sampling error in B**. Main approaches:

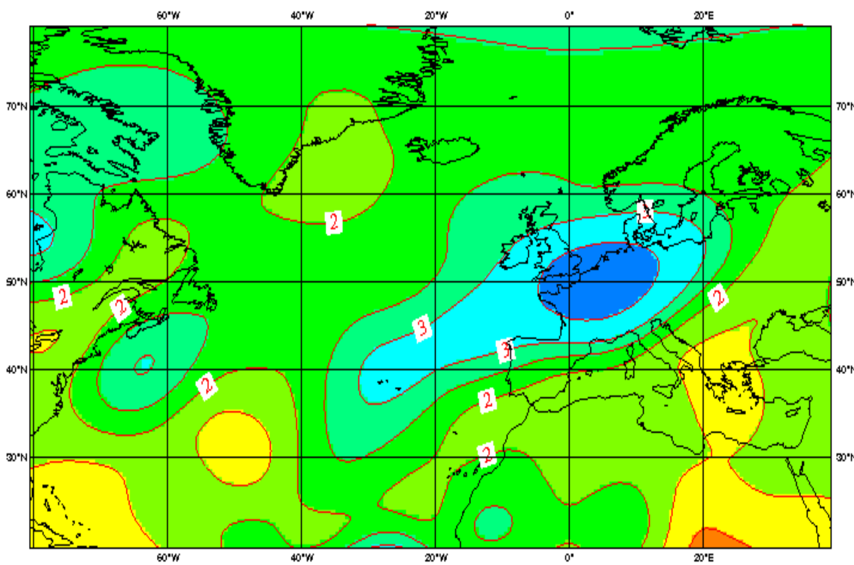
- **Spectrally filtered σ_b** (Raynaud et al, 2009) **and wavelet correlations** (Fisher (2003), Pannekoucke et al. (2007)). At Météo-France, the global Arpege 4DVar uses operationally these « σb's of the day » for all CV deduced from 6 perturbed global members with 4DVar (Berre, 2009)

- **CVT in ensemble sub-space** (En3DVar, En4DVar (Lorenc (2003), Liu (2008), Buehner (2008)) **using localizations with Schur operators** to reduce the analysis noise (Buehner and Charron, 2007)

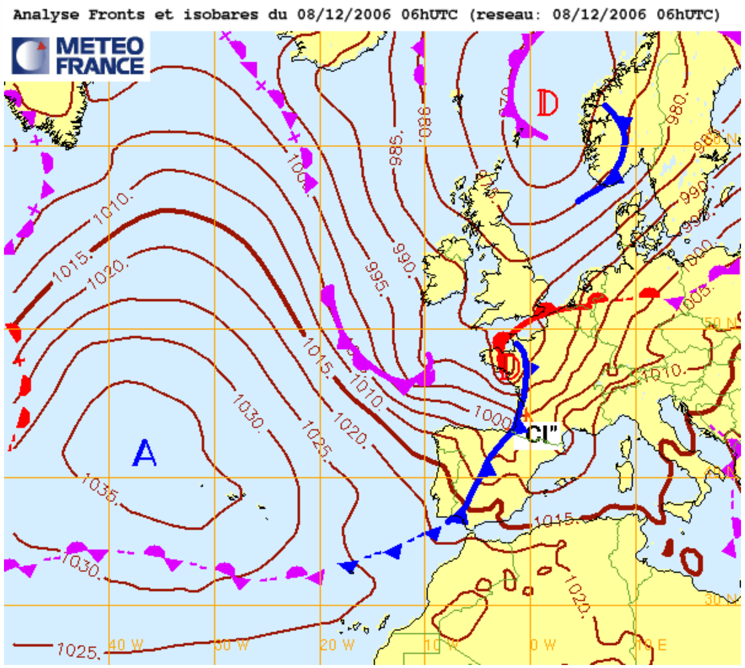
Other spatialization methods could be used for that purpose: **Diffusion operators** (Weaver and Courtier, 2001) or **recursive filters** (Purser et al, 2003)

How to get cloud and precipitation-dependent statistics?
Ensemble-based flow-dependent B

Example of filterred standard deviation « of the day »:



Filtered σ_b for ζ at 500hPa

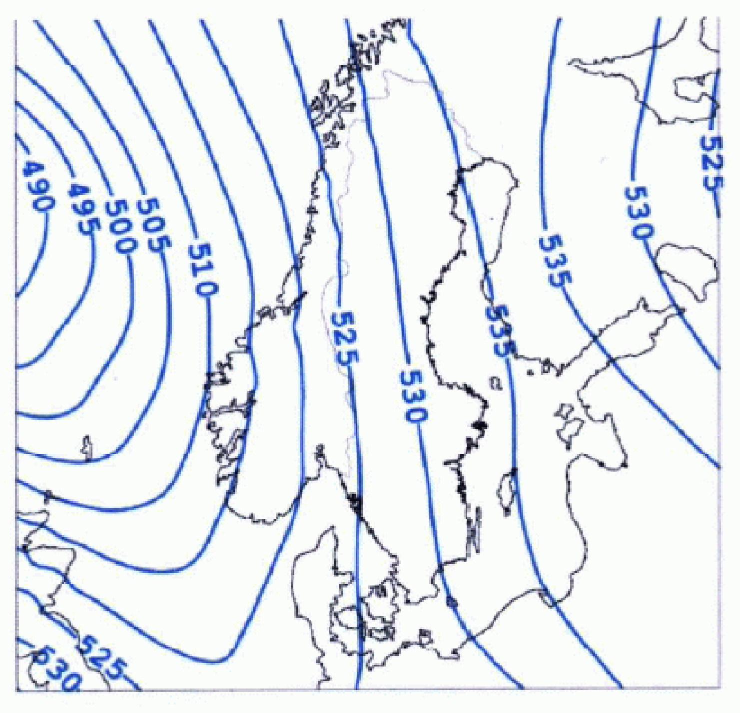


P_{surf}

Ensemble spread: large σ_b associated with a storm over France

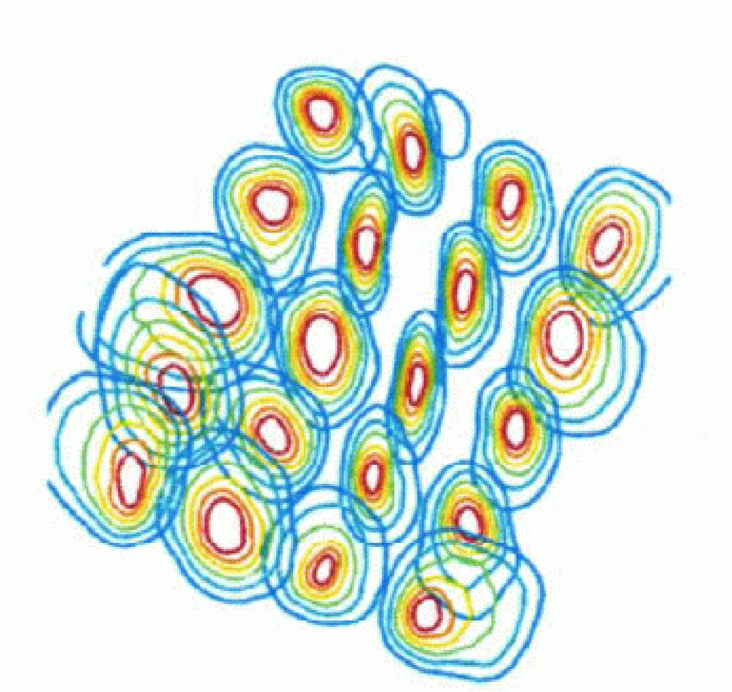
How to get cloud and precipitation-dependent statistics?
Ensemble-based flow-dependent B

Wavelet filtering of flow-dependent correlations



Synoptic situation

(geopotential near 500 hPa)



Anisotropic wavelet based correlation functions

(Lindskog et al 2007, Deckmyn et al 2005)

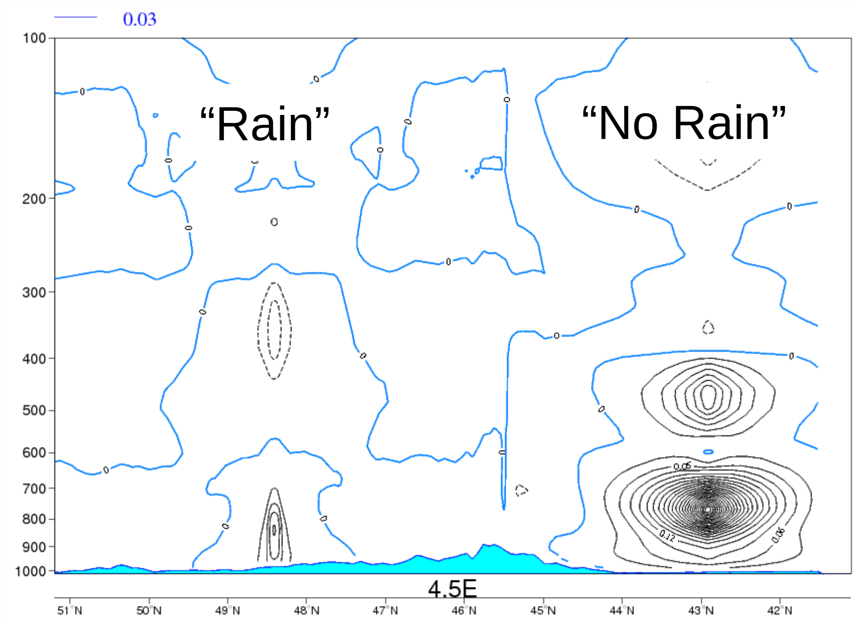
⇒ Correlation lengths are sharper in regions of strong dynamical gradients

How to get cloud and precipitation-dependent statistics?
Heterogeneous B

Adapting ideas of Courtier (1998) and Buehner (2008), to use more suitable background error statistics in precipitating and non-precipitating areas in CVT:

$$\delta x = \alpha^{1/2} B_{np}^{1/2} \chi_1 + \beta^{1/2} B_p^{1/2} \chi_2$$

α and β based on grid point masks
$\mathbf{B_p}$ and $\mathbf{B_{np}}$ being precipitating and non-precipitating background error covariances respectively.



Vertical Cross section of q increments
4 obs exp: Innovations of – 30% RH
At 800 and 500 hPa

⇒ Allows to consider simultaneously very different covariances that are representative of different weather regimes

⇒ Could be used in an ensemble flow-dependent B

Montmerle and Berre (2010)

Outlines

1. Introduction

2. Diagnosis of covariances in clouds and precipitation

3. How to get cloud and precipitation-dependent statistics?

4. Conclusions and Possible strategies

Possible strategies for B matrix modelling in clouds and precipitation

<u>Balance operators:</u>

• **Use more adequate balance relationships involving dynamical fields and a humidity-linked variable**, based on analytical diabatic formulations (Pagé 2007) and/or statistical regressions (Berre 2000). More realistic hydrometeors will be produced after spin-up.

• If hydrometeors are considered in the CV, multivariate formalism may be needed

<u>Spatial transforms</u>

• **Flow dependence can be achieved by using ensemble assimilation** related methods. **Sample noise is the big issue**, but can be addressed efficiently using filtering techniques

• **Heterogeneous covariances** can inherit from these developments, simply by adding masks in the forecast differences step

⇒ One big issue may be to incorporate spatial localization deduced from observations and/or background directly in balance operators

Thank you for your attention…

METEO FRANCE
Toujours un temps d'avance

VAR system	Reference(s)	Control variables	Mass/wind balance	Spatial filter
ECMWF	Courtier *et al.* (1998), Derber and Bouttier (1999)	$\zeta, \eta_u, (T, p_s)_u, q$	NLBE/QG omega	Spectral (horiz.), EOF (vert.)
Met Office	Lorenc *et al.* (2000), Met Office (1995), Ingleby (2001), Rawlins *et al.* (2007)	ψ, χ, p_u, μ	NLBE	Spectral (horiz.), EOF (vert.)
NMC/NCEP	Parrish and Derber (1992), Wu Purser and Parrish (2002), de Pondeca *et al.* (2007)	ζ, η, ϕ_u, q or $\psi, \chi_u, (T, p_s)_u, \mu_{pseudo}$	LBE	Spectral (horiz.), EOF (vert.) or Recursive filter
Météo-France (Arpège)	Sadiki and Fischer (2005)	$\zeta, \eta_u, (T, p_s)_u, q$	NLBE/ QG omega	Spectral (horiz.), EOF (vert.)
Météo-France (Aladin)/AROME	Berre (2000), Fischer *et al.* (2005)	$\zeta, \eta_u, (T, p_s)_u, q_u$	NLBE/ QG omega	Spectral (horiz.), EOF (vert.)
WRF	Skamarock *et al.* (2008), Sun *et al.* (2008)	$\psi, \chi_u, (T, p_s)_u, \mu_{pseudo}$	Regression	Spectral (global), Recursive filter (regional)
NCAR (MM5)	Barker *et al.* (2003), Barker *et al.* (2004)	ψ, χ, p_u, q or μ	Hybrid NLBE/ regression	Recursive filter (horiz.) EOF (vert.)
JMA (global)	JMA (2007)	$\zeta, \eta_u, (T, p_s)_u, \log q$	Regression	Spectral (horiz.), EOF (vert.)
JMA (regional)	Honda *et al.* (2005), JMA (2007)	$(\theta, p_s)_b, u_u, v_u, T_u, \mu_{pseudo}$	Regression	Recursive filter (horiz.), EOF (vert.)
HIRLAM	Gustafsson *et al.* (1999, 2001)	$T, u_u, v_u, \log p_s, q$	LBE	Spectral (horiz.), EOF (vert.)
Canadian MAM	Polavarapu *et al.* (2005)	$\psi, \chi_u, (T, p_s)_u, \log q$	Regression	Spectral (horiz.)
CMC	Gauthier *et al.* (1999), Laroche *et al.* (1999)	ψ, χ, ϕ_u, q	LBE	Spectral (horiz.)
RAMDAS	Zupanski *et al.* (2005)	$u, v, w, \Pi, \theta, q_t$	None	Convolutions (horiz./vert.)

Bannister (2008)

References

Amerault, C. and X. Zou, 2006: Comparison of Model-Produced and Observed Microwave Radiances and Estimation of Background Error Covariances for Hydrometeor Variables within Hurricanes. Mon. Wea. Rev., 134 (3), 745–758.

Barker DM, Huang W, Guo YR, Bourgeois AJ, Xiao QN. 2004. A three-dimensional variational data assimilation system for MM5: Implementation and initial results. Mon. Weather Rev. 132:
897–914.

Berre L., 2000 : Estimation of synoptic and mesoscale forecast error covariances in a limited area model, Mon. Wea. Rev., 128, 644-667.

Berre L., 2009 : Use of ensemble assimilation to represent flow-dependent B in 3D/4D-Var. 8th International Workshop on Adjoint Applications in Dynamic Meteorology 18-22 May 2009, Tannersville, PA, USA

Buehner, M, 2008: Towards an Improved Use of Flow-Dependent Background Error Covariances in a Variational Data Assimilation System. Banff, Canada. February 3-8.

Caron, J.-F. and L. Fillion, 2010: An Examination of Background Error Correlations between Mass and RotationalWind over Precipitation Regions. Mon. Wea. Rev., 138 (2), 563–578.

Chevallier, F., P. Bauer, J.-F. Mahfouf, and J.-J. Morcrette, 2002: Variational retrieval of cloud liquid profile from ATOVS observations. Quart. J. Roy. Meteor. Soc., 128, 2511–2526.

Courtier P, Andersson E, Heckley W, Pailleux J, Vasiljevic D, Hollingsworth A, Fisher M, Rabier F. 1998. The ECMWF implementation of three-dimensional variational assimilation (3D-Var). I: Formulation. Q. J. R. Meteorol. Soc. 124: 1783–1807.

Deckmyn, A., and L. Berre, 2005 : A wavelet approach to representing background error covariances in a limited area model. Mon. Wea. Rev., 133, 1279-1294.

Ehrendorfer M. 2007. A review of issues in ensemble-based Kalman filtering Meteorol. Z. 16: 795–818.

Fisher, M., 2003 : Background error covariance modelling. Proceedings of the ECMWF seminar on recent developments in data assimilation for atmosphere and ocean, 45-63.

METEO FRANCE
Toujours un temps d'avance

References

Liu, C., Q. Xiao, and B. Wang, 2008: An ensemble-based fourdimensional variational data assimilation scheme: Part I: Technical formulation and preliminary test. Mon. Wea. Rev., 136, 3363-3373.

Lorenc AC, Roulstone I, White A. 2003. On the choice of control fields in VAR. Forecasting Research Tech. Report 419 (available from Met Office, Fitzroy Road, Exeter, Devon, EX8 3PB, UK).

Lorenc, A., 2007: Ideas for adding flow-dependence to the Met Office VAR system. ECMWF Workshop on Flow-dependent aspects of data assimilation.

Michel, Y., T. Auligné and T. Montmerle, 2010: diagnosis of convective scale background error covariances within and outside of precipitating areas. MWR, submitted.

Montmerle, T. and Berre, L., 2010: Diagnosis and use of heterogeneous background error covariances at mesoscale. QJRMS, in press.

Moreau, E., P. Bauer, and F. Chevallier, 2003: Variational retrieval of rain profiles from spaceborne passive microwave radiance observations. J. Geophys. Res., 108, 4521, doi:10.1029/2002JD003315.

Pagé C, Fillion L, Zwack P. 2007. Diagnosing summertime mesoscale vertical motion: implications for atmospheric data assimilation. Mon. Weather Rev. 135: 2076–2094.

Pannekoucke, O, L. Berre and G. Desroziers, 2007 : Filtering properties of wavelets for the local background error correlations. Quart. Jour. Roy. Meteor.Soc.133, 363-379

Purser RJ, WuWS, Parrish DF, Roberts NM. 2003. Numerical aspects of the application of recursive filters to variational analysis. Part I: Spatially homogeneous and isotropic Gaussian covariances. Mon. Weather Rev. 131: 1524–1535.

Raynaud L., L. Berre et G. Desroziers, 2009 : Objective filtering of ensemble-based background error variances. Quart. Jour. Roy. Meteor. Soc., 135A, 1177-1199

Weaver A, Courtier P. 2001. Correlation modelling on the sphere using a generalized diffusion equation. Q. J. R. Meteorol. Soc. 127: 1815–1846.

Xiao, Q et al, 2007: An Approach of Radar Reflectivity Data Assimilation and Its Assessment with the Inland QPF of Typhoon Rusa (2002) at Landfall. JAMC, 46, 14-22.

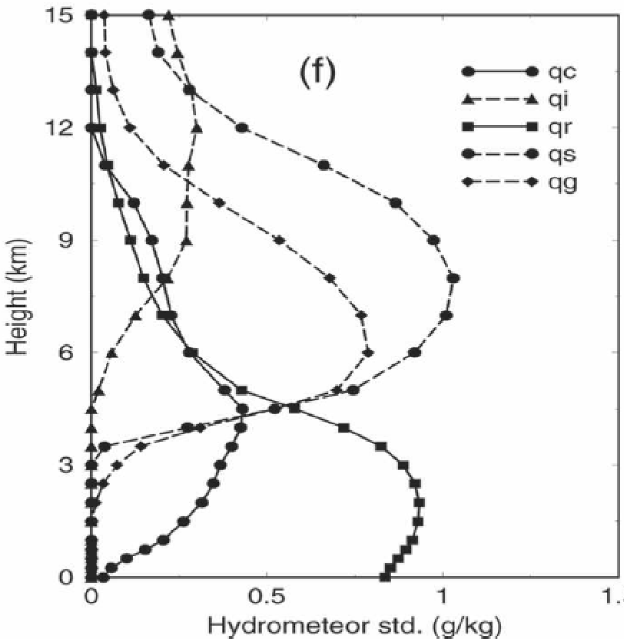
Diagnostics of covariances in clouds and precipitation

1D Error covariances:

• Some work has been done to define *vertical* background error covariance matrices for q_c and q_r to be used in 1D-Var applied to ATOVS, SSMI or TRMM radiances.

⇒ In Moreau et al. (2003), the vertical covariances were calculated *at each forecast point* in the horizontal domain of a global model, **by perturbing profiles of T and q** used as input for the model's moisture scheme.

• Amerault and Zou (2006) has performed statistics on differences of forecasts that were using **different explicit moisture schemes**.



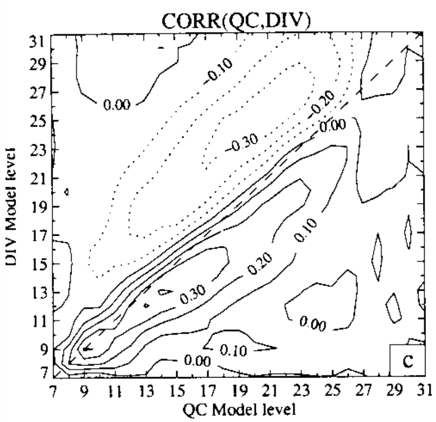
⇒ Retrieved profiles of hydrometeors can then potentially be assimilate in 3D/4DVar, assuming that their *3D* error covariances are known.

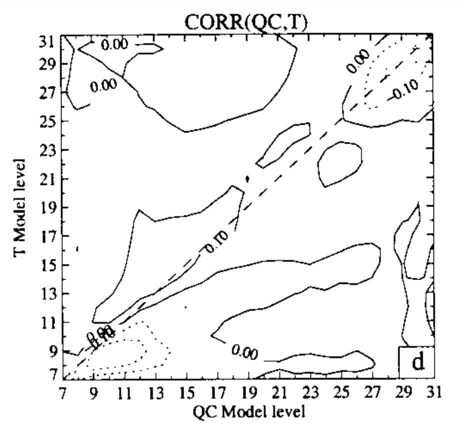
3D Error covariances:

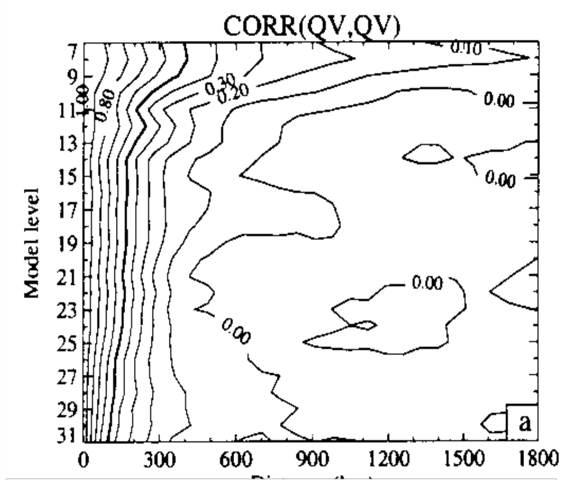
P. Lopez in his PhD Thesis (2001) shows some auto- and cross-correlations between q_c, q_r and the other traditional control variables for ARPEGE global model, using forecast differences for one meteorological situation. Main conclusions for extratropics:

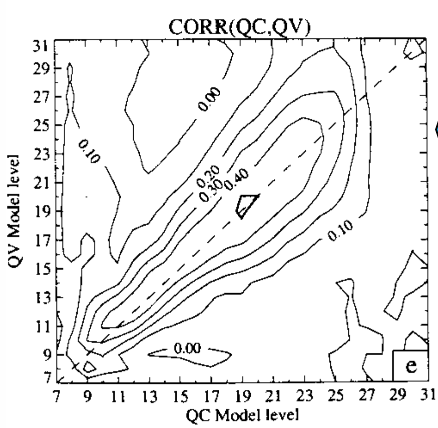
PBL

Top



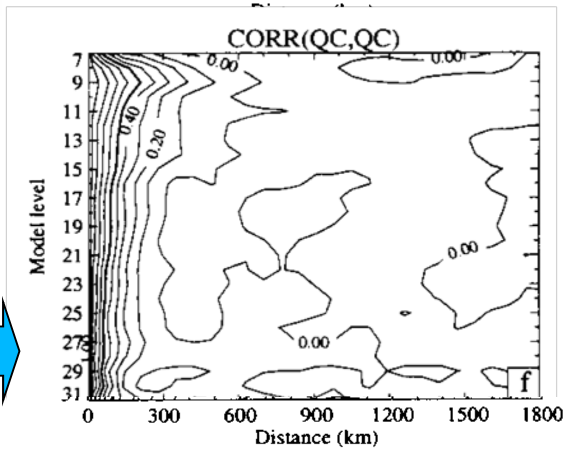






q_c (and q_r) seems decorrelated to T, but correlated to q_v and to divergence

horizontal lengthscales are shorter at all levels for q_c (and q_r)



Top PBL

Spatial and spectral localization of correlations

Needed to reduce the sampling error in ensemble assimilation

Wavelet formulation (Fisher, 2003) allowing to model simultaneously scale and position-dependent aspects of covariances

$$\mathbf{B}_u^{1/2}\chi = \left(\mathbf{S}^{-T}\mathbf{V}^{1/2}\mathbf{S}^{T}\right)\left(\mathbf{E}\mathbf{D}^{1/2}\right)\chi = \mathbf{S}^{-T}\mathbf{V}^{1/2}\sum_{j=1}^{K}\psi_j(r)\otimes\mathbf{C}_j^{1/2}\chi_j$$

• Instead of having one control vector χ ,which is function of wave number and vertical mode, there are now K sub-control vectors χ_i function of (lat,lon,z) per parameter.

• A wavelet $\psi_j(r)$ acts as a bandpass filter when convolved with correlations.

⇒ This approach allows inhomogeneity and anisotropy and thus seems attractive for mesoscale application

⇒ Curvelets may be more adapted for LAM

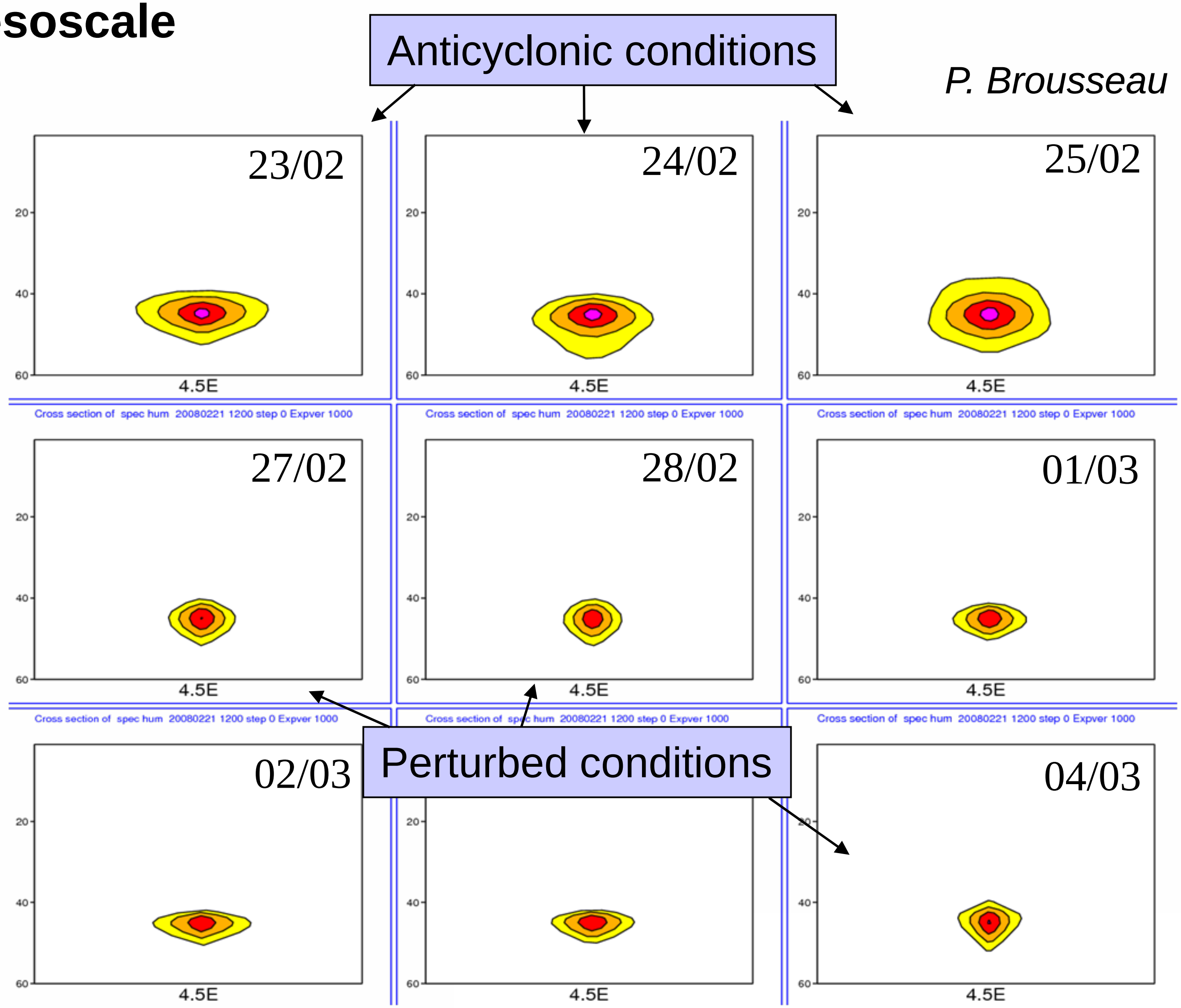
Ensemble-flow dependent B: Example at mesoscale

Daily evolution of mesoscale
δq increments
Vertical cross-sections

P. Brousseau

1 obs experiment
T innovation at 850 hPa

Daily covariances are computed using 3h AROME forecasts from 6 members at each of the 8 analysis times of the day before



⇒ Strong variabilities depending on weather regimes

Spatial and spectral localization of correlations

- **Recursive filters, based on a convolution (in grid space) of a field with a Gaussian-shaped kernel** (Purser et al, 2003a). Used in WRF, MM5 and JMA regional models to simulate isotropic and homogeneous horizontal correlations (More details in Yann Michel's talk).

Inhomogeneity can be added by varying geographically smoothing scales

To stretch covariance functions (and thus to add anisotropy), nonstandard grid lines have to be included among the set of direction along which recursive smoothing operators apply (Purser et al, 2003b)

At convective scales, inhomogeneity can be assessed through non trivial normalization and specific tunings, and anisotropy can be obtained but is technically difficult

Spatial and spectral localization of correlations

• **Diffusion operators** (Weaver and Courtier, 2001): based on replacing the Laplacian operator in the generalized diffusion equation by a polynomial in the Laplacian.

$$\frac{\partial \eta}{\partial t} - \kappa \nabla^2 \eta = 0$$

The integral solution of this equation can be interpreted, after appropriate normalization, as a covariance operator on the sphere.

The shape (spectrum) of the correlation function can be controlled by adjusting the relative weights of the different Laplacian terms.

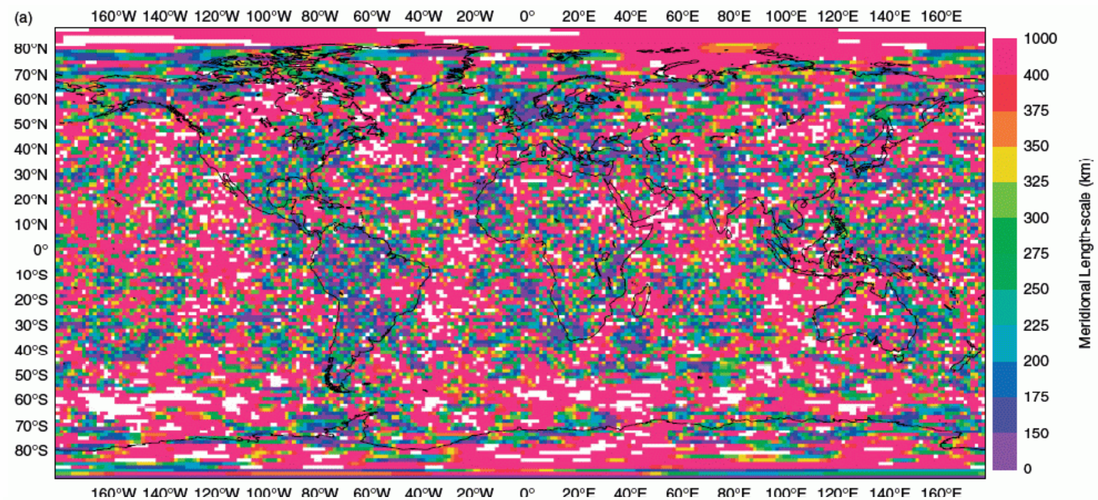
The correlation functions can be made anisotropic by stretching and/or rotating the computational coordinate system via a ‘diffusion’ tensor.

Could be adapted for LAM applications. « Playing » on diffusion coefficients to get weather-dependent length-scales could be interesting. Filtering of sampling noise still needs to be addressed

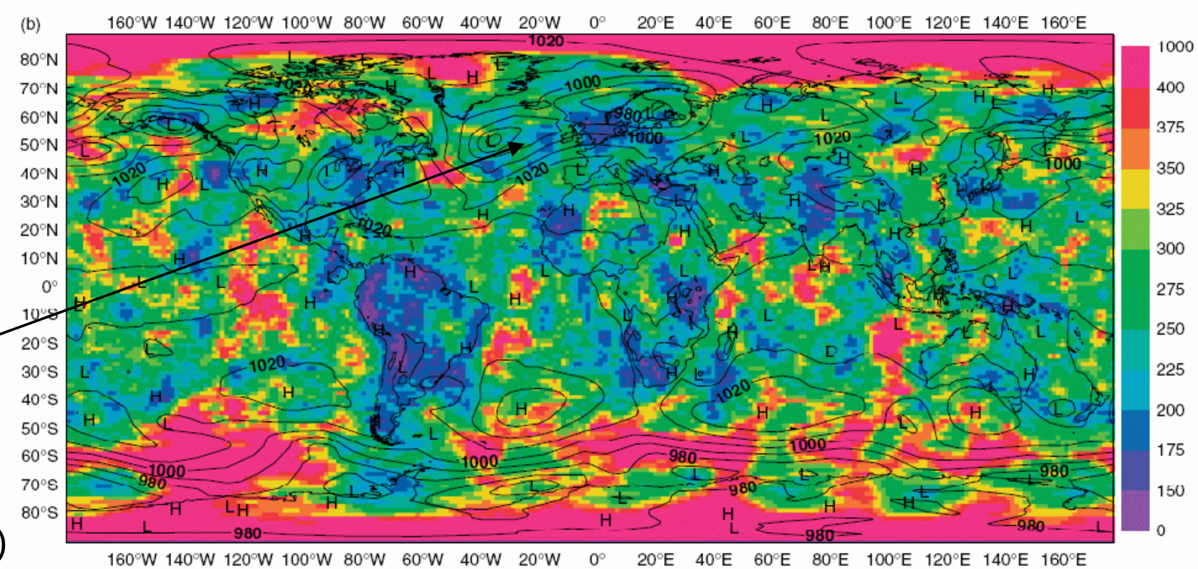
Ensemble-flow dependent B: wavelet correlations

Meridional length-scales (km) for surface pressure on 10 February 2002 at 12 UTC, computed from 6 ensemble members

Raw length-scales



Wavelet-implied length-scales, superimposed on the background field of sea-level pressure



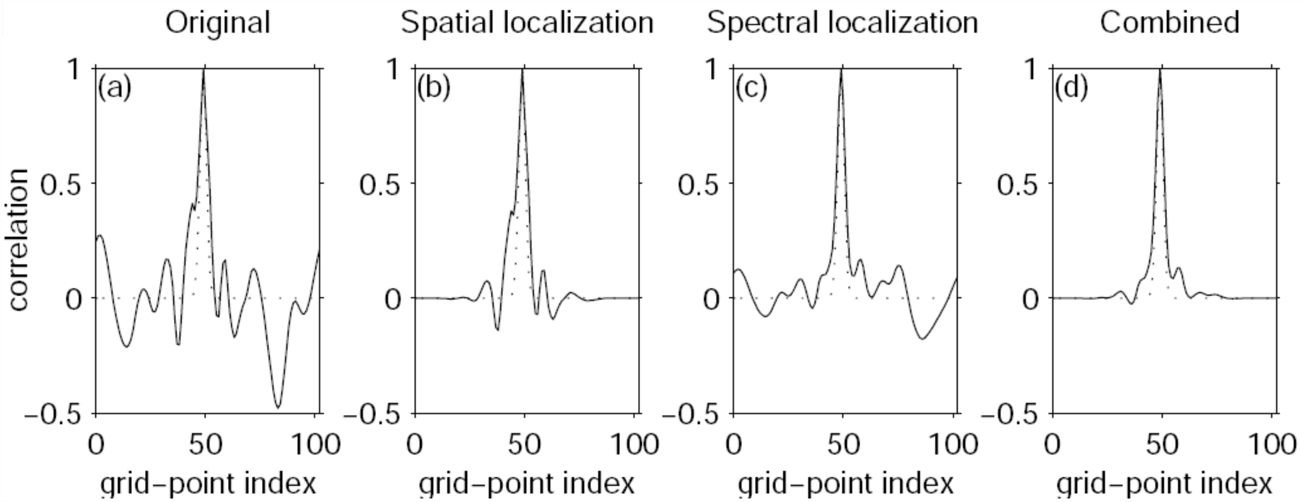
Shorter correlation lengths in regions of large gradients

Pannekoucke, et al. (2007)

Spatial and spectral localization of correlations

Main approaches:

• **Schur product of correlations with localization functions in the spatial and spectral domains** (Buehner and Charron, 2007). These functions are simple “correlation” matrices with monotonically decreasing values as a function of separation distance

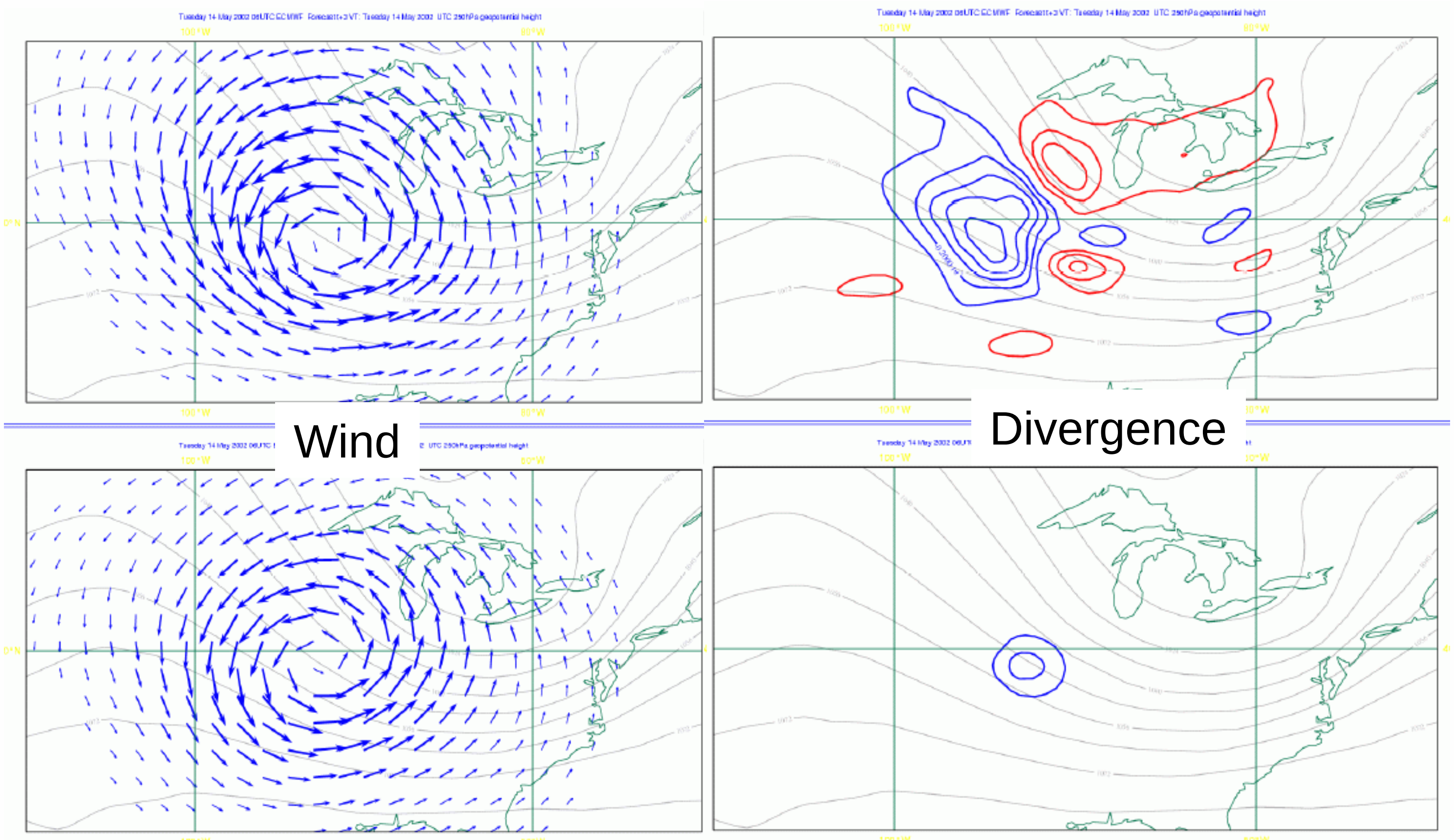


Spatial and spectral localizations improve long and short ranges respectively. Combination seems to give best results.

They show that localization of correlations in spectral space (multiplication) is equivalent with spatial averaging of correlations in grid-point space (convolution) => mainly usefull to reduce sampling error in an ensemble.

How to get weather-dependent statistics?
Non linear balance relationships

Increments at level 31 from a single height observation at 300hPa



Top panels: Jb includes Nonlinear balance and QG omega equations
Bottom panels: Linear balance only

Fisher (2003)

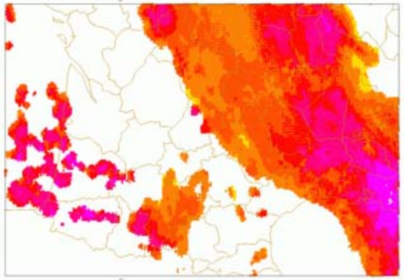
⇒ Enforcement of coherent extratropical divergent flow structures

Real case experiment

CNTRL: AROME oper + Reflectivities

EXP: CNTRL using simultaneously (B_p, B_{np})

Mask deduced from observed reflectivities (zoom)



δT_{950hPa}

δq_{800hPa}