Recent Advances in Global Nonhydrostatic Modeling at NCEP

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- Global models approaching limits of hydrostatic approximation

- Experience with nonhydrostatic models on cloud scale not directly or entirely applicable in NWP, NWP deals with wider range of temporal and spatial scales

- Nonhydrostatic models for (mostly regional) NWP developed and implemented (e.g. Davies et al. 2005; Doms and Schaettler 1997; Janjic et al. 2001; Janjic 2003; Room et al. 2006; Saito et al. 2007; Skamarock and Klemp 2008; Steppeler et al. 2003; Yeh et al., 2002)

- General criteria for a successful nonhydrostatic NWP model
  - Accuracy must not be inferior to that of mature hydrostatic models
  - Computational efficiency, uncertainties concerning the benefits from nonhydrostatic dynamics at transitional resolutions
  - Nonhydrostatic dynamics should be capable of reproducing strongly nonhydrostatic flows at very high resolutions as proof of concept

- Novel, evolutionary, approach in the last two generations of NCEP models (Janjic et al., 2001; Janjic, 2003)
  - Regional Nonhydrostatic *Mesoscale* Model (WRF NMM) within Weather Research and Forecasting (WRF)
  - Unified Nonhydrostatic *Multiscale* Model on the Arakawa B grid (NMMB) (Janjic, 2005; Janjic and Black, 2007)
    - Within NOAA Environmental Modeling System (NEMS)
    - For a broad range of spatial and temporal scales extending from LES to NWP and climate studies on regional and global scales

- Hydrostatic approximation relaxed in model formulations based on modeling principles proven in practice through several model generations (e.g., Janjic 1977, 1979, 1984) (although numerical schemes evolved over time and over two orders of magnitude in resolution)
  - Applicability extended to nonhydrostatic motions
  - Favorable features of the hydrostatic formulation preserved
  - Nonhydrostatic effects introduced through an on/off add-on module
    - Comparison of hydrostatic and nonhydrostatic solutions at fine resolutions
    - Computational efficiency at coarse resolutions

- Vertical coordinate based on mass (hydrostatic pressure)
  - Nondivergent flow remains on constant pressure surfaces
  - Conservation properties easily maintained
- Generalized terrain-following vertical coordinate s that varies from 0 at the model top to 1 at the surface (Simmons and Burridge, 1981)
- No linearizations or additional approximations required, fully compressible system consistent with Laprise (1992)
- Nonhydrostatic equations split into two parts
  - Hydrostatic part, except for higher order terms due to vertical acceleration
  - The part that allows computation of the corrections in the first part

Inviscid adiabatic equations, hydrostatic except for p and ε

π Hydrostatic pressure

p Nonhydrostatic pressure

$\mu = \pi_S - \pi_T$ Difference between hydrostatic pressures at surface and top

$\alpha = RT/p$ Gas law

$$\frac{\partial \Phi}{\partial \pi} = -\alpha$$ Hypsometric (not "hydrostatic") Eq.

$$\left[\frac{\partial}{\partial t}\left(\frac{\partial \pi}{\partial s}\right)\right]_s + \nabla_s \cdot \left(\mathbf{v}\frac{\partial \pi}{\partial s}\right) + \frac{\partial}{\partial s}\left(\dot{s}\frac{\partial \pi}{\partial s}\right) = 0$$ Hydrostatic continuity Eq.

Continued …

$$w \equiv \frac{dz}{dt} = \frac{1}{g}\left[\left(\frac{\partial \Phi}{\partial t}\right)_s + \mathbf{v}\cdot\nabla_s \Phi + \left(\dot{s}\frac{\partial \pi}{\partial s}\right)\frac{\partial \Phi}{\partial \pi}\right]$$

Vertical velocity definition, nonhydrostatic continuity Eq.

$$\varepsilon \equiv \frac{1}{g}\frac{dw}{dt} = \frac{1}{g}\left[\left(\frac{\partial w}{\partial t}\right)_s + \mathbf{v}\cdot\nabla_s w + \left(\dot{s}\frac{\partial \pi}{\partial s}\right)\frac{\partial w}{\partial \pi}\right]$$

$$\frac{\partial p}{\partial \pi} = 1 + \varepsilon$$

Third Eq. of motion

$$\frac{d\mathbf{v}}{dt} = -(1+\varepsilon)\nabla_s \Phi - \alpha \nabla_s p + f\mathbf{k}\times\mathbf{v}$$

Momentum Eq.

$$\frac{\partial T}{\partial t} = -\mathbf{v}\cdot\nabla_s T - \left(\dot{s}\frac{\partial \pi}{\partial s}\right)\frac{\partial T}{\partial \pi} + \frac{\alpha}{c_p}\left[\frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla_s p + \left(\dot{s}\frac{\partial \pi}{\partial s}\right)\frac{\partial p}{\partial \pi}\right]$$

Thermodynamic Eq.

- “Novel approach”?
  - Mass based vertical coordinate (compared to MM5, DWD LM …)
  - Only one extra prognostic equation (instead of usual two)
    - z can be computed from temperature, hydrostatic and nonhydrostatic pressures, and therefore w (and dw/dt) is not independent variable
  - No awkward computation of 3D divergence
  - Preservation of successful hydrostatic “asymptote”
  - Add-on nonhydrostatic module
  - Simple integration algorithm
  - Fast, (10-20%) of the dynamics or less in global case

- Time stepping
  - Explicit, except for vertical advection and vertically propagating sound waves
  - Fast, no redundant computations
  - Different schemes for different processes:
    - Modified Adams-Bashforth for horizontal advection of u, v, T and Coriolis force
      - Weak linear instability stabilized by slight off-centering

$$\frac{y^{\tau+1}-y^{\tau}}{\Delta t}=1.533 f(y^{\tau})-0.533 f(y^{\tau-1})$$

- Crank-Nicholson for vertical advection of u, v, T

$$\frac{y^{\tau+1} - y^{\tau}}{\Delta t} = \frac{1}{2}[f(y^{\tau+1}) + f(y^{\tau})]$$

- Forward-Backward (Ames, 1968; Gadd, 1974; Janjic and Wiin-Nielsen, 1977; Janjic 1979, Beitrage) for fast waves

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x}, \frac{\partial h}{\partial t} = -H\frac{\partial u}{\partial x}$$

$$h^{\tau+1} = h^{\tau} - \Delta t\, H\delta_x u^{\tau}, u^{\tau+1} = u^{\tau} - \Delta t\, g\delta_x h^{\tau+1}$$

- Implicit for vertically propagating sound waves (Janjic et al., 2001; Janjic, 2003)

- Hydrostatic continuity Eq. forward

$$\dot{s} \equiv ds/dt = 0 \text{ at } s = 0,\ s = 1$$

$$\left(\dot{s}\frac{\partial \pi}{\partial s}\right)_s^n = -\left(\frac{\partial \pi}{\partial t}\right)_s^n - \int_0^s \nabla_{s'} \cdot \left(\mathbf{v}\frac{\partial \pi}{\partial s'}\right)^n ds'$$

$$\mu^{n+1} = \mu^n - \Delta t \int_0^1 \nabla_{s'} \cdot \left(\mathbf{v}\frac{\partial \pi}{\partial s'}\right)^n ds'$$

Continued …

- First fractional step

$$p_1 = p^n + (1+\varepsilon^n)\Delta t\left(\frac{\partial \pi}{\partial t}\right)^n$$

$$\omega_1 = \mathbf{v}^n \cdot \nabla_s p^n - (1+\varepsilon^n)\int_0^s \nabla_{s'} \cdot \left(\mathbf{v}\frac{\partial \pi}{\partial s'}\right)^n ds'$$

$$T_1 = T^n + \frac{\Delta t}{c_p}\frac{RT^n}{p^n}\omega_1 - \Delta t\left[\mathbf{v}^n \cdot \nabla_s T^{n+1/2} - \left(\dot{s}\frac{\partial \pi}{\partial s}\right)^n \frac{\partial T^{n+1/2}}{\partial \pi^n}\right]$$

Continued …

$$\Phi_1 = \Phi_S + \int_s^1 \frac{RT_1}{p_1}\left(\frac{\partial \pi}{\partial s'}\right)^{n+1} ds'$$

$$\Phi^{n+1} - \Phi_1 \le O(\Delta t)$$

$$g\,w_1 = \frac{\Phi_1 - \Phi^n}{\Delta t} + \mathbf{v}^n \cdot \nabla_s \Phi_1 + \left(\dot{s}\,\frac{\partial \pi}{\partial s}\right)^n \frac{\partial \Phi_1}{\partial \pi^n}$$

$$g\,\varepsilon_1 = \frac{w_1 - w^{n-1/2}}{\Delta t} + \mathbf{v}^n \cdot \nabla_\sigma w_1 + \left(\dot{s}\,\frac{\partial \pi}{\partial s}\right)^n \frac{\partial w_1}{\partial \pi^n}$$

- Finalization step

$$\Phi^{n+1} = \Phi_S + \int_s^1 \frac{RT^{n+1}}{p^{n+1}} \left(\frac{\partial \pi}{\partial s'} \right)^{n+1} ds'$$

$$T^{n+1} - T_1 = \frac{1}{c_p} \frac{RT_1}{p_1} (p^{n+1} - p_1)$$

$$w^{n+1/2} - w_1 = \frac{\Phi^{n+1} - \Phi_1}{g\Delta t}$$

$$w^{n+1/2} - g\Delta t \frac{\partial p^{n+1}}{\partial \pi^{n+1}} = w_1 - g\Delta t (1 + \varepsilon_1)$$

Four coupled equations with four unknowns!

Continued …

Eliminate all unknowns except p^{n+1}

Discretize in the vertical

$$l-1/2,\pi,p \qquad l,T \qquad l+1/2,\pi,p$$

$$\frac{\Delta p^*}{\Delta\pi^{n+1}} \equiv (1+\varepsilon_1) \Rightarrow p^{n+1} \approx p_1 \approx p^*$$

$$\frac{R(1-\kappa)}{(g\Delta t)^2}\sum_{k=l}^{k=lm} T_{1l}\left(\frac{\overline{p_1}^{\pi}_l - \overline{p^{n+1}}^{\pi}_l}{\overline{p^{n+1}}^{\pi}_l\,\overline{p_1}^{\pi}_l}\right)\Delta\pi_l^{n+1} = \frac{\Delta p_l^{n+1} - \Delta p_l^*}{\Delta\pi_l^{n+1}}$$

Empirical, fast convergence if iterated from

$$p_{2l+1/2} = p_{1l+1/2} + 0.35\left(p_{2l-1/2} + \varepsilon_1\Delta\pi_l - p_{1l+1/2}\right)$$

Well justified approximation, not needed if iterating

$$\frac{R(1-\kappa)}{(g\Delta t)^2}\sum_{k=l}^{k=lm} \frac{T_{1l}}{\overline{p}^2_{2l}}\left(\overline{p_1}^{\pi}_l - \overline{p^{n+1}}^{\pi}_l\right)\Delta\pi_l^{n+1} = \frac{\Delta p_l^{n+1} - \Delta p_l^*}{\Delta\pi_l^{n+1}}$$

Continued …

$l-1/2$

l

$l+1/2$

$$\frac{R(1-\kappa)}{(g\Delta t)^2}\frac{T_{1l}}{\overline{p}_{2l}^{2}}\left[\overline{p_1}^{\pi}_{l} - \frac{1}{2}\left(p_{l-1/2}^{n+1}+p_{l+1/2}^{n+1}\right)\right]\Delta\pi_l^{n+1} =$$

$$\frac{p_{l+1/2}^{n+1}-p_{l-1/2}^{n+1}-\Delta p_l^{*}}{\Delta\pi_l^{n+1}} - \frac{p_{l+3/2}^{n+1}-p_{l+1/2}^{n+1}-\Delta p_{l+1}^{*}}{\Delta\pi_{l+1}^{n+1}}$$

Three-diagonal system for p^{n+1}

$p=\pi$ at $s=0$,

$$\frac{\Delta(p-p^{*})}{\Delta\pi}=0 \text{ at } s=1$$

Backsubstitute p^{n+1} into T^{n+1}, w^{n+1} and ε^{n+1}

Complete the time step by updating the horizontal momentum Eq.

- What is the final step doing?
  - Consider equations linearized around a horizontally homogenous atmosphere at rest and in hydrostatic equilibrium
  - Consider only the solutions that preserve horizontal homogeneity (eliminates motions that belong to the first part of the time stepping procedure, only solutions of the linearized set of the four coupled equations left)
  - Vertically propagating sound waves

$$\frac{p'^{n+1} - 2p'^{n} + p'^{n-1}}{\Delta t^2} = \frac{c_p}{c_v} R T_0 \frac{\partial^2 p'^{n+1}}{\partial z_0^2}$$

- Conservation of important properties of continuous system (Arakawa, 1966, 1972, …; Jacobson, 2001; Janjic, 1977, …; Sadourny, 1968, … ; Tripoli, 1992 … aka "mimetic" approach in Comp. Math)
  - Nonlinear energy cascade controlled through energy and enstrophy conservation
  - "Finite volume"
  - A number of first order and quadratic quantities conserved
  - A number of properties of differential operators preserved
  - Omega-alpha term, consistent transformations between KE and PE
  - Errors associated with representation of orography minimized

- Coordinate system and grid
  - Global lat-lon, regional rotated lat-lon, more uniform grid size
  - Other geometries considered (Purser and Janjic, 2005)
  - Arakawa B grid (in contrast to the WRF-NMM E grid)

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h   h   h
  v   v
h   h   h
  v   v
h   h   h
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  - Pressure-sigma hybrid (Simmons and Burridge 1981)
  - Lorenz vertical grid

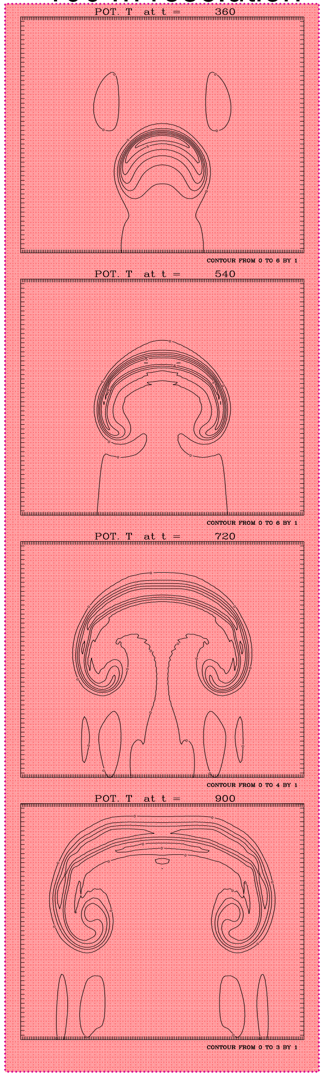
- Polar filter configuration
  - “Decelerator”
  - Tendencies of T, u, v, Eulerian tracers, divergence, dw/dt, deformation
  - Physics not filtered
- Polar filter formulation
  - Waves in the zonal direction faster than waves with the same wavelength in the latitudinal direction slowed down
  - Filter response function multiple quasi 1-2-1 (on filtered part of spectrum)

- NCEP WRF NMM "standard" physical package (more options will be available)
- Further upgrades in NMMB
  - Affordable Eulerian positive definite, conservative and monotone tracer advection scheme
  - Gravity wave drag (Kim & Arakawa 1995; Lott & Miller 1997; Alpert, 2004)
  - RRTM radiation (Mlawer et al. 1997, implemented by Carlos Perez)

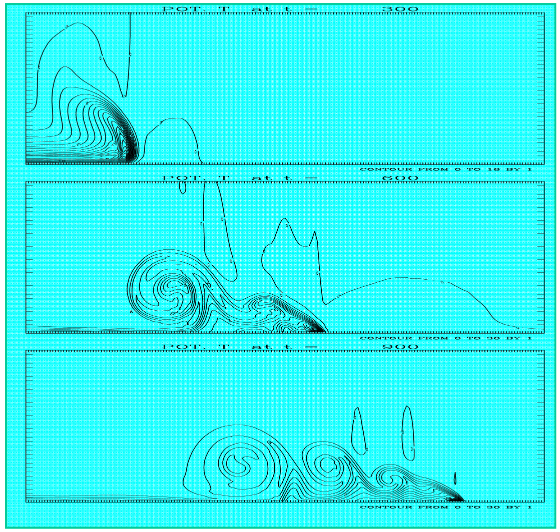
- WRF NMM 4km runs in support of Storm Prediction Center since 2004
- WRF NMM the main NWS regional short range forecasting model for North America since June 2006
- WRF NMM dynamics in Hurricane WRF (HWRF), operational since 2007
- Regional NMMB pre-operationally tested, planned to replace the WRF NMM in operations in the second quarter of 2011
- Global NMMB tested in parallel runs for over a year with moderate resolution

2D very high resolution tests

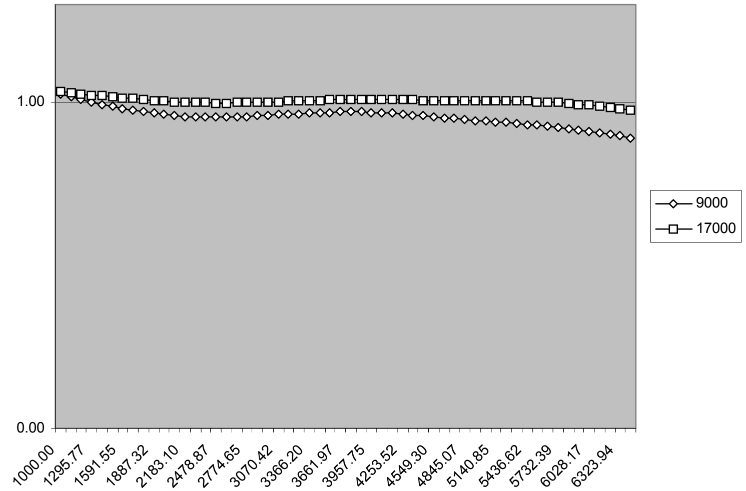
Warm bubble,
100 m resolution



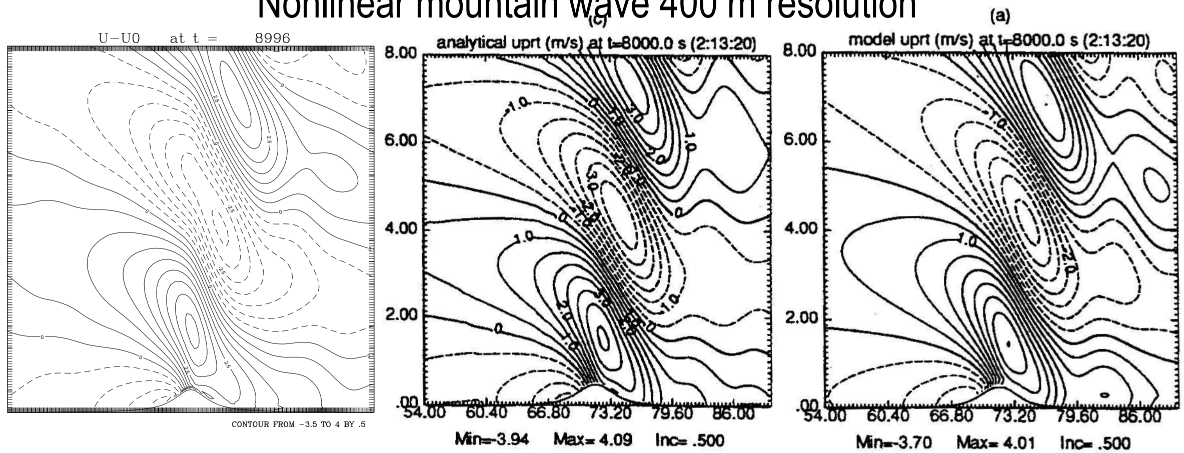
Cold bubble, 100 m resolution



Normalized vertical momentum flux,
400 m resolution



Nonlinear mountain wave 400 m resolution

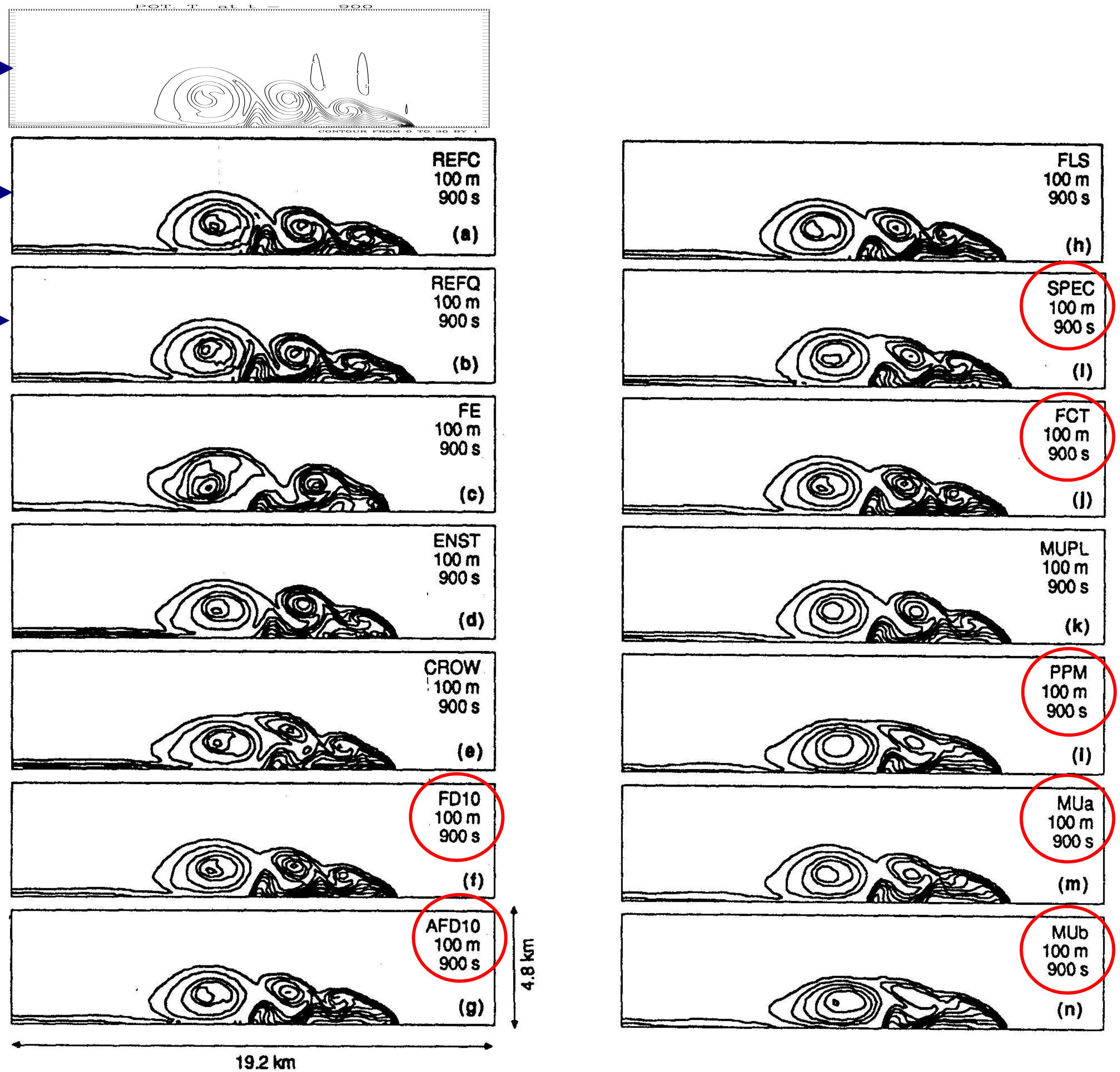


Full compressible NMM Analytical (Boussinesque) ARPS (Boussinesque)

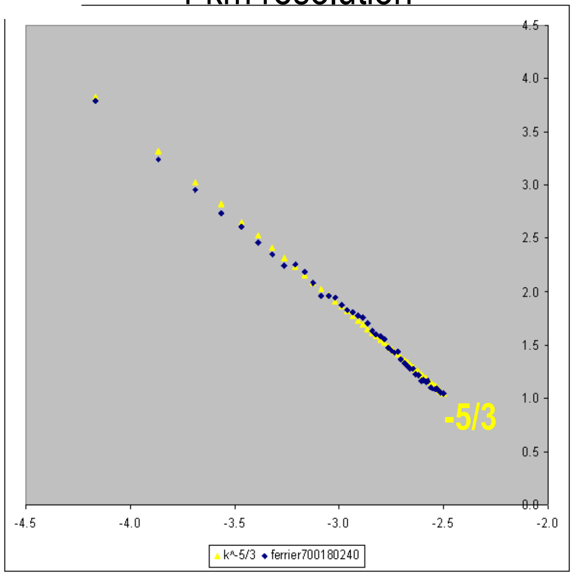
Janjic et al. 2001

Reference

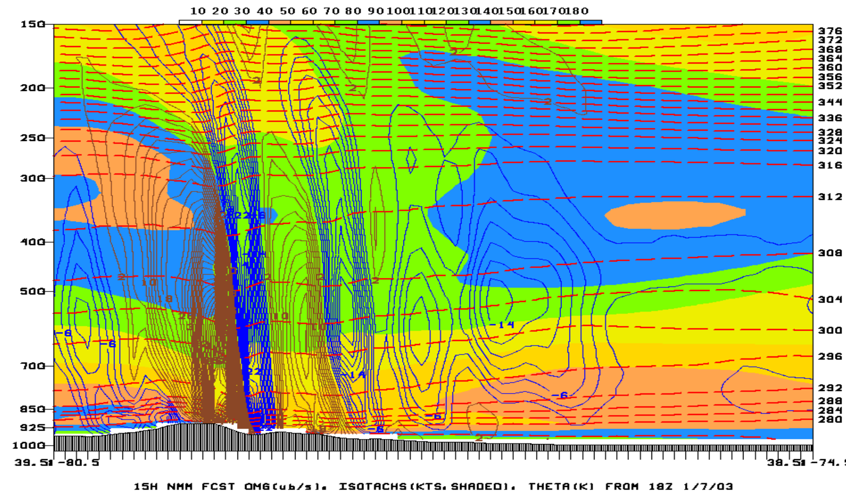
Reference



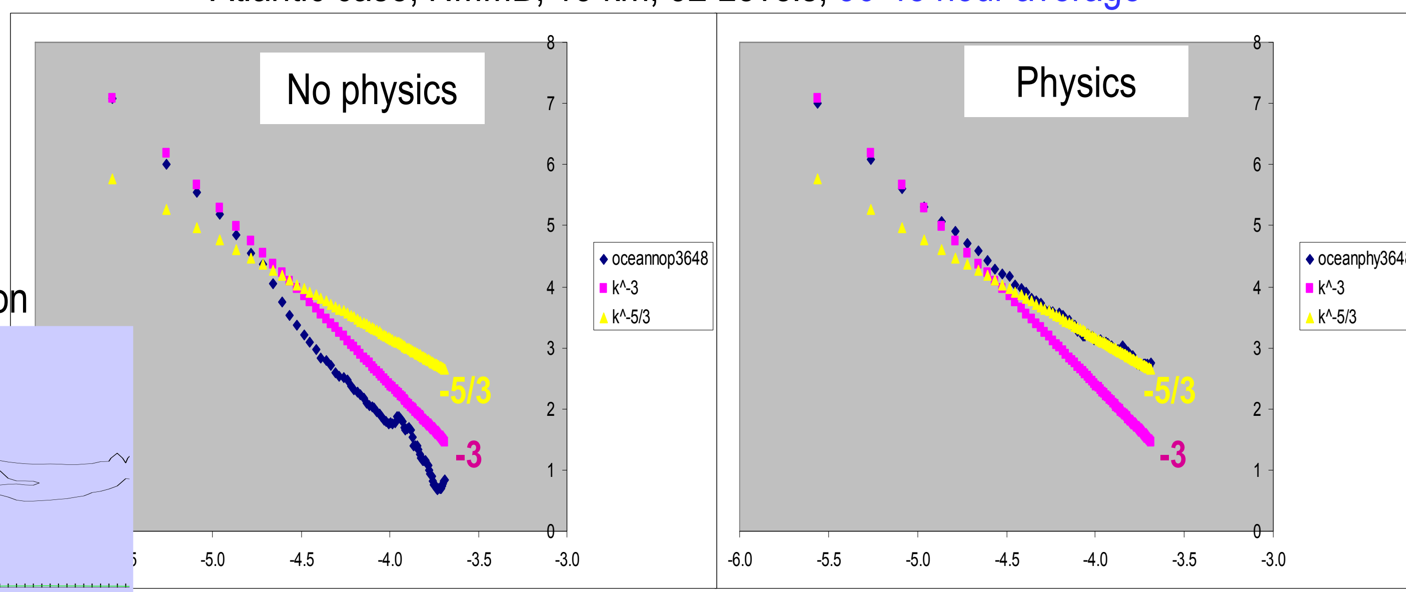
Decaying 3D turbulence,
1 km resolution



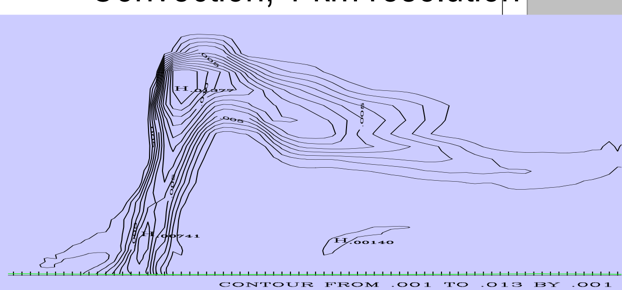
Mountain waves, 8 km resolution



Atlantic case, NMMB, 15 km, 32 Levels, 36-48 hour average



Convection, 1 km resolution



4km resolution, no parameterized convection

2004

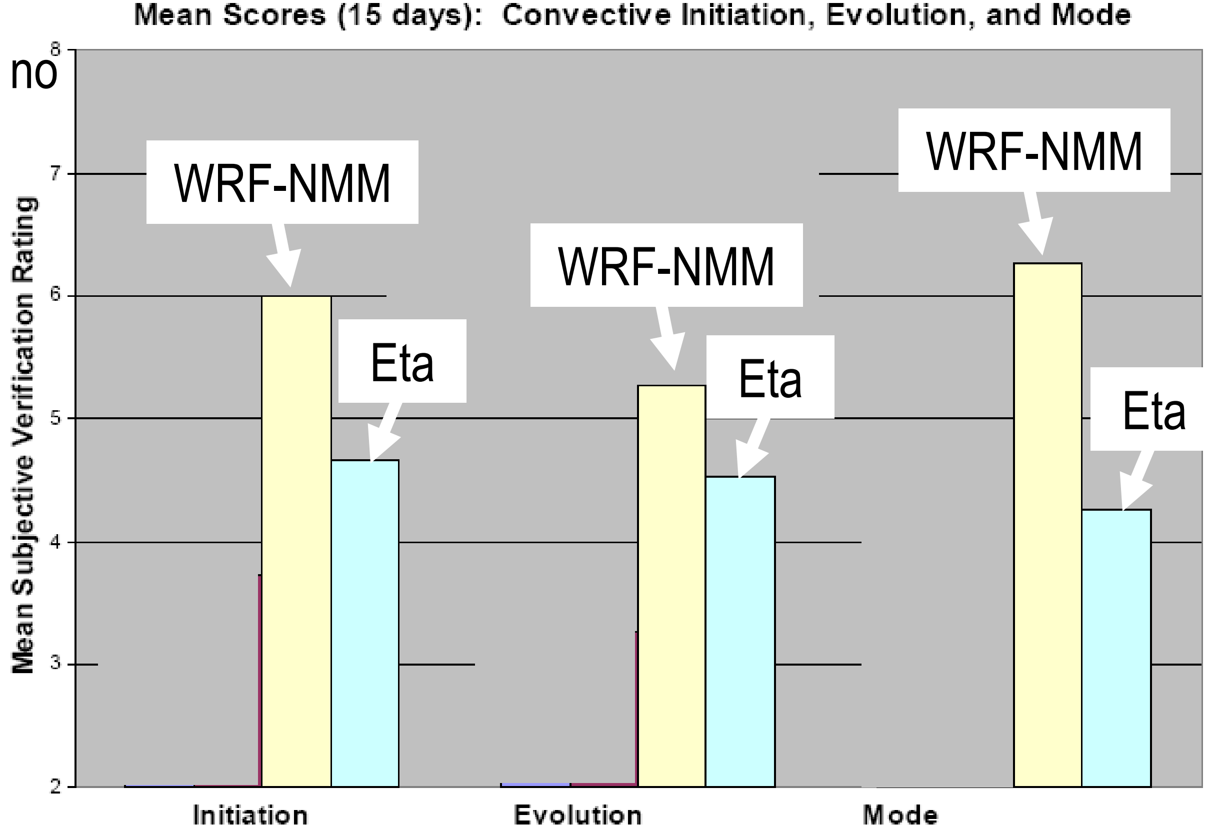


Fig. 5. Mean subjective verification ratings for the operational Eta model and the 3 high-resolution configurations of the WRF model, for categories of convective initiation, evolution, and mode, for the 15 days when all 4 models were available

17.1 EXAMINATION OF SEVERAL DIFFERENT VERSIONS OF THE WRF MODEL FOR THE PREDICTION OF SEVERE CONVECTIVE WEATHER: THE SPC/NSSL SPRING PROGRAM 2004

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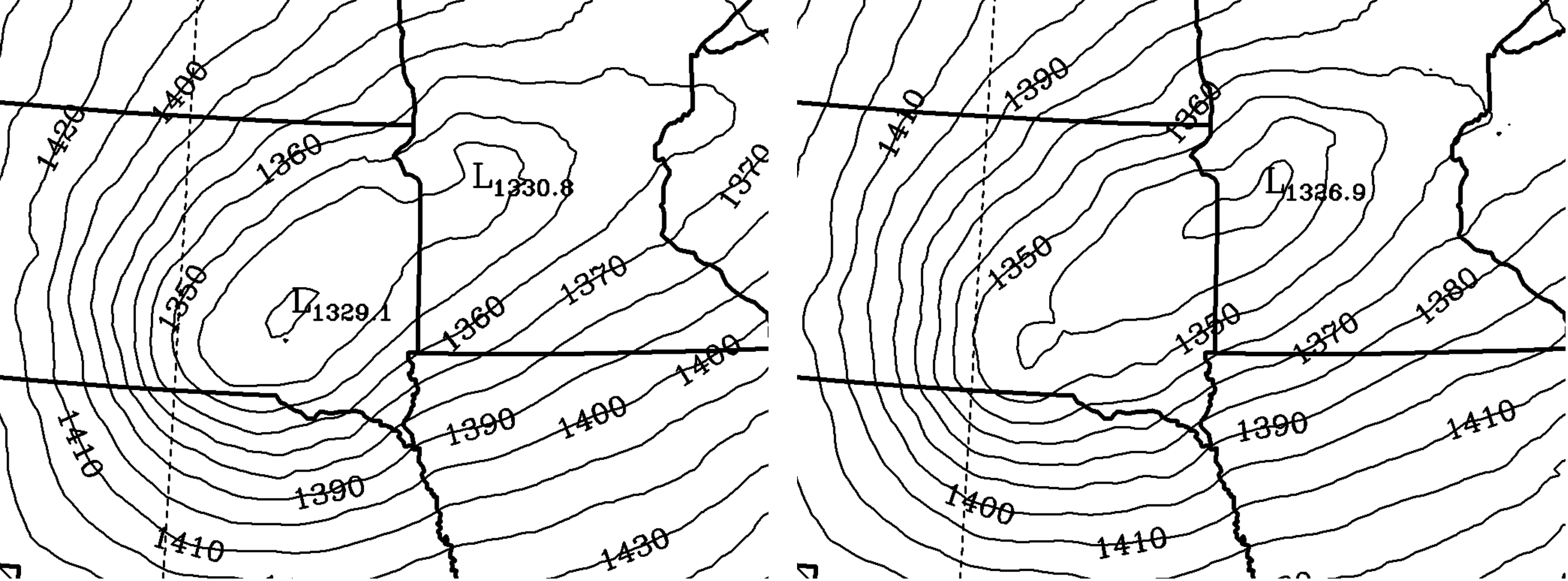
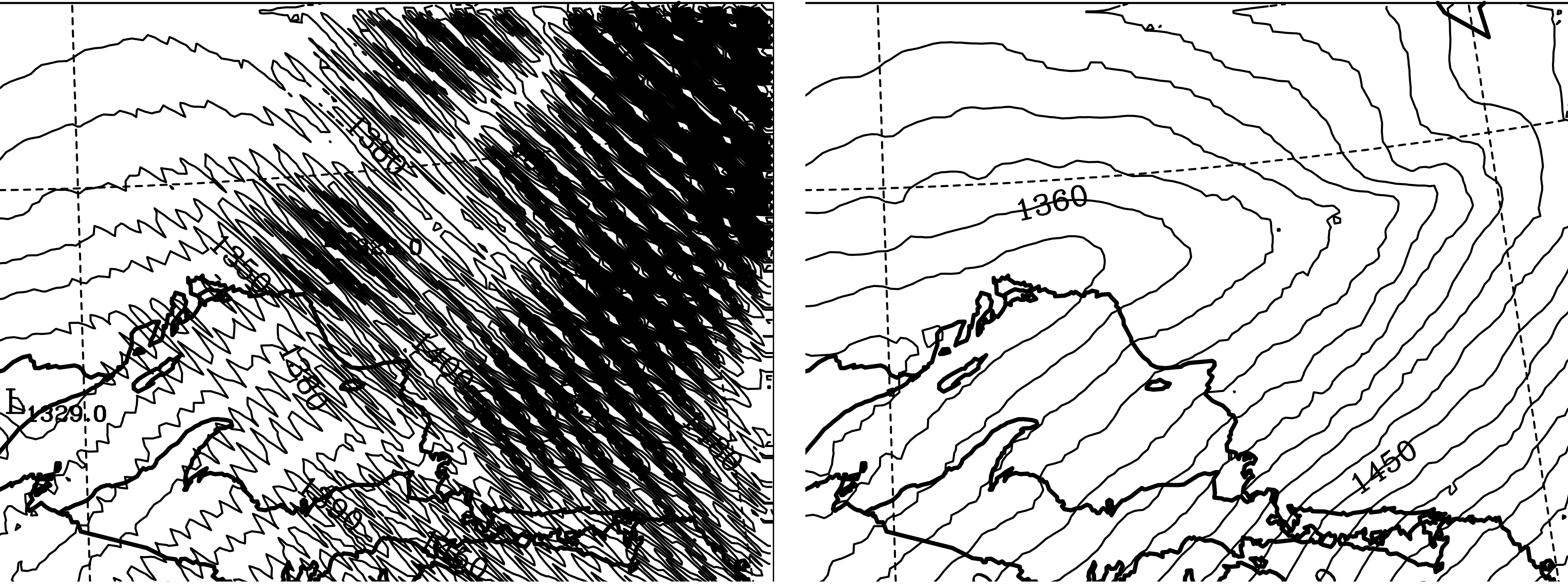
[2]University of Oklahoma/CIMMS/National Severe Storms Laboratory

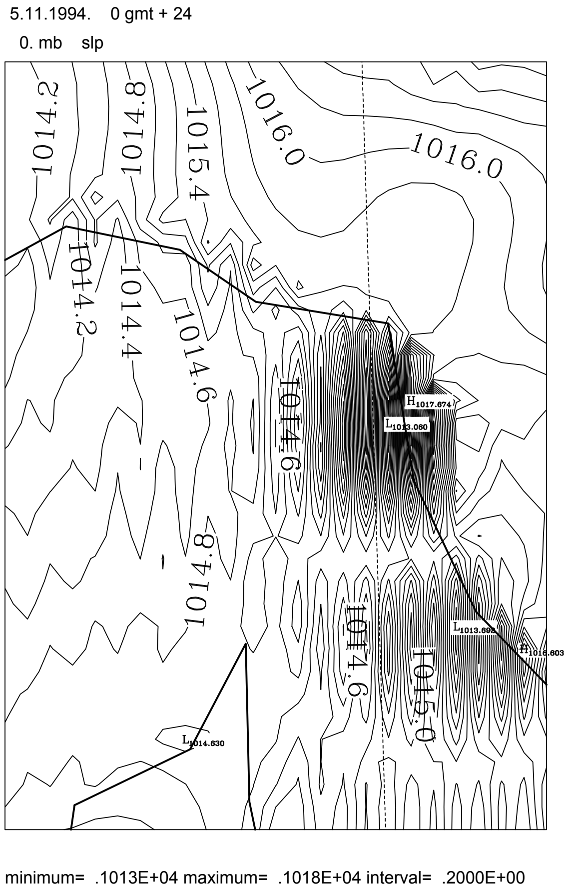
22nd Conference on Severe Local Storms, October 3-8, 2004, Hyannis, MA.

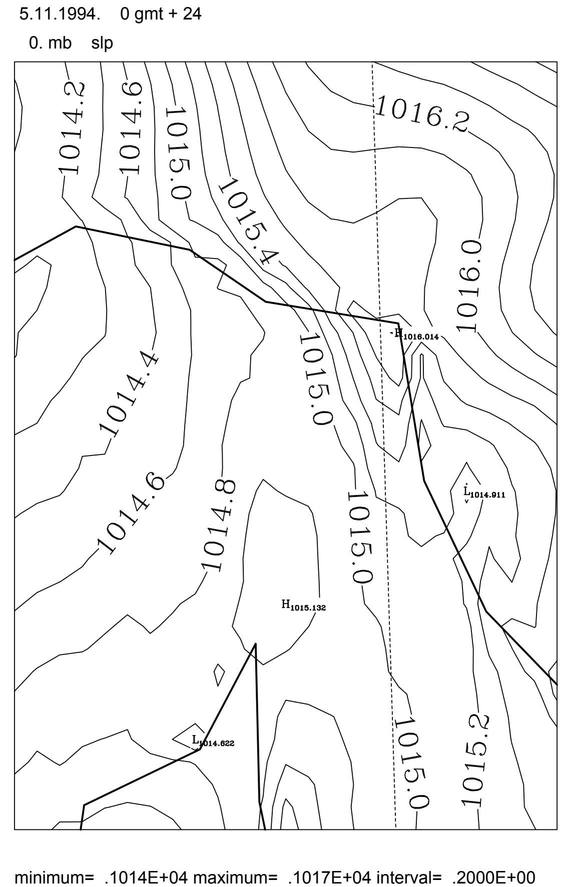


WRF NMM, 8 km, 50 levels, 24 hour hydrostatic (left panel) and nonhydrostatic (right panel) forecasts of 850 hPa heights starting from 12 UTC, April 20, 2001 (Janjic, 2003). The contour interval is 10 m. Differences sensitive to diffusion.

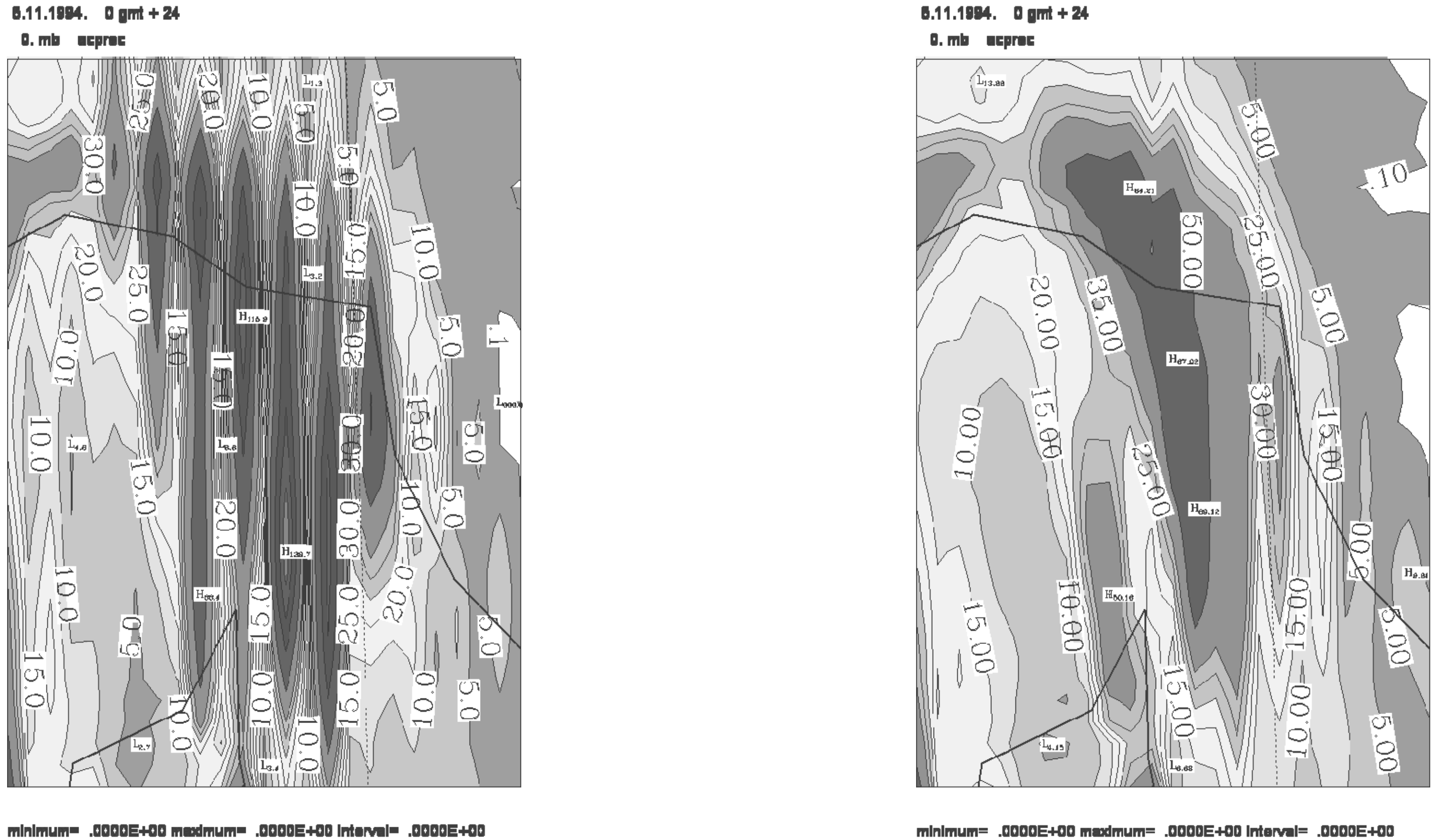


WRF NMM, no lateral diffusion, minimum divergence damping, 8 km, 50 levels, 24 hour hydrostatic (left panel) and nonhydrostatic (right panel) forecasts of 850 hPa heights starting from 12 UTC, April 22, 2001 (Janjic, 2003). The contour interval is 10 m.

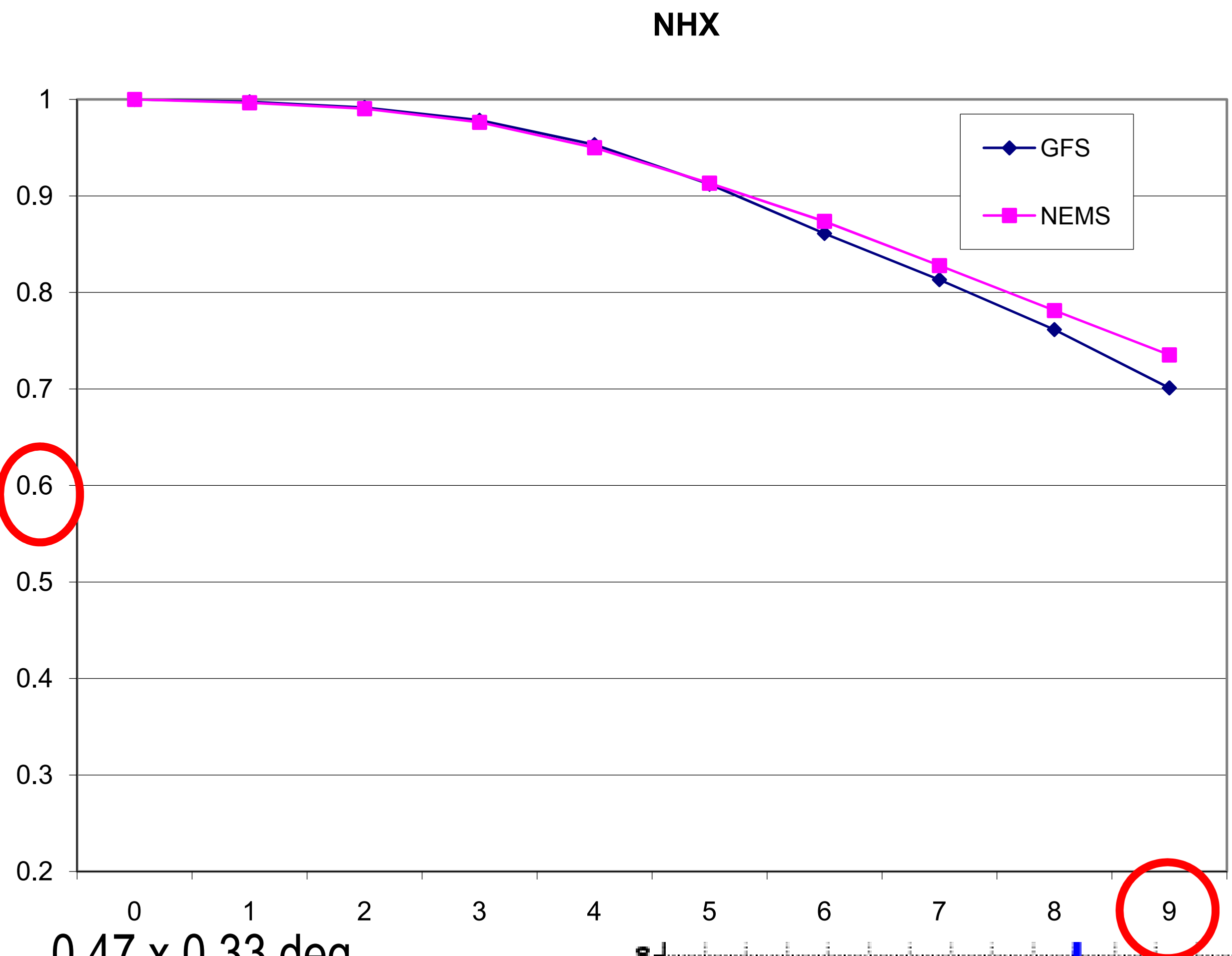




Heavy precipitation event, Gulf of Genoa, NMMB, 8 km, 32 levels, 24 hour hydrostatic (left panel) and nonhydrostatic (right panel) forecasts of the sea level pressure starting from 00 UTC, November 5, 1994 (Janjic, 2003). The contour interval is 20 Pa.

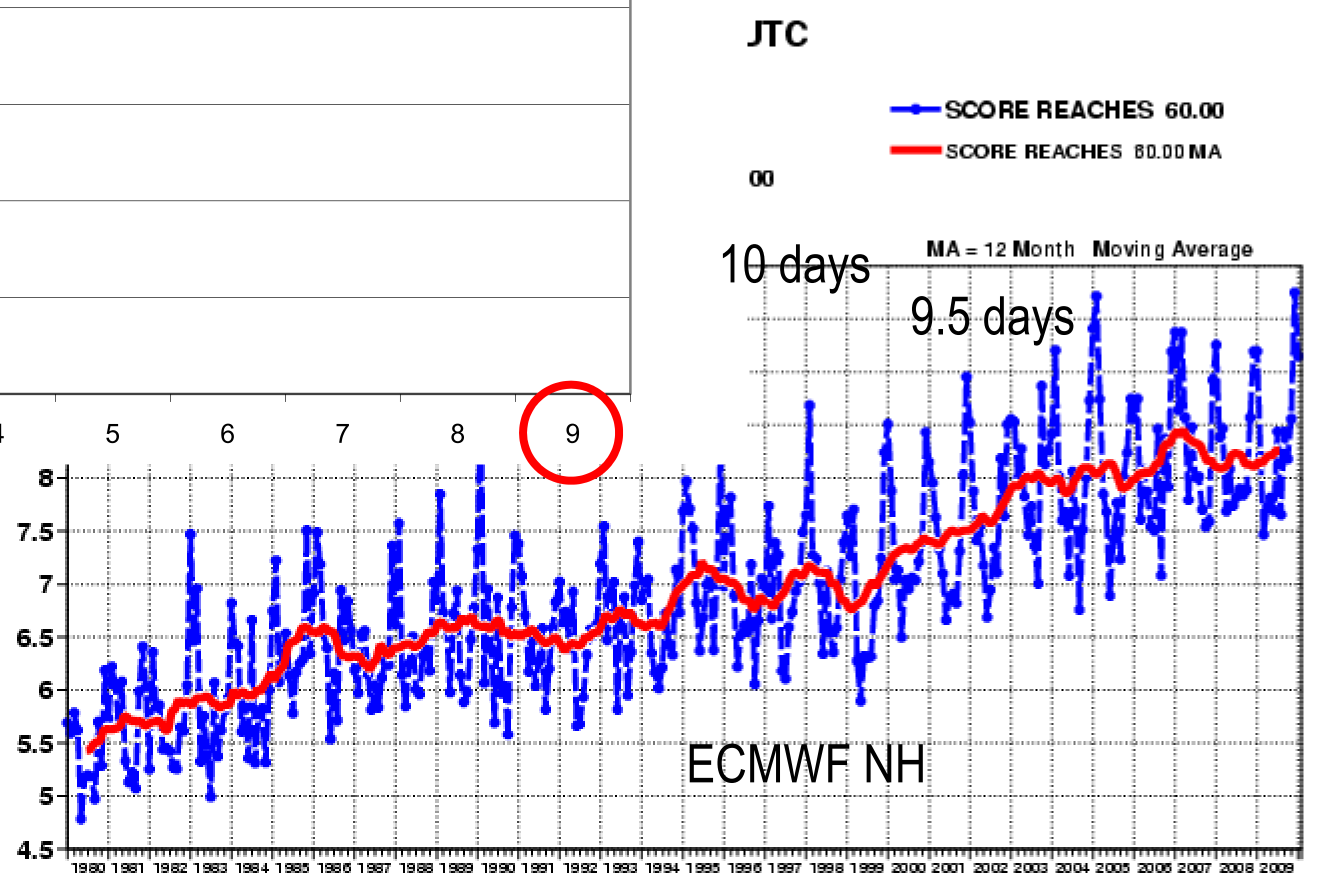


Heavy precipitation event, Gulf of Genoa, NMMB, 8 km, 32 levels, 24 hour hydrostatic (left panel) and nonhydrostatic (right panel) forecasts of precipitation accumulated over 24 hours starting from 00 UTC, November 5, 1994. The contours are drawn at 0, 5, 10, 15 20, 25, 30, 35, 50, 75, 100 etc. mm, and the grayscale filling is used to enhance the main features (Janjic, 2003).



NMMB, end of February 2010, 21 case, North Hemisphere

NMMB initialized and verified by GFS analyses



0.47 x 0.33 deg
64 levels

No statistical difference between hydrostatic and nonhydrostatic solutions at this resolution

Zavisa Janjic, ECMWF 2010

- Conclusions
  - Internally consistent
  - Passed difficult nonhydrostatic tests
  - Well tested in NWP practice on various scales
  - Fast
  - Computationally robust
  - Can be implemented in existing hydrostatic models