GRAVITY WAVES IN THE STABLE PLANETARY BOUNDARY LAYER

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ORIGINS OF GRAVITY WAVES OBSERVED IN THE STABLE PBL

GLOBAL:

WAVES OUTSIDE THE MODEL DOMAIN

LOCAL:

WAVES THAT CAN BE EITHER RESOLVED BY THE MODEL PHYSICS OR PARAMETERIZED AS FUNCTIONS OF THE MODEL PHYSICS.

EXAMPLES

GLOBAL:

1. SOLITARY WAVES
2. TROPOSPHERIC UNDULAR BORES
3. JET STREAKS
4. INERTIA-GRAVITY WAVES
5. LARGE-SCALE LEE WAVES

LOCAL:

1. DUCTED MODES
2. MESOSCALE UNDULAR BORES
3. LEE WAVES
4. CANOPY WAVES
5. VISCOUS WAVES


PBL

WE DESCRIBE THREE WAVE MECHANISMS WHICH CAN BE CONSIDERED LOCAL:

1. LEE WAVE DECAY
   (SMITH *ET AL* 2002, *JAS*)

2. DISSIPATIVE WAVES
   (HOOKE & JONES 1986, *JAS*)

3. STABLE PBL INTERNAL WAVES
   (CHIMONAS, 2002, *BLM*)

LEE WAVE DECAY

OBSERVATIONS AND ANALYSES OF WAVES OVER *MONT BLANC* BY SMITH *et al* (2002)[1] SHOW THAT THE AMPLITUDES OF TRAPPED LEE WAVES DECAY DOWNSTREAM DUE TO THE ABSORPTION OF WAVE ENERGY IN THE PBL.

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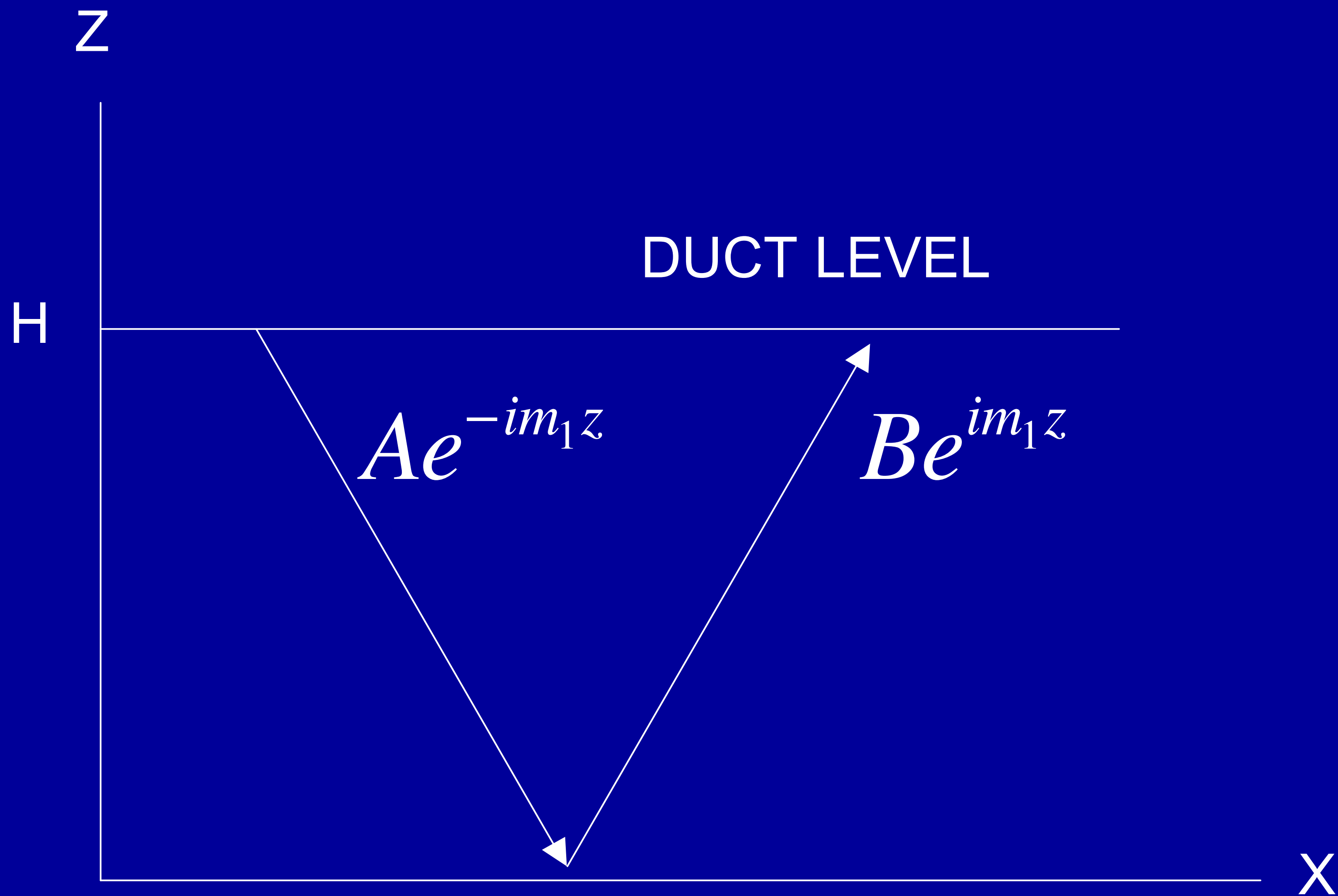
[1]Smith etal 2002: *J. of Atmos. Sci. 2073-2092*

THEORY



FOR PERFECT REFLECTION
AT THE GROUND SURFACE

$$A + B = 0$$

FOR WAVE ABSORPTION

$$A + qB = 0$$

$$0 < q < 1$$

q = REFLECTION FACTOR

RESULTS FROM COAMPS
(COUPLED OCEAN/ATMOSPHERIC MESOSCALE PREDICTION SYSTEM)

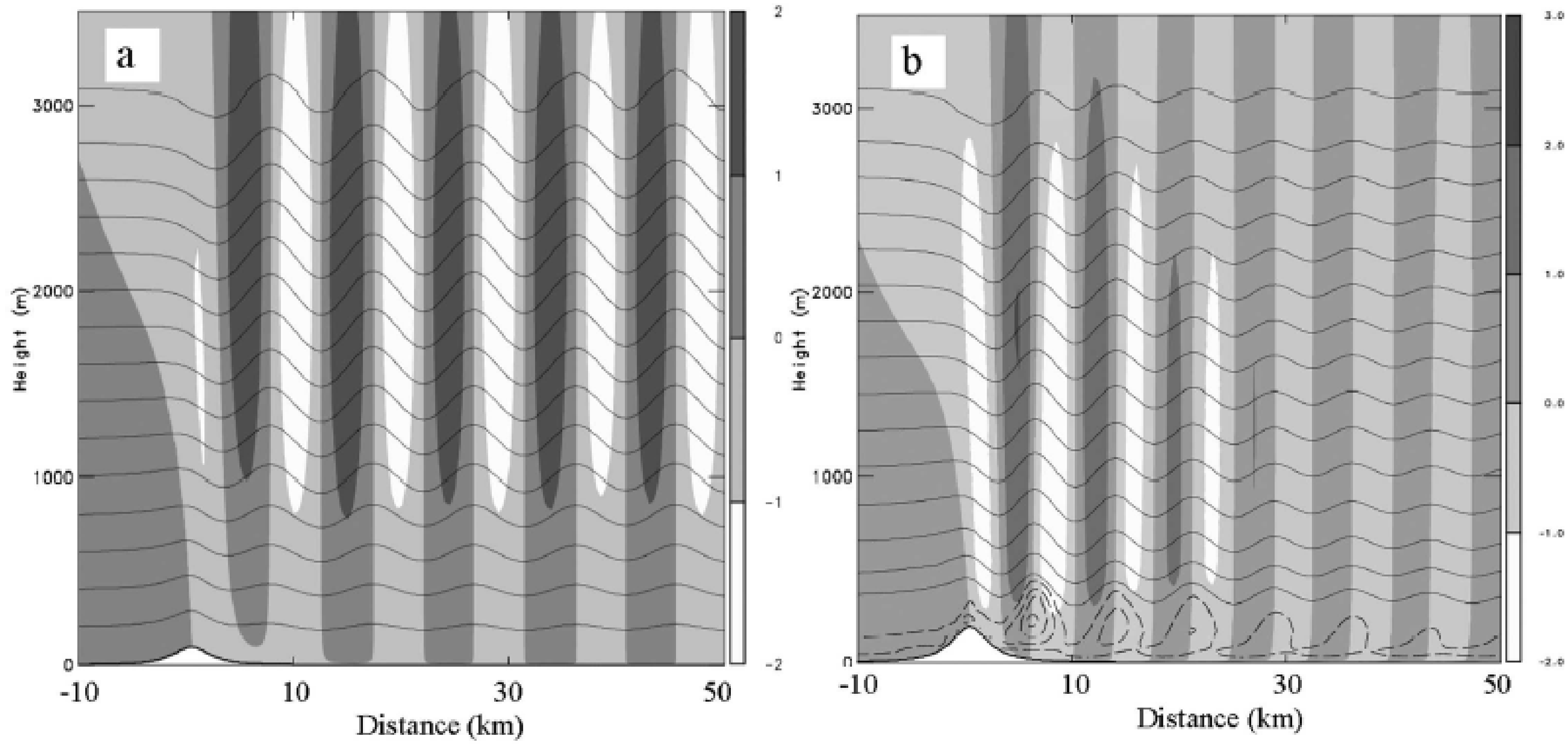


FIG. 2. Cross section of vertical velocity component (in grayscale), isentropes (interval: 4 K), and eddy viscosity coefficient (dashed contours, interval: 5 $m^2 s^{-2}$) derived from simulations with (a) h_m = 100 m and a free-slip condition and (b) h_m = 200 m and a no-slip condition with z_o = 1 m.

FROM Jiang et al (2006): *J. Atmos. Sci. 617-633*

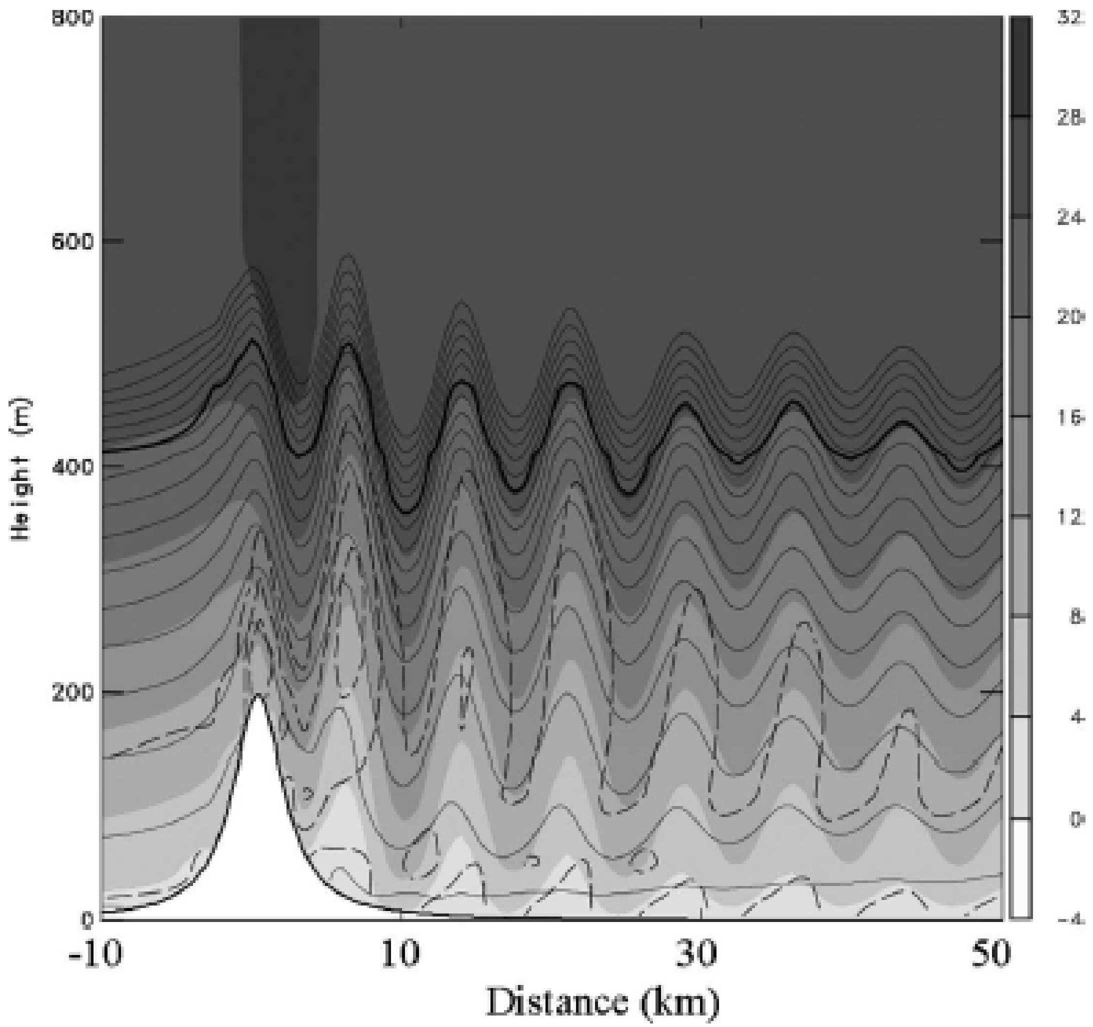
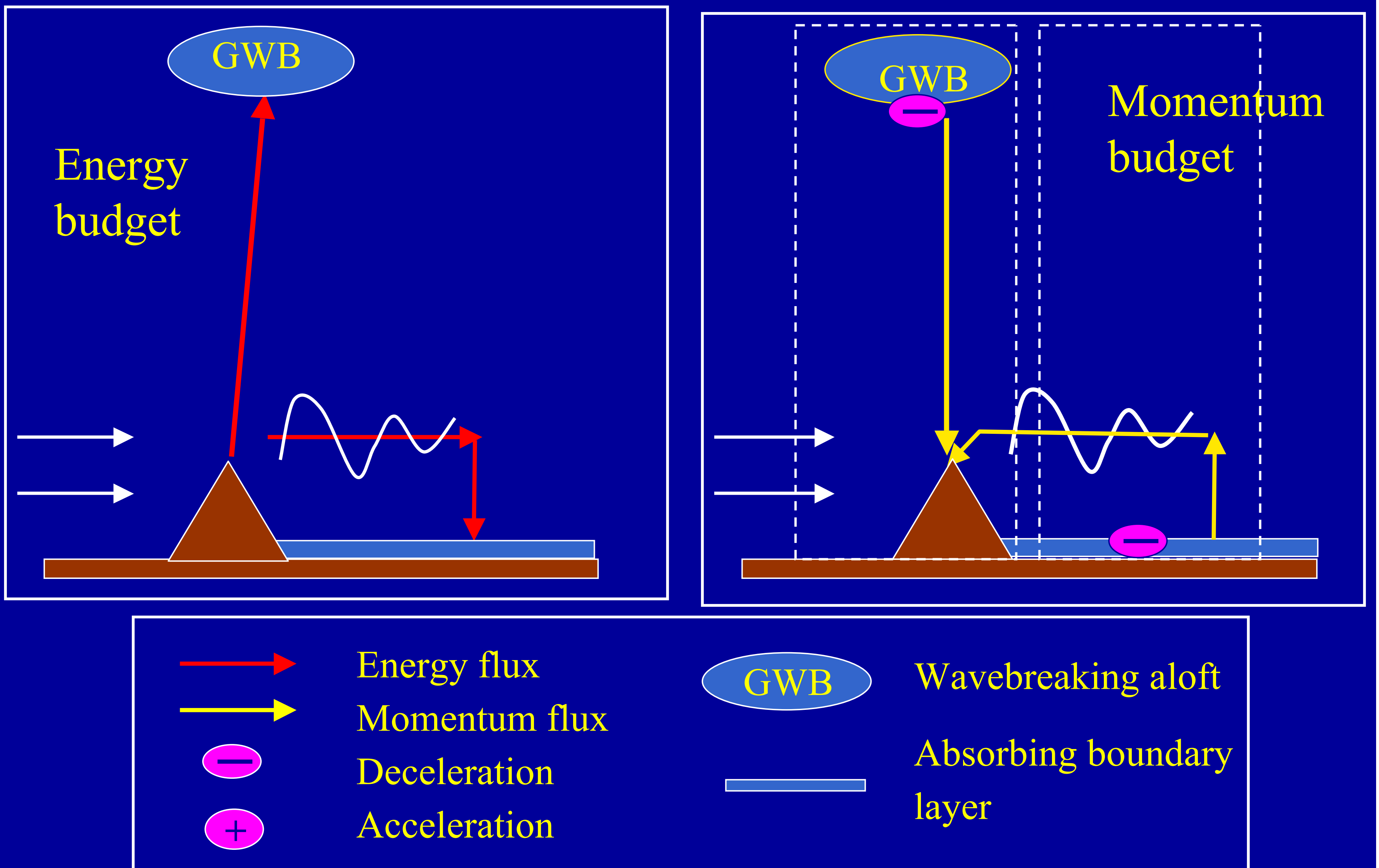
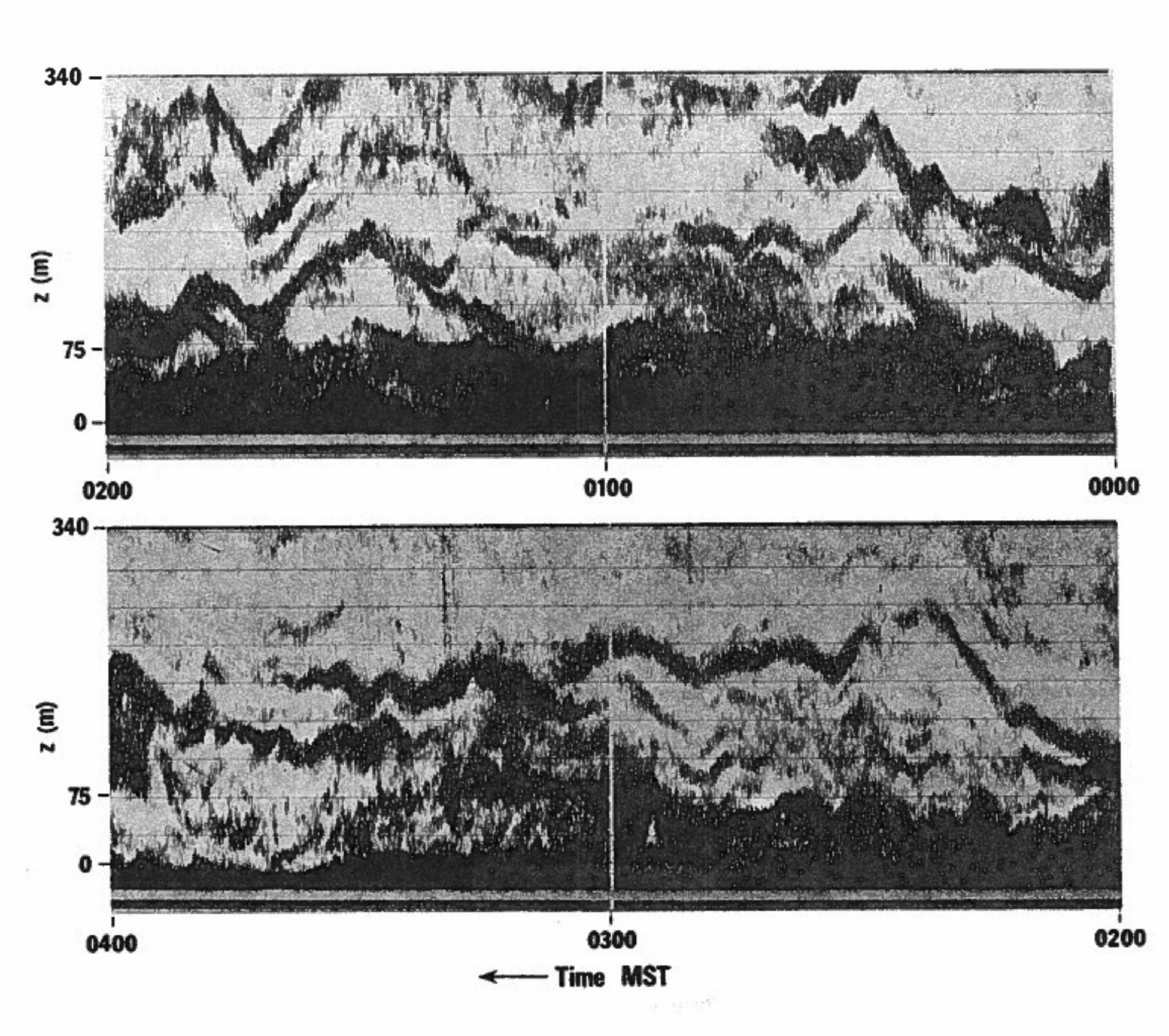


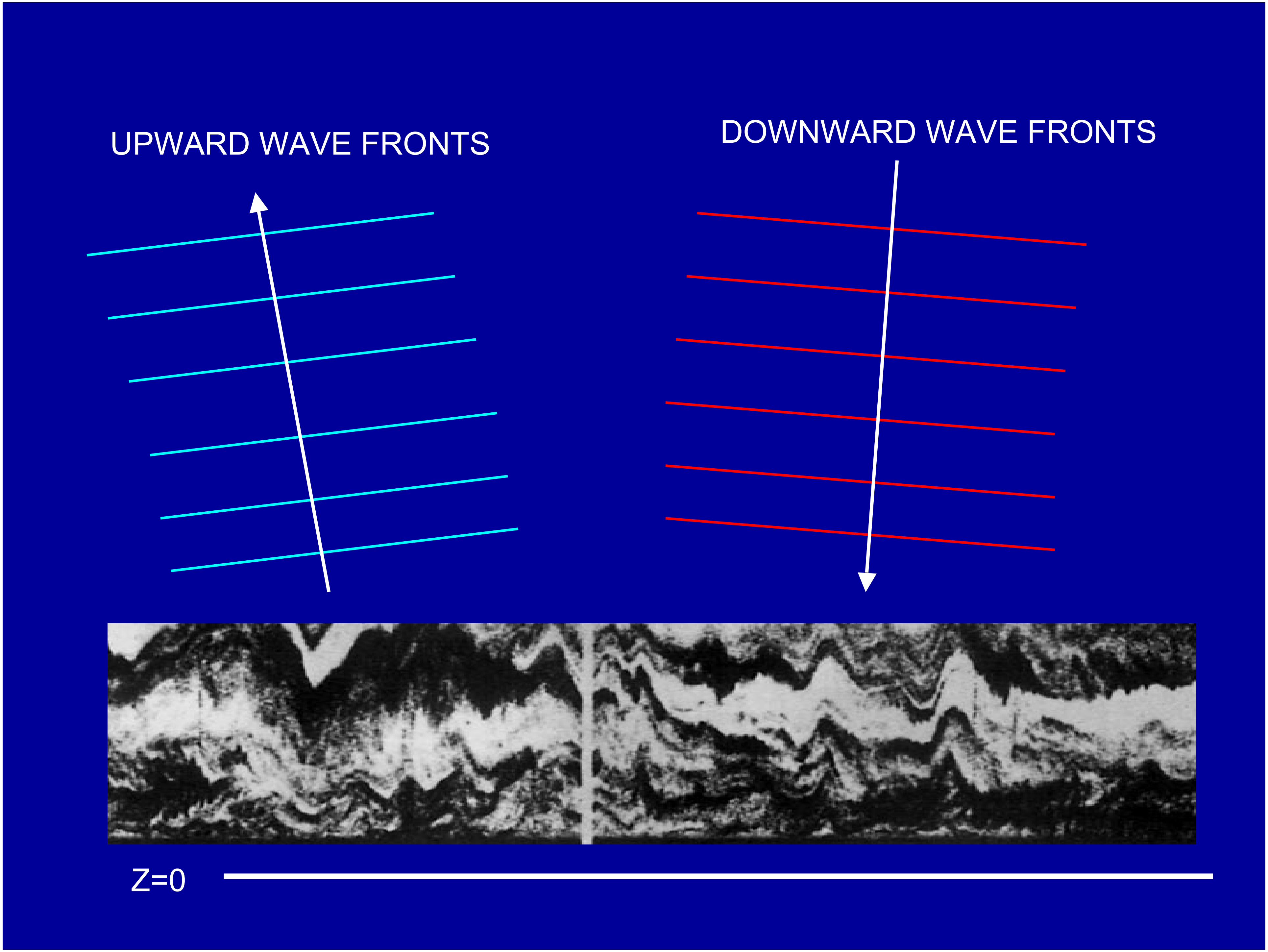
FIG. 3. Cross section of the horizontal wind component (grayscale), isentropes (solid contours, interval: 1 K), and TKE (dashed contours, interval: 1 $m^2 s^{-2}$) from the same no-slip simulations as in Fig. 2b. The bold contour corresponds to Richardson number Ri = 0.5.

MECHANISM (JIANG *PC*)



DISSIPATIVE WAVES IN THE STABLE PBL




UPWARD WAVE FRONTS
DOWNWARD WAVE FRONTS
Z=0

VISCOSITY

$$\rho_0 \frac{\partial u'}{\partial t} = -\frac{\partial p'}{\partial x} + \mu \frac{\partial^2 u'}{\partial x^2} + \mu \frac{\partial^2 u'}{\partial z^2}$$

$$\rho_0 \frac{\partial w'}{\partial t} = -\frac{\partial p'}{\partial z} - \rho' g + \mu \frac{\partial^2 w'}{\partial x^2} + \mu \frac{\partial^2 w'}{\partial z^2}$$

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0$$

$$\frac{\partial \rho'}{\partial t} + w' \frac{\partial \rho_0}{\partial z} = \frac{\kappa}{\rho_0}\left(\frac{\partial^2 \rho'}{\partial x^2} + \frac{\partial^2 \rho'}{\partial x^2}\right)$$

THERMAL CONDUCTIVITY

WAVE EQUATIONS

INVISCID CASE:

$$w = a_i \exp(\omega t - kx - nz) + a_r \exp(\omega t - kx + nz)$$

$$w(0) = 0 \Rightarrow a_i + a_r = 0$$

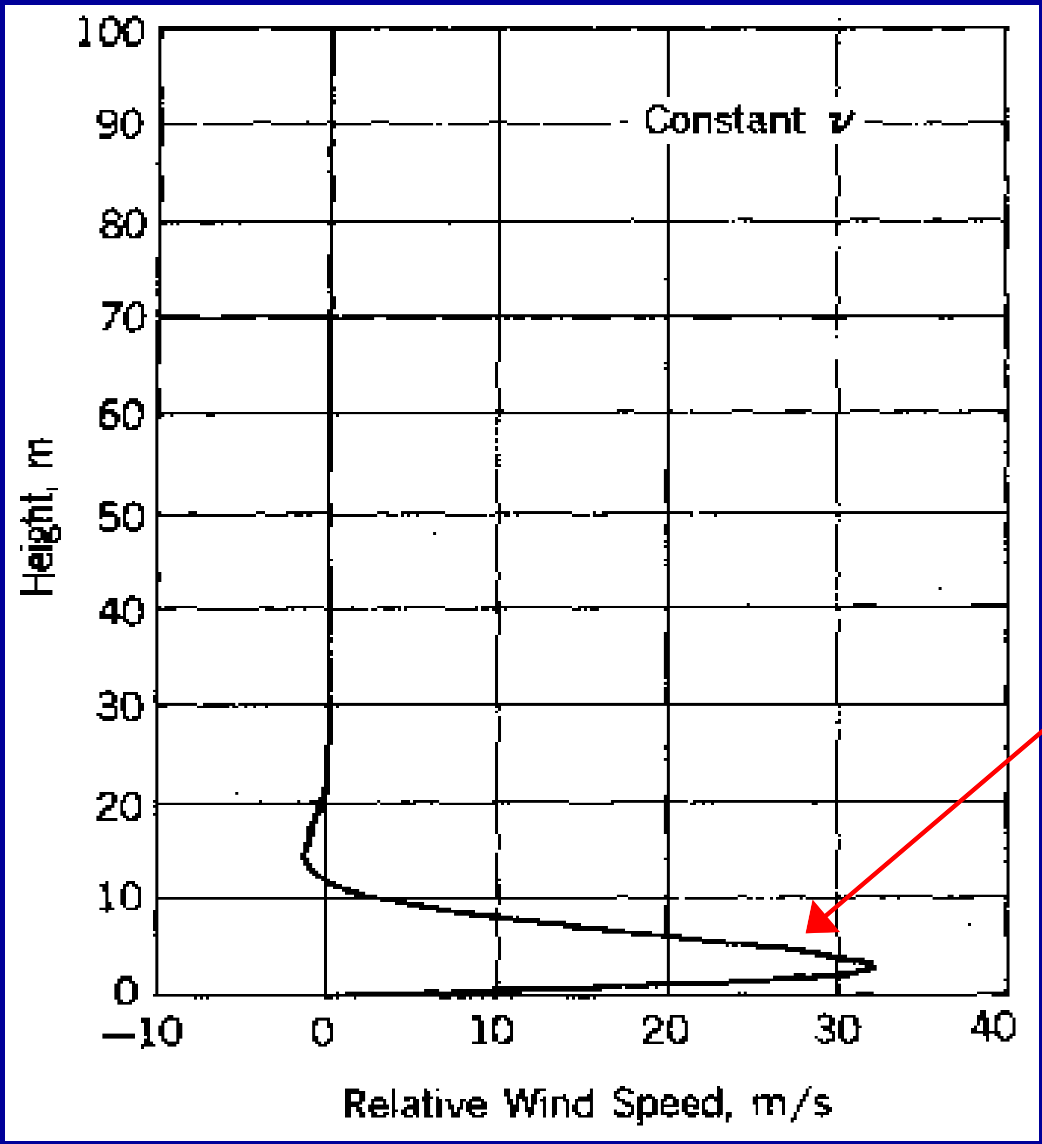
DISSIPATIVE CASE:

$$w = a_i \exp\left(\omega t - kx - n_g z\right) + a_r \exp\left(\omega t - kx + n_g\right)$$

$$+ \; a_v \exp\left(\omega t - kx + n_v z\right) + a_t \exp\left(\omega t - kx + n_t z\right)$$

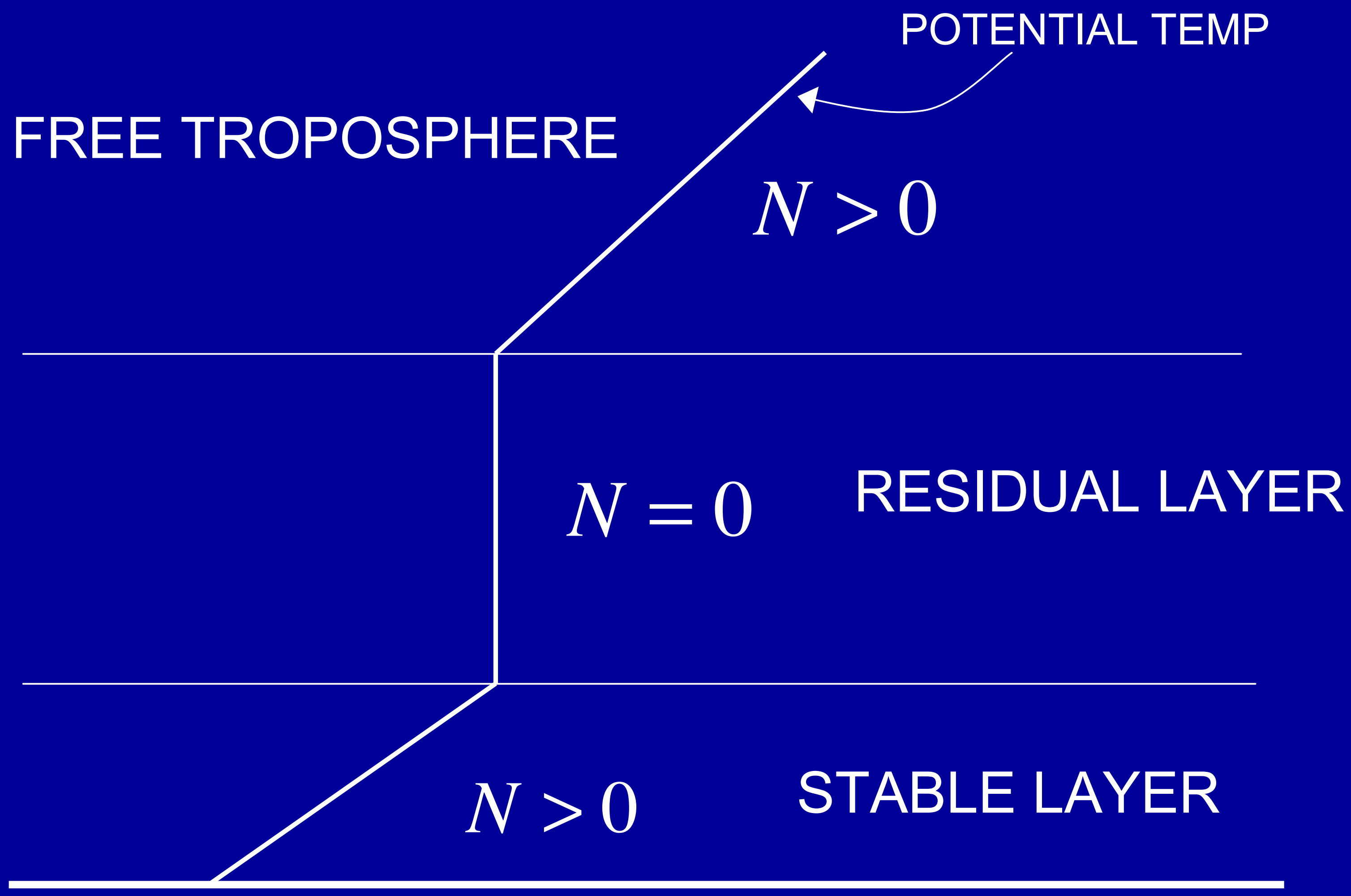
$$w(0) = 0 \Rightarrow a_i + a_r + a_v + a_t = 0$$

THE VISCOUS WAVE

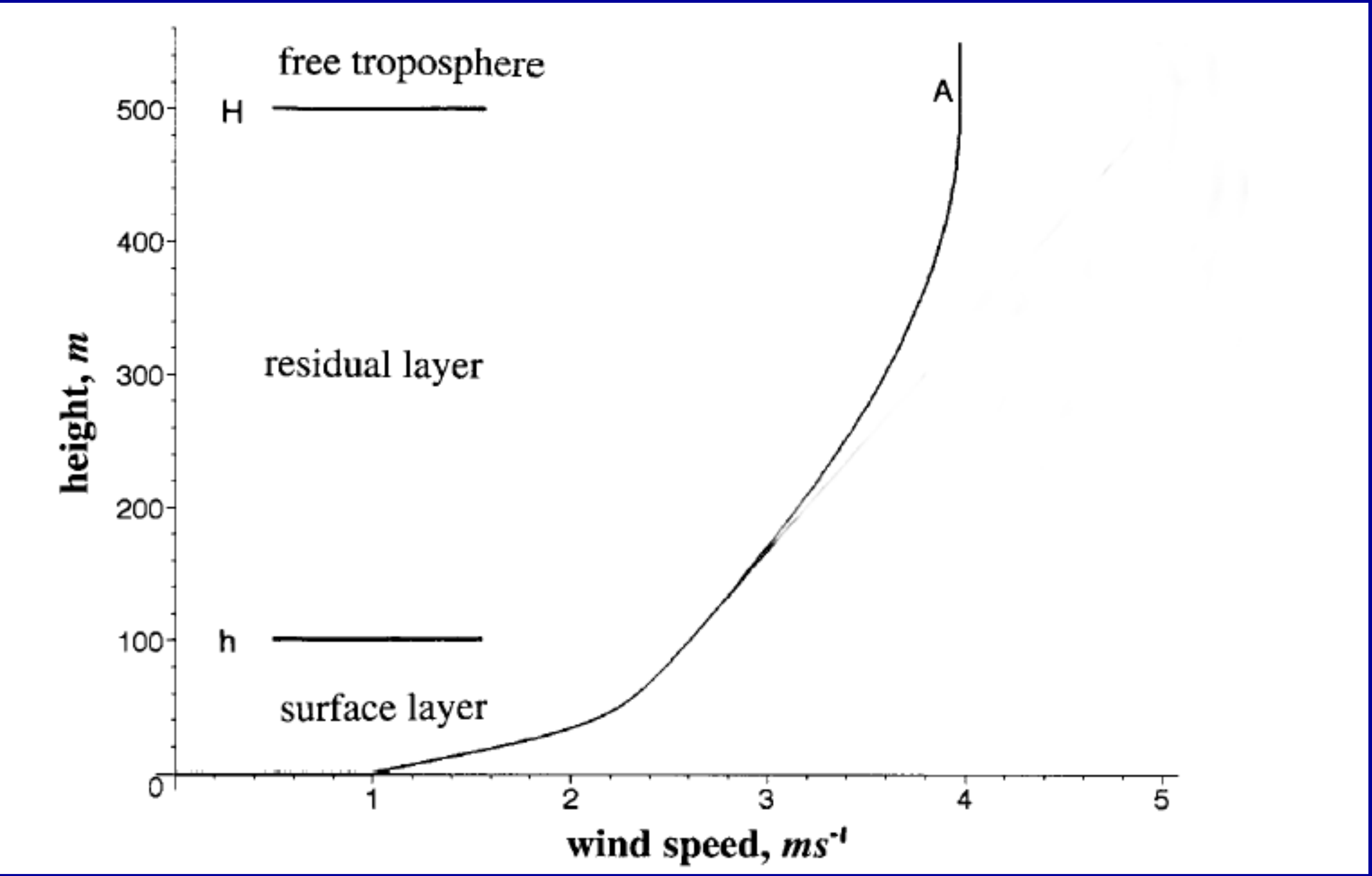


GENERATION OF SHEAR INSTABILITY ?

LOCAL WAVE MODES IN THE STABLE PBL



ANALYTICAL WIND PROFILE



GOVERNING EQUATION:

$$\frac{d^2 w}{dz^2} + \left[\frac{N^2}{(c-U)^2} + \frac{1}{(c-U)} \frac{d^2 U}{dz^2} - \kappa^2 \right] = 0$$

MODAL BOUNDARY CONDITIONS:

$$w(0) = 0 \qquad \mathrm{w}(z \to \infty) = 0$$

REQUIREMENTS FOR A GROWING MODE:

COMPLEX PHASE SPEED: $c = c_r + ic_i$

A CRITICAL LEVEL: $c_r = U(z_c)$

CRITICAL LEVEL RICHARDSON NUMBER: $0 \le \mathrm{Ri}(\mathrm{z}_c) \le 0.25$

IN THE RESIDUAL LAYER:

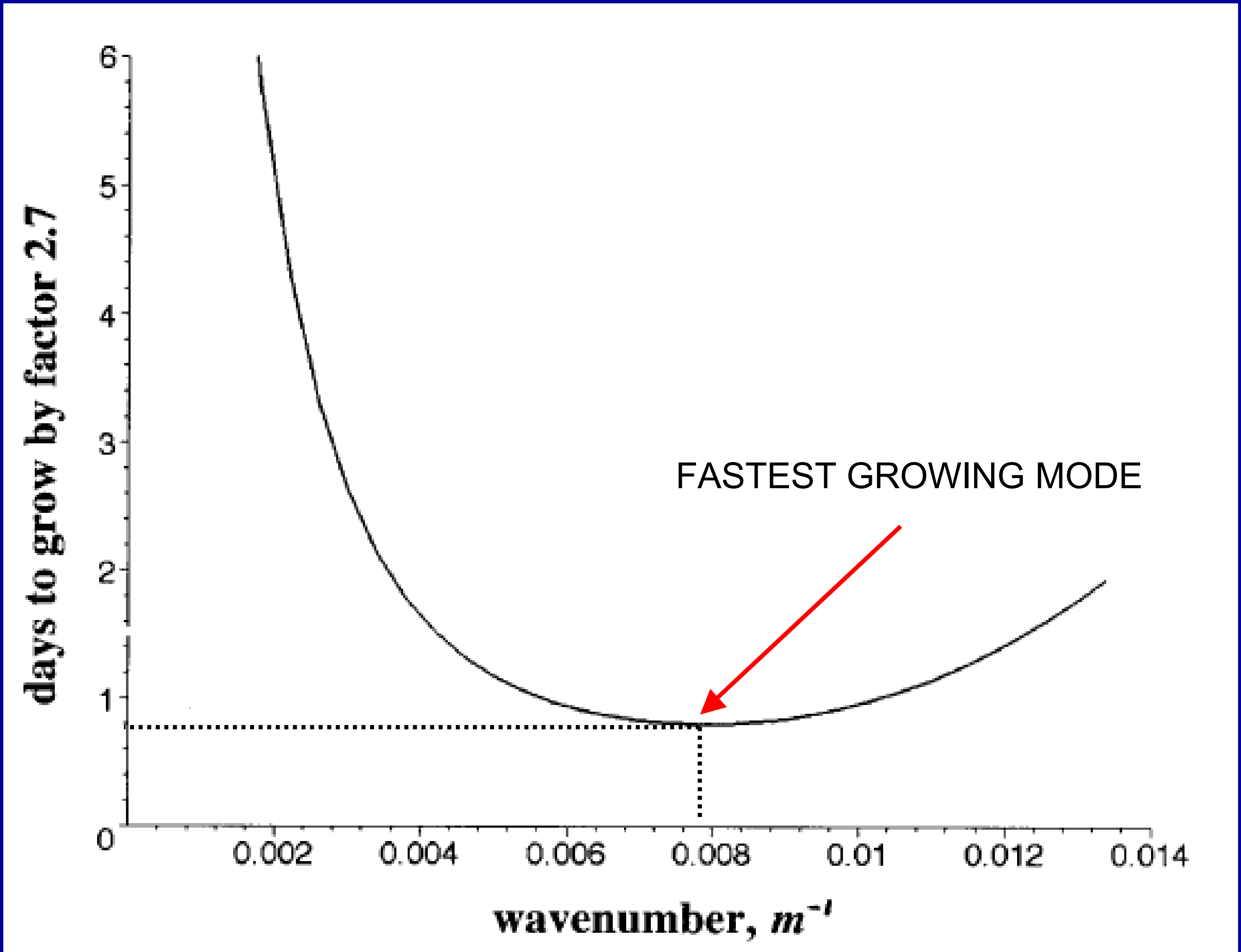
$$\frac{d^2w}{dz^2}+\left[\frac{1}{(c-U)}\frac{d^2U}{dz^2}-\kappa^2\right]=0$$

A CRITICAL LEVEL IS VERY LIKELY TO EXIST.

THEN WAVE AMPLITUDE GROWS AS:

$$A(t)=A(0)e^{c_i\kappa t}$$

GROWTH RATE



6
5
4
3
2
1
0
days to grow by factor 2.7
FASTEST GROWING MODE
0.002
0.004
0.006
0.008
0.01
0.012
0.014
wavenumber, m⁻¹

ABOVE THE RESIDUAL LAYER:

$$\frac{d^2w}{dz^2} + \left[\frac{N_{trop}^2}{\left(c - U_{trop}\right)^2} - \kappa^2 \right] w = 0$$

AT SOME HEIGHT THE VERTICAL WAVENUMBER BECOMES COMPLEX

$$m = m_r + i m_i$$

NOW SOLUTION IS:

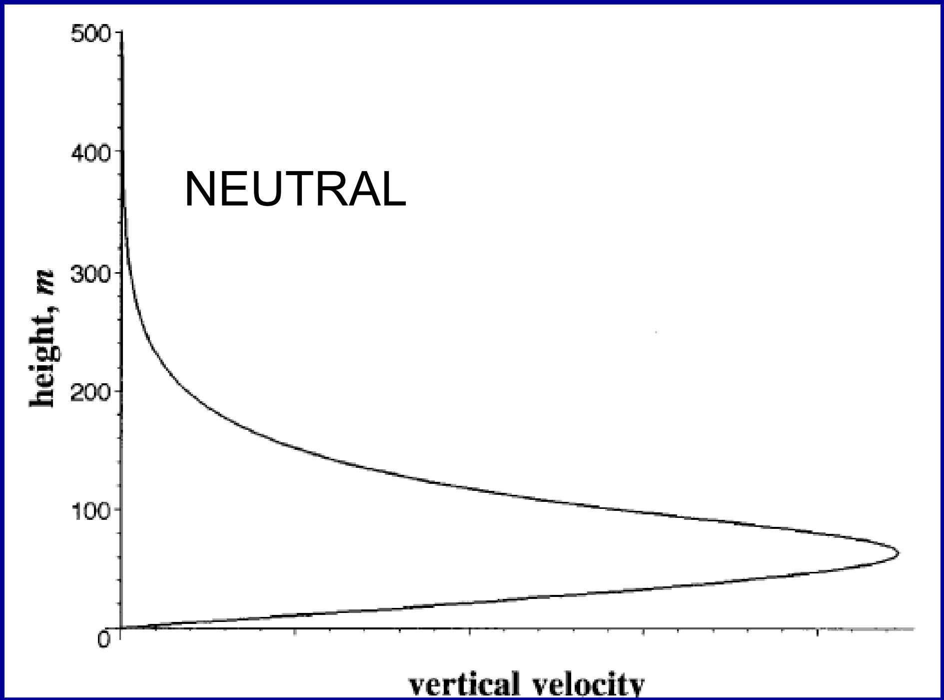
$$w(z > H) = A(0) \exp\left[i\kappa \left(c_r t - x \right) \right] \exp\left[c_i t - m_i z \right]$$

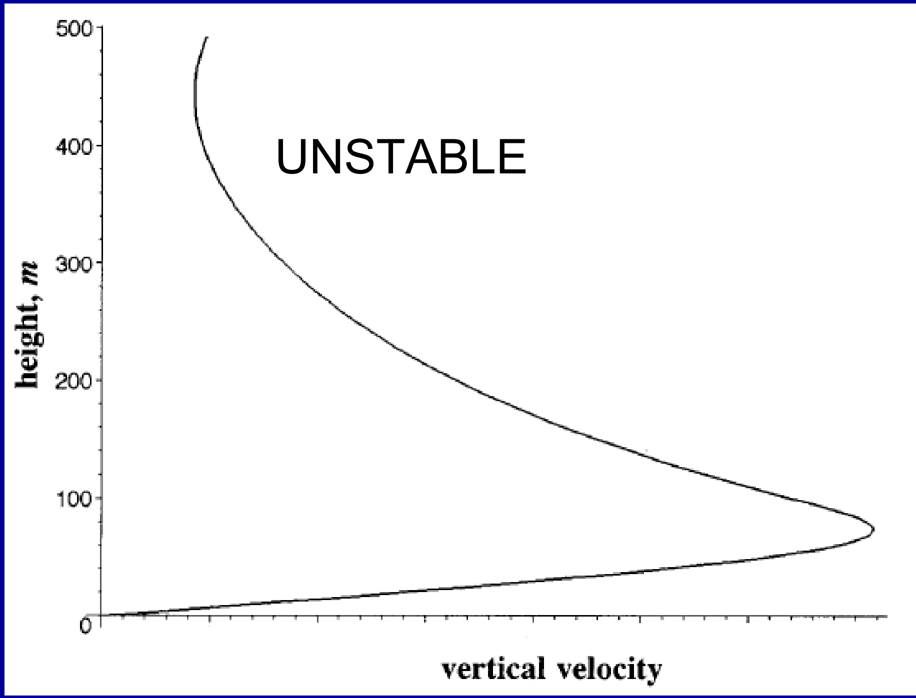
PERIODIC

GROWTH

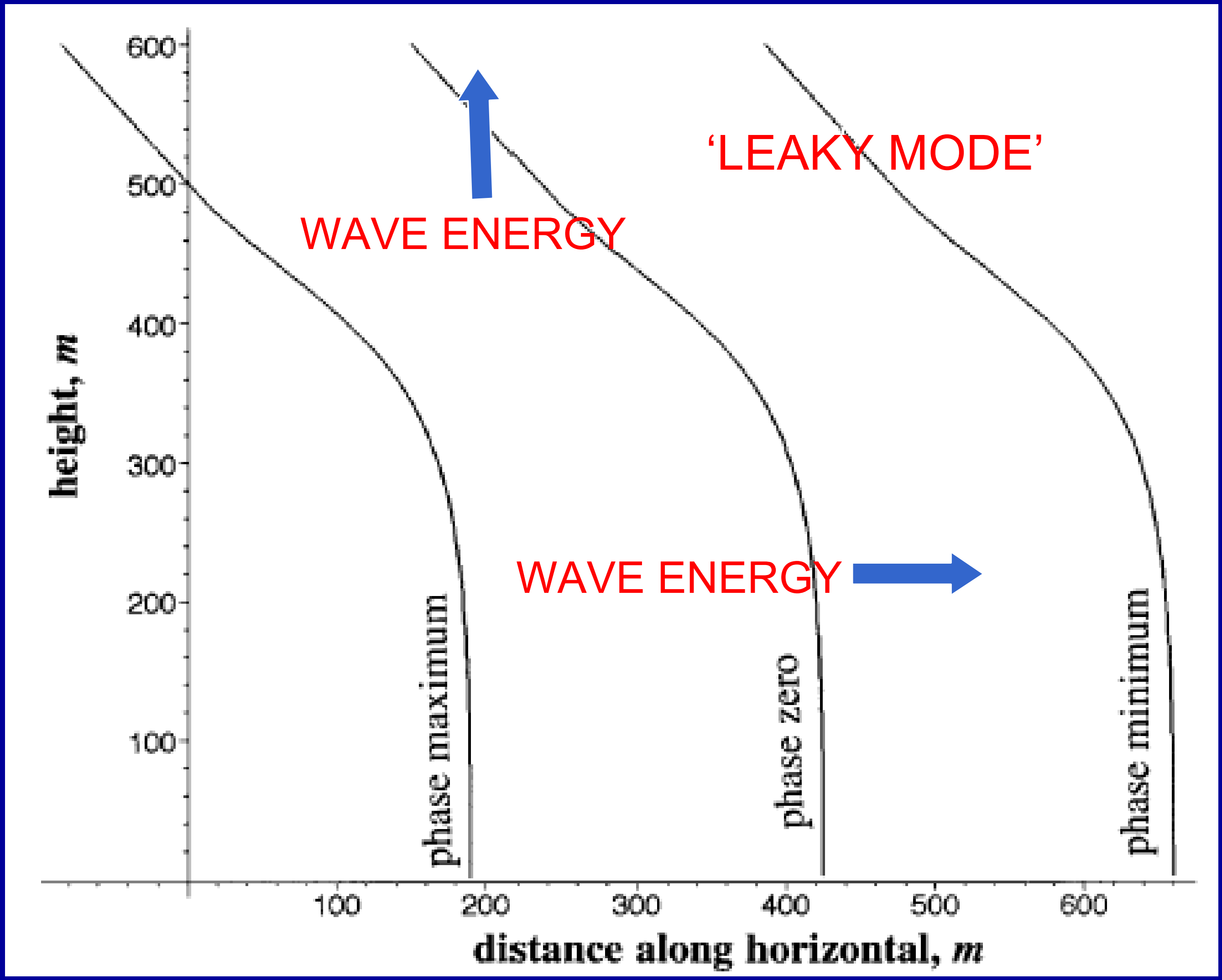
DECAY

FUNDAMENTAL MODE



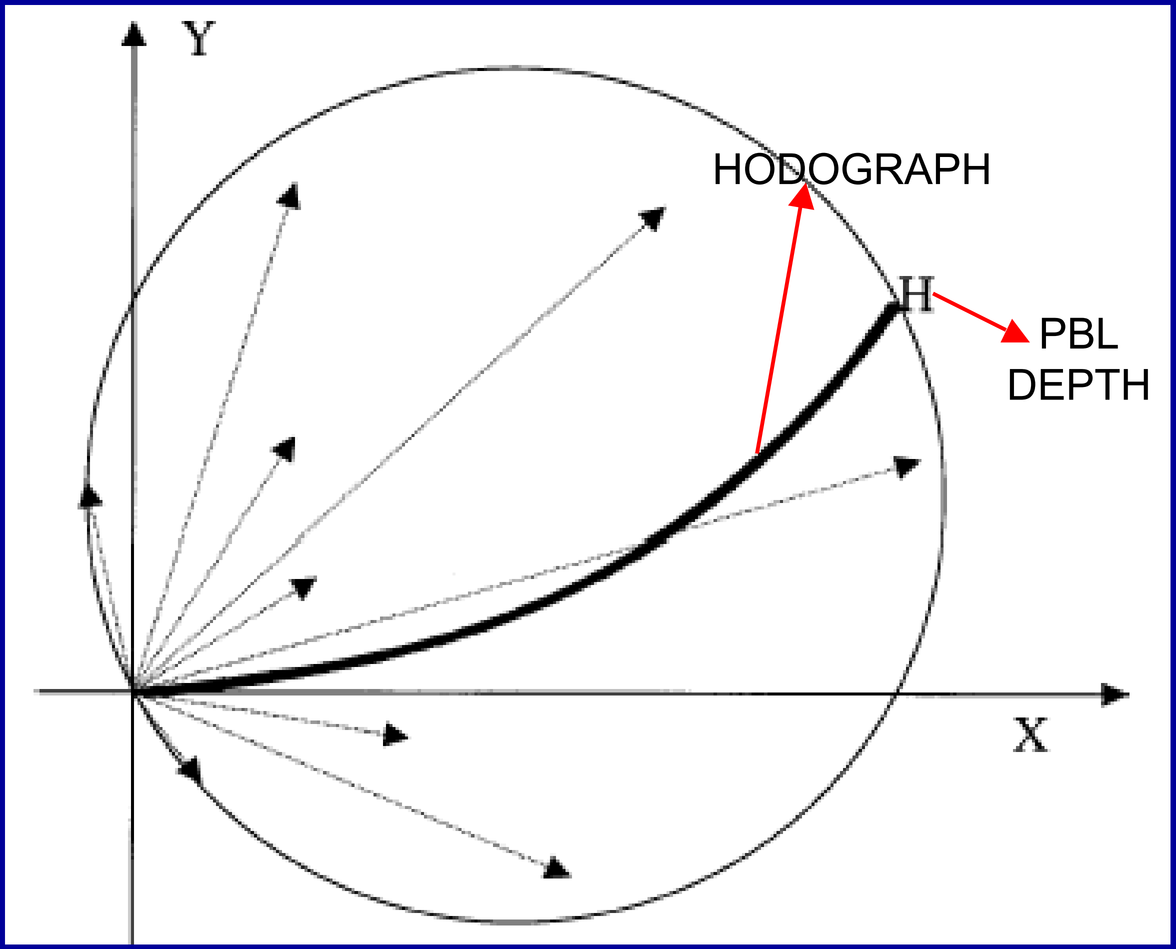


WAVE FRONTS



'LEAKY MODE'
WAVE ENERGY
WAVE ENERGY
600
500
400
300
200
100
height, m
100
200
300
400
500
600
distance along horizontal, m
phase maximum
phase zero
phase minimum

DOMAIN OF POSSIBLE PHASE-VELOCITIES OF BOUNDARY LAYER MODES IF THE PROJECTION OF THE WIND PROFILE INTO THE PLANE OF THE MODE HAS A CRITICAL LEVEL.



CONCLUSIONS

WE HAVE DESCRIBED THREE DIFFERENT MECHANISMS FOR GENERATING LOCAL GRAVITY WAVES IN THE STABLE PBL.

THESE MECHANISMS LEND THEMSELVES TO PARAMETERIZATIONS WITHIN A MESOSCALE MODEL.

AT ANY TIME ALL THREE MECHANISMS CAN BE ACTIVE.

OR

IT'S PURE WEAPON'S GRADE BALONIUM