# Effect of ocean waves on ocean circulation

Peter Janssen

European Centre for Medium-Range Weather Forecasts

<peter.janssen@ecmwf.int>

# INTRODUCTION

Apart from the traditional benefits of sea state forecasting (e.g. for shipping, fisheries, offshore operations and coastal protection) it is now known that knowledge of the sea state is also important for a more accurate description of air-sea interaction, e.g. the sea state affects

- the momentum transfer,
- the heat transfer and
- the ocean surface albedo.

Here, I will briefly study sea state effects on the upper ocean dynamics and upper ocean mixing.

Starting point is the Mellor-Yamada scheme where the turbulent velocity is determined from the **turbulent kinetic energy equation**. The turbulent velocity is then used to determine the eddy viscosities in the equations for momentum and heat.

The turbulent kinetic energy equation describes the balance between the production of turbulent kinetic energy by work against the **shear** in the current and the Stokes drift (produces wave-induced and Langmuir turbulence), production by gravity **wave dissipation**, **buoyancy** and **dissipation of turbulence**.

Note that also momentum transport is directly affected by the waves through the so-called **Stokes-Coriolis** force.

The mixed layer model has been run for a one year period from the 16th of October 1994 for a location in the Arabian Sea where extensive observations of temperature profile, current profile, solar insolation, wind speed, etc. were collected during the Arabian Sea mixed layer dynamics Experiment (ASE).

Verification of modelled temperature profiles against these observations shows that dissipating waves and the Stokes-Coriolis force play a considerable role in the upper ocean mixing in that area.

Today, I discuss briefly the following items:

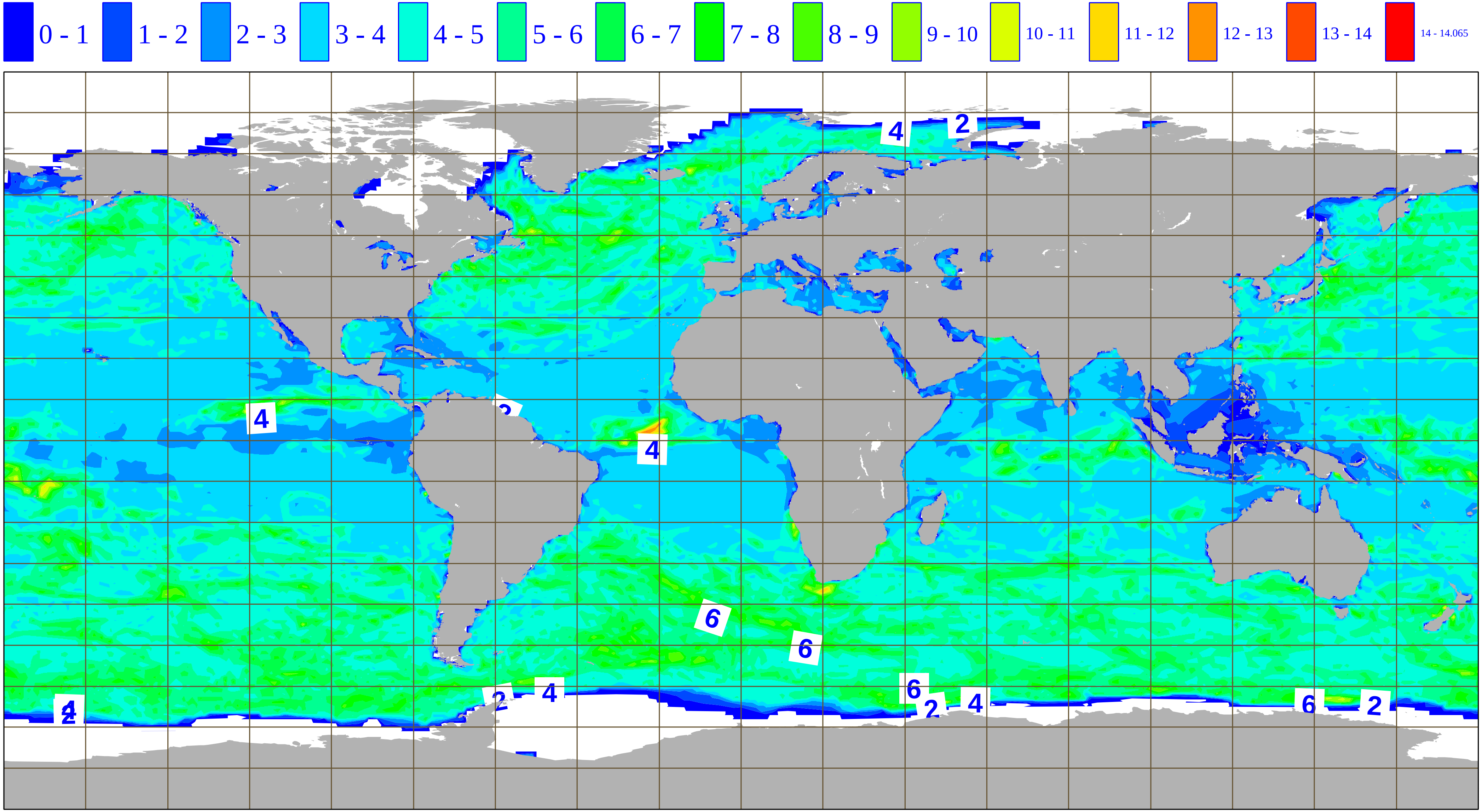
- **MIXED LAYER MODEL**

  **Monin-Obukhov similarity** does not work for the upper ocean mixing because surface wave dissipation and Langmuir turbulence produce large deviations from the usual balance between production, buoyancy and dissipation.

- **WAVE BREAKING, LANGMUIR CIRCULATION AND MIXED LAYER**

  Energy flux $\Phi_{oc}$ from atmosphere to ocean is controlled by wave breaking. Gives an energy flux of the form $\Phi_{oc} = m\rho_a u_*^3$ where $m$ depends on the sea state. Wave breaking penetrates into the ocean at a scale of the order of the significant wave height $H_S$. Also, the shear in the Stokes drift gives an additional production of turbulent kinetic energy which penetrates into the ocean at a scale of the order the typical wavelength of the surface waves.

# ENERGY FLUX TO OCEAN FOR MAY 1995



- **BUOYANCY EFFECTS**

  Buoyancy effects are essential for modelling diurnal cycle. I have used the approach of Noh and Kim (1999) but adapted the relevant coefficients based on knowledge of stratification effects from the atmospheric community.

- **DIURNAL CYCLE IN SST**

  Diurnal cycle in SST follows from the balance between absorption of Solar radiation in the ocean column on the one hand and transport of heat by turbulence on the other hand. Since Solar absorption is fairly well-known, the study of the diurnal cycle is a good test of our ideas of mixing in the upper ocean. Also, using the ASE observations I will show that sea state effects are relevant for upper ocean mixing.

# MIXED LAYER MODEL

## TKE EQUATION

If effects of advection are ignored, the TKE equation describes the rate of change of turbulent kinetic energy $e$ due to processes such as shear production (including the shear in the Stokes drift), damping by buoyancy, vertical transport of pressure and TKE, and turbulent dissipation $\varepsilon$. It reads

$$\frac{\partial e}{\partial t} = \nu_m S^2 + \nu_m \mathbf{S} \cdot \frac{\partial \mathbf{U}_S}{\partial z} - \nu_h N^2 + \frac{1}{\rho_w} \frac{\partial}{\partial z}(\overline{\delta p \delta w}) + \frac{\partial}{\partial z}(\overline{e \delta w}) - \varepsilon,$$

where $e = q^2/2$, with $q$ the turbulent velocity, $\mathbf{S} = \partial \mathbf{U}/\partial z$ and $N^2 = -g\rho_0^{-1}\partial\rho/\partial z$, with $N$ the Brunt-Väisälä frequency, $\rho_w$ is the water density, $\delta p$ and $\delta w$ are the pressure and vertical velocity fluctuations and the over-bar denotes an average taken over a time scale that removes linear turbulent fluctuations.

Following Grant and Belcher (2009) and Huang and Qiao (2010) wave-induced turbulence is modelled by introducing work against the shear in the Stokes drift. Here $U_S$ is the magnitude of the Stokes drift for a general wave spectrum $F(\omega)$,

$$\mathbf{U}_S = 2\int_0^\infty d\omega\ \omega\mathbf{k}F(\omega)e^{-2k|z|},\ k = \omega^2/g.$$

In stead of this I will use the approximate expression

$$\mathbf{U}_S = \mathbf{U}_S(0)e^{-2k_S|z|},$$

where $U_S(0)$ is the value of the Stokes drift at the surface and $k_S$ is an appropiately chosen wavenumber scale.

The dissipation term is taken to be proportional to the cube of the turbulent velocity divided by the mixing length

$$\varepsilon = \frac{q^3}{Bl},$$

Here, $B$ is dimensionless constant.

It is customary (see e.g. Mellor and Yamada, 1982) to model the combined effects of the pressure term and the vertical transport of TKE by means of a diffusion term. However, the pressure term can also be determined by explicitely modelling the energy transport caused by wave breaking. The correlation between pressure fluctuation and vertical velocity fluctuation at the surface is

$$I_w(0) = +\frac{1}{\rho_w}\overline{\delta p \delta w}(z=0) = g\int_0^\infty S_{diss}(\mathbf{k})d\mathbf{k} = m\frac{\rho_a}{\rho_w}u_*^3 = m\frac{\rho_w^{1/2}}{\rho_a^{1/2}}w_*^3 = \alpha w_*^3$$

and the main problem is how to model the depth dependence of $\overline{\delta p \delta w}$. Assume depth scale is controlled by significant wave height $H_S$:

$$I_w(z) = +\frac{1}{\rho_w}\overline{\delta p \delta w} = \alpha w_*^3 \times \hat{I}_w(z),\ \hat{I}_w(z) = e^{-z/z_0}$$

where the depth scale $z_0 \sim H_S$ will play the role of a roughness length. Thus, the TKE equation becomes

$$\frac{\partial e}{\partial t} = \frac{\partial}{\partial z}\left(lqS_q\frac{\partial e}{\partial z}\right) + \frac{\partial I_w(z)}{\partial z} + \nu_m S^2 + \nu_m \mathbf{S}\cdot\frac{\partial \mathbf{U}_S}{\partial z} - \nu_h N^2 - \frac{q^3}{Bl(z)}.$$

At the surface there is no direct conversion of mechanical energy to turbulent energy and therefore the turbulent energy flux is assumed to vanish. Hence the boundary conditions become

$$lqS_q\frac{\partial e}{\partial z} = 0 \qquad \text{for} \qquad z = 0,$$

$$\frac{\partial e}{\partial z} = 0 \qquad \text{for} \qquad z \to \infty.$$

## MOMENTUM EQUATION

To simplify the problem, the wind/wave driven water velocity is assumed to be uniform without any pressure gradients in the horizontal directions. Taking into account effects from the wave-induced stresses the momentum equations then reduce to

$$\frac{\partial \mathbf{u}}{\partial t} = \frac{\partial}{\partial z}\left(\nu_m \frac{\partial \mathbf{u}}{\partial z}\right) + (\mathbf{u} + \mathbf{u}_{Stokes}) \times \mathbf{f}.$$

Here, $\nu_m$ is the eddy viscosity for momentum which depends on the turbulent velocity $q = (2e)^{1/2}$, i.e.,

$$\nu_{m,h} = l(z)q(z)S_{M,H}$$

where $l(z) = \kappa z$ is the turbulent mixing length, and $S_M$ and $S_H$ are dimensionless parameters which may still depend on stratification.

Note that the Stokes-Coriolis force is obtained from the radiation stress on a rotating sphere.

# HEAT EQUATION

The heat equation describes the evolution of the temperature $T$ due to radiative forcing and turbulent diffusion. Using the depth variable $z$, the temperature evolves according to

$$\frac{\partial T}{\partial t} = \frac{1}{\rho_w c_w} \frac{\partial R}{\partial z} + \frac{\partial}{\partial z} \nu_h \frac{\partial T}{\partial z},$$

where $\nu_h$ is the eddy viscosity for heat, while the solar radiation profile $R(z)$ is parametrized following the work of Soloviev (1982), i.e.

$$R(z) = a_1 \exp(z/z_1) + a_2 \exp(z/z_2) + a_3 \exp(z/z_3), \; z < 0,$$

with

$(a_1, a_2, a_3) = (0.28, 0.27, 0.45)$ and $(z_1, z_2, z_3) = (0.013986, 0.357143, 14.28571)$.

## BUOYANCY

Extreme events in diurnal cycle typically arise for low winds. Buoyancy effects are then important, because they reduce mixing giving rise to heating up of the top layer of the ocean.

Effects of stratification are modelled using the approach of Noh and Kim (1999). Under very stable conditions one would expect that turbulence is characterized by the Brunt-Väisälä frequency' $N$ ($N^2 = -g\rho_0^{-1}\partial\rho/\partial z$). This suggests that the mixing length is limited by an additional length scale $l_b = q/N$. The eddy viscosity can then be estimated by

$$\nu \sim q l_b \sim q l Ri_t^{-1/2}$$

where

$$Ri_t = (Nl/q)^2$$

is the Richardson number for turbulent eddies and the mixing length $l$ is chosen as the usual one for neutrally stable flow, i.e. $l(z) = \kappa z$, with $\kappa = 0.4$ the von Kármán constant.

# STEADY STATE PROPERTIES

## NEUTRALLY STABLE

The properties of the steady state version of the TKE equation were studied extensively. Without presenting any of the details, for neutral stratification the following **'1/3'-rule** is found. Introducing the dimensionless turbulent velocity $Q = \left(S_M/B\right)^{1/4} \times q/w_*$ the approximate solution of the TKE equation becomes

$$w(z) = Q^3 \approx 1 - \alpha\kappa z \frac{d\hat{I}_w}{dz} - La^{-2}\kappa z \frac{d\hat{U}_S}{dz},$$

where $La = (w_*/U_S(0))^{1/2}$ is the turbulent Langmuir number. So in terms of $Q^3$ there is a **superposition principle**, i.e. contributions due to wave dissipation and Langmuir turbulence may be added to the shear production term.

The next graph shows the contributions of wave dissipation and Langmuir turbulence to the turbulent velocity

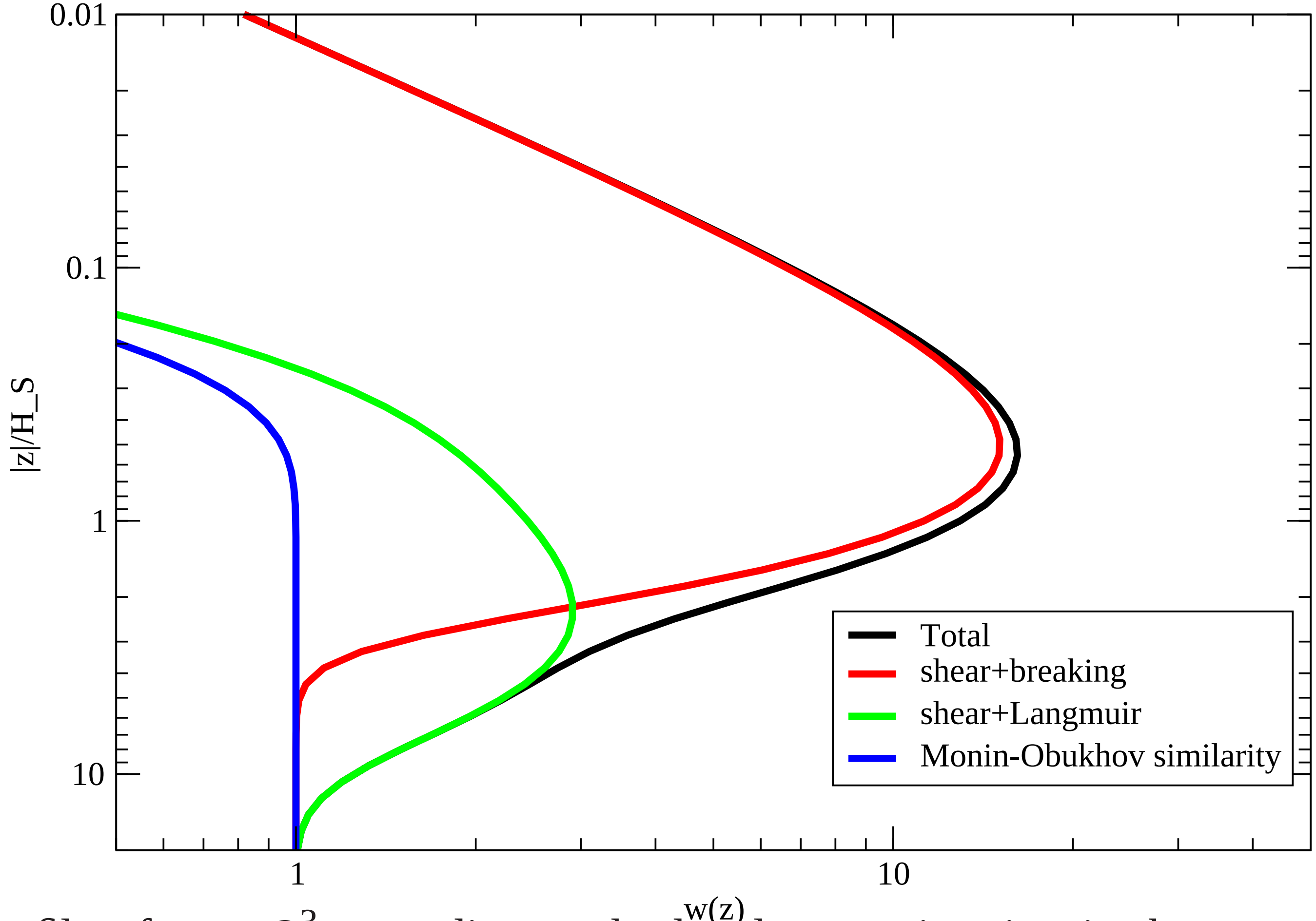



Figure 1: Profile of $w = Q^3$ according to the local approximation in the ocean column near the surface. The contributions by wave dissipation (red line) and Langmuir turbulence (green line) are shown as well. Finally, the $w$-profile according to Monin-Obukhov similarity, which is basically the balance between shear production and dissipation, is shown as the blue line.

# SIMULATION OF DIURNAL CYCLE

The mixed layer model was solved using the boundary condition that at depth $D = 20$ m current velocity $u(z)$, $v(z)$ and temperature $T(z)$ are given. The equations for momentum, heat and turbulent kinetic energy are discretized in such a way that the fluxes are conserved, while the quantities are advanced in time using an implicit scheme with a time step of 5 minutes. The vertical discretization is obtained using a logarithmic transformation. Because such a transformation is so efficient in capturing the relevant details only 25 layers are required.

The model was driven by observations of fluxes and solar insolation for a period of one year at a location in the Arabian Sea. The ocean wave parameters were obtained from wave spectra produced by the ERA-Interim reanalysis.

Modelled Diurnal SST Amplitude (DSA) is compared with observations. A good agreement is obtained, while errors are bigger when wave effects are ignored.

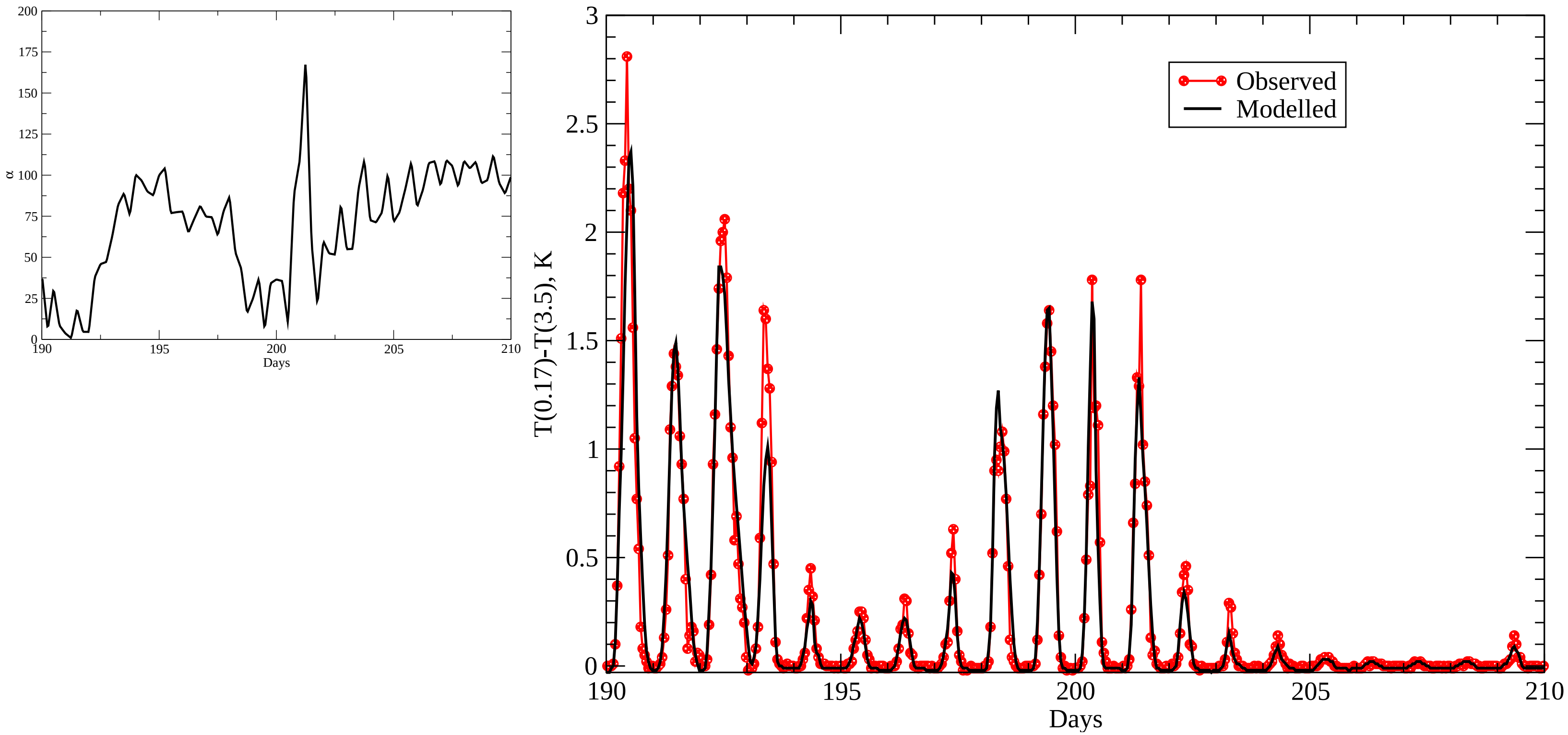



Figure 2: Left Panel: Timeseries of normalized energy flux $\alpha$. Right Panel: Observed and simulated ocean temperature $\Delta T = T(0.17) - T(3.5)$ at 15$^o$30' N, 61$^o$30' E in the Arabian Sea for 20 days from the 23rd of April.

Finally, the ASE has a large volume of high quality observations. I took the opportunity to validate the modelled temperature profiles against the hourly observations. This allowed me to check the impact of the wave breaking parametrization and the Stokes-Coriolis force.

- Wave breaking parametrization results in a considerable reduction of errors near the surface (depth $\simeq H_S$).
- Stokes-Coriolis force gives a positive impact in the deeper layers of the ocean ($\simeq 1/k$)
- Could not validate the impact of Langmuir turbulence as the average turbulent Langmuir number $La \simeq 2$ so at this site turbulent Langmuir mixing is not important.

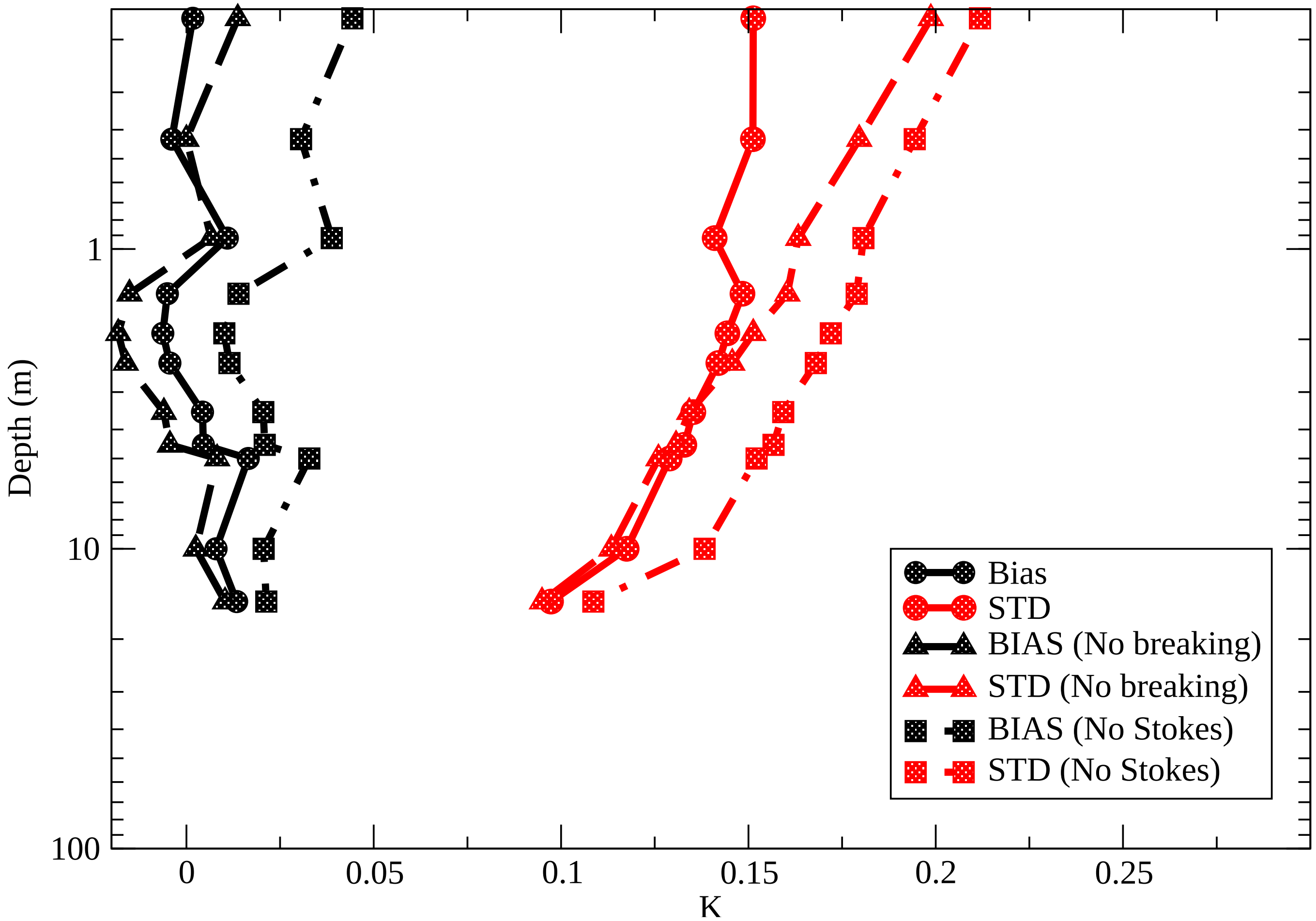



Figure 3: Comparison of modelled temperature data against hourly observations as function of depth at 15$^o$30’ N, 61$^o$30’ E in the Arabian Sea for one year from the 16th of October 1994.

# CONCLUSIONS

- Wave effects, such as wave breaking and Stokes-Coriolis forcing, are important even under low-wind conditions.
- Wave breaking and buoyancy play a key role in understanding the diurnal cycle in SST and in the current.
- Simulations of SST are possible with high accuracy as the random error is just above 0.15 K.